Efficient Samplings in Stochastic Optimization for Reactive Power Planning Problems

Résumé
Dans cet article, un optimal power flow stochastique est développé afin de résoudre un problème de planification de puissance réactive (RPP) sujet à des productions de puissance intermittentes et des charges stochastiques. Le problème RPP est utilisé pour gérer de manière optimale les écoulements de puissance réactive sur le réseau électrique et consiste en un problème d’optimisation non-linéaire à variables binaires (MINLP) dont la difficulté de résolution est élevée. Une décomposition de Benders est donc utilisée pour obtenir une solution faisable. Les sources d’incertitude sont modélisées par des densités de probabilité (pdf) à partir desquelles des échantillons sont tirés (approche par scénario). L’approche classique de Monte Carlo nécessite un nombre trop élevé d’échantillons si des cas peu probables doivent être identifiés ce qui justifie le développement de techniques d’échantillonnage plus efficace permettant de trouver les échantillons critiques tout en réduisant la variance des résultats. Cet article vise donc à identifier les cas menant à des violations des limites de tension due à une gestion inefficace de la puissance réactive afin de concevoir la compensation réactive nécessaire au bon fonctionnement du réseau. Un processus itératif est développé afin de concentrer les échantillons dans les zones les plus critiques pour le RPP. Cette méthodes est testée sur un réseau à 9 nœuds et permet de trouver la compensation réactive optimale tout en réduisant le nombre d’échantillons par rapport à la méthode de Monte Carlo. Une formulation robuste et une formulation stochastique du RPP sont implémentées. Une extension de cette méthode à de plus grands réseaux nécessitera une adaptation de la stratégie d’échantillonnage stratifié due à l’augmentation du nombre de paramètres incertains.

Summary
In this paper, a stochastic optimal power flow is developed to solve the reactive power planning (RPP) problem while dealing with uncertainties such as intermittent power generation and loads. RPP problem is used to optimally manage the reactive power flows on the electrical grid. The RPP problem is formulated as a mixed integer non-linear problem (MINLP) which is a hard problem to solve. Therefore, Benders decomposition is used in order to get feasible solutions. The uncertainty sources are characterized by probability density functions from which samples are generated (scenario approach). If rare events are considered, conventional Monte Carlo method gives rise to a too large number of samples. Efficient sampling techniques must be developed to reduce the sample set while considering critical cases to decrease the variance of the results. The main objective of this paper is to identify the most critical samples leading to possible voltage limits violations due to a lack of reactive power supply and find the required optimal reactive power compensation. An iterative procedure is developed and aims to concentrate the samples in the zones having the highest influence on the RPP. The iterative sampling process is assessed on the 9-node test case and allows finding the optimal robust reactive power compensation with a reduced number of samples compared to direct Monte Carlo Simulation (MCS). This method is tested both on a robust and a stochastic RPP problem. Extension to larger grids requires modifications of the stratified sampling strategy due to the increasing number of uncertainties.

1. Introduction and objectives

The electrical grid is used as the mean to transport the electricity generated in power plants to the final user of this electricity, i.e. houses and industries which are called loads. This grid is subject to injection and consumption of electrical power and must be designed to supply all the loads that are connected while ensuring a correct voltage level for the users. In the traditional context, the power generation could be considered as deterministic, i.e. the power produced by each power plant (nuclear, gas, coal power plants) was perfectly known. The conventional grid operation is now complicated by the growing share of renewable power production sources (wind farms, photovoltaic panels…) which are connected to it. The grid is then subject to uncertain power flows that have to be managed by the grid operators. The stochastic behavior of the intermittent renewable power generation and the uncertain load has to be taken into account in all aspects of the electrical grid management from operation to planning. Useful tools for grid planning problems are the Optimal Power Flows (OPF) which are designed to optimize the management of the grid. This tool has now to integrate the stochastic dimension to consider planning of electrical grids subject to uncertainties. The development of such probabilistic tools is thus necessary for future grid expansion and for the optimization of existing grids (e.g. managing reactive power). The development of Stochastic Optimal Power Flows (SOPF) is thus a relevant and interesting problem to tackle.

One particular application of OPF is the reactive power planning whose objective is the optimization of the reactive power flows on the grid. Reactive support provides an optimal management of the reactive power in the grid by maximizing the individual margins of generators’ reactive power and improves the voltage profile by local injection or consumption of reactive power. Reactive power compensation, consisting in capacitor and reactor banks, is added in the grid to provide or consume some reactive power in order to control the voltage levels in the grid. Due to the uncertain context, as previously explained, these RPP problems become stochastic.
problems and need to be considered in a stochastic way. There is a need to develop a stochastic RPP problem aiming to optimally locate and size the reactive power support equipment for the reactive power planning.

