NUMERICAL STUDY OF MIXED CONVECTION 
AND THERMAL RADIATION IN A SQUARE 
CAVITY WITH AN INSIDE INCLINED HEATER

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Abstract :

The present paper analyses the inclination effect of a thin heated plate located inside a square cavity on thermal field, fluid flow and heat transfer under mixed convection conditions and with account for thermal surface radiation. The finite volume method is adopted to solve the mixed convection governing equations, and the surface to surface method (S2S) is used to model radiative transfer in the grey, diffuse and opaque cavity walls with no-participating medium. Flow parameters such as Grashof Number (Gr) and Richardson number (Ri), and Geometrical parameters like heated plate inclination are varied to show their effect on heat transfer enhancement, while the walls emissivity was kept constant. The obtained results show that there is not a limited angle which boost heat transfer for all the considered cases, but for every Gr and Ri value there is a specific inclination which gives the more efficient rate of heat transfer regardless from wall emissivity.

Keywords: Heat Transfer, Thermal Radiation, Mixed Convection ; Electronic Cooling

1 Introduction

Mixed and natural convection in cavities filled with air, has been the subject of many studies due to the importance of this phenomenon for facilitating effective heat transfer in order to obtain a specified temperature, or a more efficient heat removal in many engineering fields. Indeed, it is commonly used for cooling electronic modules encapsulated inside cabinet, solar collectors, thermal design of buildings, air conditioning and many others disciplines. A literature review on the subject shows that many authors have considered numerous theoretical and experimental studies related to natural convection and mixed convection in cavities containing internal partitions or heated plates.

One of the systematic numerical investigation in natural convection was directed by A.K Abdur Hakeem [1,2], where the authors studied the effect of isothermal temperature or isoflux boundary condition, and the effect of different locations of the heat generating baffles inside a square cavity. They found that the type of boundary conditions and the location of the baffles affects sorely the fluid flow, thermal field and the overall heat transfer inside the cavity. They also concluded that baffle
location depends on the applied thermal boundary condition and for each case, there is an optimal baffle location. A. Kumar De [3] likewise studied effect of boundary conditions and the effect of heated cylinder location. He observed that for the case where the heated body is placed at the center of the cavity with isolated top and bottom walls, and cold sidewalls, the isothermal boundary condition gave a higher heat transfer while the cylinder location does not play an important role. A. Ben-Nakhi [4] studied the case of a finned pipe placed in the center of a square enclosure with constant internal heat generation; he concluded that the temperature and the stream function can be controlled through the finned pipe inclination angle and fins length. The effect of combined natural convection and thermal radiation in a square cavity with heated plate was studied by S. Saravanan [5]. It was found that radiation homogenize the temperature fields and makes the whole cavity thermally active. He also found that the effect of emissivity is more perceptible at high Rayleigh numbers. In the case of mixed convection with an inside heat source, many studies [6-8] considered the effect of cavity inlet and outlet position on heat transfer rate and concluded that these last depend of the heat source emplacement, dimension and orientation. They further concluded that the fluid flow and thermal field is still dependent of Reynolds number, Prandtl number and other dimensionless governing numbers.  

For the case of combined mixed convection and thermal radiation in rectangular cavity, A.Bahlaoui [9] studied several effects for thermal enhancing in rectangular cavity with isolated partition at the bottom. They concluded that the presence of radiation reduces the quality of convective heat transfer (Convective Nusselt) and increases radiative heat transfer (radiative Nusselt). He also found that as emissivity of heat source increased, the cavity average temperature decreased. C. Balaji [10] studied mixed convection with surface radiation from a horizontal channel with protruding heat sources. The results showed that the mean temperature inside the channel decreased as the Reynolds number increased. He further noticed that the effect of radiation interaction decreased as Reynolds number increased. Many researchers have investigated natural and mixed convection, both conjugated with surface radiation, for multiple geometrical parameters and flow parameters. Despite the number of investigations carried out, data about conjugated mixed convection and surface radiation in vented cavities containing an inside thin heater, are still scarce. Therefore, the essential objective of this work is to examine the effects of thin orientation and flow parameters on the overall heat transfer and fluid flow inside a square enclosure.

2 Problem Statement

A schematic view of the square cavity with boundary conditions, together with the system of coordinates is depicted in Figure 1. The height and the width of the cavity are denoted H. The length of the cavity perpendicular to its plane is assumed to be long. Hence, the problem is considered to be two-dimensional. The top and bottom wall are kept insulated whereas the sidewalls are maintained at cold temperature $T_c$. A thin finned plate with a length $h$, an inclination $\phi$ and an isothermal temperature $T_{H} > T_c$ is located at the center of the cavity. The system is submitted to an incoming flow with a uniform velocity $U_o$, with an ambient temperature $T_c$. The inlet opening is located on the bottom of the left vertical wall, whereas the outlet opening is located on the top of the opposite side wall, both with a height $w$. Air is considered as the working fluid and is assumed to be a non-participating medium. The cavity walls and plate surfaces are assumed opaque, grey and diffuse emitters and reflectors of radiation with an emissivity $\epsilon$. The flow is assumed to be laminar, incompressible and steady state.  

In the light of assumptions mentioned above, the non-dimensional continuity, Navier-Stokes and energy equations are written as follows:
The dimensionless variable in the above equations (1-4) are defined as:

\[
x = \frac{X}{H}, \quad y = \frac{Y}{H}, \quad u = \frac{U}{u_0}, \quad v = \frac{V}{u_0}, \quad p = \frac{P}{\rho u_0^2}, \quad \theta = \frac{T - T_c}{T_H - T_c}, \quad Pr = \frac{\nu}{\alpha}
\]

The variable mentioned in the previous equations (1-4), represents: (X,Y) Dimensional Cartesian coordinates (m), (x,y) Dimensionless Cartesian coordinates, (U,V) Dimensional X and Y velocity respectively (m.s\(^{-1}\)), (u,v) Dimensionless x and y velocity component respectively, P dimensional pressure, p dimensionless pressure, \(\theta\) dimensionless temperature, \(\rho\) the density of the fluid (kg.m\(^{-3}\)), \(\alpha\) thermal diffusivity of the fluid, \(\beta\) Thermal expansion coefficient (K\(^{-1}\)), \(\nu\) Kinematic viscosity diffusivity of the fluid (m\(^2\).s\(^{-1}\)) and Pr the Prandtl number which is dimensionless.

Mixed convection is identified by three dimensionless parameters, namely the Grashof number \(Gr\), the Reynolds number \(Re\), and the Richardson number \(Ri\) expressed as:

\[
Gr = \frac{8 \beta \Delta T H^4}{\nu^2}, \quad Re = \frac{u_H H}{\nu}, \quad Ri = \frac{Gr}{Re^2}
\]

### 2.1 Radiation Formulation

In the present work, there is an interaction between convection and radiation in an enclosure containing transparent medium, calculation of radiative heat transfer is carried out using the radiosity formulation. the method used is the same as that used by A.Bahlaoui [9].
2.2 Heat Transfer

For the present study, contributions of both convection and radiation to heat transfer should be considered. Hence, the mean total Nusselt number (\(Nu_{tot}\)) at the walls, is defined as the sum of the corresponding convective and radiative Nusselt numbers, respectively (\(Nu_{conv}\)) and (\(Nu_{rad}\)).

\[
Nu_{tot} = Nu_{conv} + Nu_{rad} = \int_{\text{wall}} \frac{\partial \theta}{\partial x} dY + \int_{\text{wall}} N_r \, Q \, dY
\]

(7)

3 Method of Solution

The numerical solution of the continuity, momentum and energy equations (1)-(4) with the corresponding boundary conditions was obtained by using the finite volume method. The computational domain has been discretized with a regular and uniform space grid in the Cartesian directions. The SIMPLE algorithm was used to couple momentum and continuity equations with Power Law discretization for the convective-diffusive terms, while the advective terms are discretized using second-order central difference scheme. The convergence criterion was set as \(10^{-6}\) for all dependent variables.

3.1 Code Validation

The numerical code was validated against the results obtained by A.Bahlaoui [9] for case of mixed convection and surface radiation in a horizontal ventilated heated cavity with an adiabatic thin partition on the heated surface. Comparisons, made in terms of total Nusselt number (\(Nu_{tot}\)), evaluated at the heated wall for various wall emissivities and various Reynolds number, are presented in Figure 2. As shown, a good agreement was obtained with a relative maximum deviation limited to 1.43 % for an emissivity \(\varepsilon=0.85\) and \(Re=3000\), providing sufficient confidence in the present computations.

4 Results and Discussion

As stated earlier, the main objective of the current study is to investigate the characteristics of flow and heat transfer in open cavity submitted to the interaction of mixed convection and radiation in presence of an inclined thin plate. The computations were carried out for \(Pr=0.71\). The characteristic temperature difference \(\Delta T\) and the average temperature of air \((T_0=(T_H+T_C)/2)\) were chosen to be 20K and 303.15K respectively. The results have been obtained for Grashof number ranging from \(10^3\) to \(10^6\), various Reynolds numbers and plate inclination from 0 to 160°. All the cavity walls were considered to have the same emissivity \(\varepsilon\) that was fixed to \(\varepsilon=0.7\) for our case. As the number of the document page is limited, only the effect of heater tilt angle on isotherms and streamlines for \(Gr=10^3\), for fixed Reynolds number and \(\varepsilon=0.7\) are presented in Figure 3.

At first glance, it can be seen that the variation of inclination angle affects sorely the flow and temperature patterns. The corresponding isotherms are denser at the vicinity of the heated plate, whereas a thermal plume appeared and raised from the top surface of the fin towards the insulated top wall, suggesting a better convective heat exchange with the surrounding fluid and with the cold walls. A path is created among the isotherms by the entering fluid that travels from the inlet then mixes with air within the cavity and finally exits through the outlet. A temperature gradient is also remarked at the
interface of the incoming flow and the primary convection cell (surrounding the heated plate), which leads to heightened heat transfer with the entering cold air. When the inclination of the plate is increased \((\phi > 20^\circ)\), the isotherms located under the heated plate are less tightened started too be distorted while the upper thermal plume became larger, indicating the predominance of convective heat transfer. For inclinations \(\phi > 90^\circ\), the thermal plume localized on the plate dissipated to reappear for an angle of \(\phi > 160^\circ\). Streamlines are presented in the second line of Figure 3. The cold air entered into the cavity through the inlet and expanded in the inside as result of pressure rise. Inter alia it can be noticed a close spacing between the streamlines in the vicinity of the incoming flow from the inlet to the outlet indicating high velocities. It can also be seen the development of two major cells, extracting heat from the two sides of the thin, while two insignificant cells appeared at the down right an up left corners of the cavity.

\[
\begin{align*}
\phi = 0^\circ & & \phi = 20^\circ & & \phi = 45^\circ & & \phi = 65^\circ \\
\phi = 90^\circ & & \phi = 110^\circ & & \phi = 135^\circ & & \phi = 160^\circ
\end{align*}
\]

FIG.3 - Isotherms and streamlines for \(Gr=10^3\), \(\varepsilon=0.7\) with various inclination angle
As the plate angle increases, the recirculating cell above the heated plate enlarged and shifted to the upper right section of the cavity, whereas the below cell shrank gradually to dissipate when the plate became in vertical position ($\phi = 90^\circ$). For $\phi = 110^\circ$, the two previous cells merged to create on major cell surrounding the plate, and then split again for superior inclinations. The same behavior as before was observed for the left upper small cell, which grew proportionally to the increase of the tilt angle until $\phi = 90^\circ$, then began shrinking again.

For higher values of Grashof Number (Figure 4), the isotherms appearance changes thoroughly indicating an increase in buoyancy mechanism effect. It is accompanied by a notable distortion in temperature contours and an enlargement of the thermal plume above the heated plate. Furthermore, a better thermal stratification at both sides of the fin was observed leading to an enhanced convective heat transfer.

FIG.4 - Isotherms and streamlines for Gr=10^6, $\varepsilon=0.7$ with various inclination angle
transfer. It can also be noticed that the thermal stratification in the lower part of the fin became more visible as the inclination angle increased until $\varphi = 110^\circ$, then gradually decreased for $\varphi \geq 135^\circ$. The Streamlines presented in the second line of Figure 4 transformed to multicellular flow as Grashof number increased, which resulted from the augmentation of the buoyancy mechanism. The couple of cell located above the heated plate gradually weakened while the lower located cell gradually spread for the augmentation of the inclination angle until $\varphi = 90^\circ$, then progressively regained its original size for superior angles. Furthermore, it can be noticed that for $\varphi > 90^\circ$, the incoming flow divide the cavity in two principal rotating cells.

5 Heat transfer rating

The Streamlines and isotherms presented previously (Figure 3,4), do not provide information about the effect of the studied parameters on the quality of heat transfer within the cavity. Therefore, the variation of convective, radiative and total Nusselt number for several geometrical and configurations is illustrated in Figure 5. For Grashof number $Gr=10^5$, it can be seen that the convective and total Nusselt number varied with heater inclination and with heat transfer mode (Richardson Number). For $Ri=0.1$ and for a heater tilt angle of $45^\circ$, the convective and total Nusselt numbers at the heated plate are higher in comparison with other orientation, while the radiative Nusselt number stayed nearly constant. For $Ri=1$ and $Ri=10$, it is seen from Figure 4 b-c that orientations of $90^\circ$ and $110^\circ$ favored heat transfer since the convective and total Nusselt number for the heated plate and for the left cold wall are higher than those corresponding to other inclinations. On the other hand, the values of the average convective Nusselt number were always bigger than the average radiative Nusselt number. This fact indicates convection heat transfer is predominant compared to radiative heat transfer, but the contribution of the latter is not negligible (40% of total heat transfer).

![Figure 5 - Nusselt variation for heated plate (Nu)$_H$ and left cold wall (Nu)$_C$](image)

(a) $Gr=10^5$, $\varepsilon=0.7$, $Ri=0.1$ (a), $Ri=1$ (b), $Ri=10$ (c)

6 Conclusion

The problem of combined mixed convection with radiation in a square vented cavity with an inside inclined thin heater has been studied numerically. The effects of inclination angle, Grashof Number and Richardson number on flow and heat transfer has been investigated. Based on the obtained results, the conclusions of this work are the following:

- The Temperature field and the flow pattern are very influenced by fin orientation angle.
- For low Grashof numbers, the variation of the inclination angle has no compelling effect on enhancing heat transfer and that for any Richardson value considered, however the behavior changed for higher value of Grashof number as there is an optimal inclination, which enhance heat transfer.
- Convective heat transfer is more prevalent than radiative heat transfer in the present study. Furthermore, fin inclination has no significant influence on Radiative Nusselt number for all the considered Grashof values.
- There is not an inclination angle that improves heat transfer for all flow and thermal configuration, but for every case there is an orientation that gives best thermal performance.

References