A classical approach to deal with stochastic optimization problem is to use samples which are generated from the probability distributions of the uncertainties such as loads or wind speed and to solve multiple deterministic problems with these samples. This is called the scenario approach. The entire set of possibilities is thus approximated by a certain set of samples with given probabilities that must be a very accurate approximation of the original set. The choice and the definition of the samples are critical points to define a high quality approximation of the stochastic problem. In the literature, sampling methods are used to deal with the uncertainties but the number of samples characterizing these uncertainties and the way they are generated are not detailed. Direct Monte Carlo sampling is a classical method to generate samples from probability density functions (pdf) of uncertainties but it requires a huge number of samples to consider situations associated with the tails of the pdf. This method is then not efficient to sample the possibly most extreme cases, i.e. the most problematic situations on the grid, and new sampling techniques have to be proposed to take these critical samples into account while keeping an acceptable computation time. This paper intends to create a method to find the samples which have an important impact on the RPP. The idea is to find these samples, which are not known a priori, from the complete sample set in order to make a robust design of the reactive power support equipment. A robust solution of a problem is a solution which is feasible and close to optimal for all the samples that are generated and which takes the extreme samples into account in the optimization. The RPP solution is then robust against the realizations of the uncertain variables which result in the most extreme values of voltages. The classical sampling process has to be modified to include this search of most impacting samples. The aim is then to find the samples leading to the most critical situations on the grid in order to size and locate the ideal reactive power compensation through a robust RPP optimization problem. An efficient sampling method is required to reduce the total sample set while finding the critical samples to reduce the variance of the results. The conventional formulation of the RPP can lead to intractable problems due to the difficulty to solve the MINLP. In this paper, a Benders decomposition (Geoffrion, 1972) of the problem is proposed to be able to get feasible solutions of the RPP.

Another type of problem is also considered with a stochastic objective. The reactive power compensation costs are minimized in parallel with the possible load shedding resulting from a deficient reactive power management. In this particular problem, the sampling process aims at finding the critical cases that could lead to an excess of cost. A tradeoff between the reactive power compensation cost and the expected load shedding is reached while considering the most critical set of samples.

This paper is structured in four distinct sections: section 2 introduces the reactive power and the reactive power planning problem as well as its mathematical model. The sampling techniques are described in section 3 which also presents a new sampling process. Section 4 shows the results of the developed technique on a test case and a comparison with the Monte Carlo approach; finally section 5 concludes the paper.

### 2. Reactive Power Planning (RPP)

#### 2.1. Reactive power

The electrical power flowing on the grid is called the apparent power and is defined as the product of the complex current and the complex voltage. It can be divided in two parts: active and reactive power. The active power is the useful component of the total power and is directly used by the loads, while reactive power is a by-product and results from the difference of angles between voltage and current in the grid. This reactive power plays a very important role in the management of the electrical grid as it has a large influence on the voltage profile.

Therefore, managing the reactive power flows is a very critical problem for the grid operators. Reactive Power Planning (RPP) problems are part of the midterm reliable planning of electrical grids and aim to optimally size and locate the elements used for reactive power support. These elements are mainly capacitor and reactive banks producing or consuming reactive power locally. Reactive support provides an optimal management of the reactive power in the grid by optimizing the generation of reactive power and improves the voltage profile by local injection or consumption of reactive power (Zhang et al., 2007). The reactive power management has to be carefully studied to avoid reaching unacceptable voltages. A RPP problem can be solved through an optimization problem called AC OPF whose objective is the minimization of the installation and operation costs of the reactive power support equipment. The problem is subject to several constraints: the Load Flow equations which are defined by the Kirchhoff's laws at each node of the system and the other physical constraints such as the bounds on the voltages at each node (minimum and maximum acceptable voltage) or the maximum current allowed to flow on a line that is limited by thermal constraints. The RPP problem is formulated as a Mixed Integer Non Linear Problem (MINLP) which can be intractable for large grids with multiple uncertainties. The location of the reactive power compensation is described by binary variables, representing the presence of

---

1 A node is defined as a point in the grid where loads and/or generators are connected. Nodes are at the intersection of several lines.
20e Congrès de maîtrise des risques et de sûreté de fonctionnement - Saint-Malo 11-13 octobre 2016

Communication 6D /1

page 3/10

The mathematical formulation of the reactive power planning problem is described below and the different terms are explained hereafter. The objective function of RPP is the minimization of the installation and operation costs of the reactive power compensation:

\[
\text{Min} \sum_{i=1}^{N} z_{C,i} \cdot (C_{f,i} + C_{o,i}Q_{C,i}^\text{max}) \quad \{1\}
\]

Where \( N \) is the number of nodes in the grid, \( C_{f,i} \) and \( C_{o,i} \) are the installation cost and the operation cost of the reactive support equipment at node \( i \), respectively. \( z_{C,i} \) is the binary variable for the location of reactive support equipment at node \( i \) and \( Q_{C,i}^\text{max} \) is the maximum reactive power that has to be installed at this node. This objective has to be minimized under some constraints consisting in the Load Flow equality equations (Eq. (2-3)) and the inequality constraints on the different variables of the problem (Eq. (4 to 9)).

The optimization problem is thus a MINLP which is extremely hard to solve with existing solvers.

\[
P_{g,i}^k - P_{l,i}^k + P_{\text{unc},g,i}^k - P_{\text{unc},l,i}^k = V_i^k \sum_{j=1}^{n} V_j^k \left( G_{ij}^k \cos \theta_{ij}^k + B_{ij}^k \sin \theta_{ij}^k \right) \quad \{2\}
\]

\[
Q_{g,i}^k - Q_{l,i}^k + Q_{\text{unc},g,i}^k - Q_{\text{unc},l,i}^k + Q_{C,i}^k \cdot z_{C,i} = V_i^k \sum_{j=1}^{n} V_j^k \left( G_{ij}^k \sin \theta_{ij}^k - B_{ij}^k \cos \theta_{ij}^k \right) \quad \{3\}
\]

\[
\begin{align*}
V_{i}^{\min} & \leq V_{i}^{k} \leq V_{i}^{\max} \quad \{4\} \\
P_{g,i}^{\min} & \leq P_{g,i}^{k} \leq P_{g,i}^{\max} \quad \{5\} \\
Q_{g,i}^{\min} & \leq Q_{g,i}^{k} \leq Q_{g,i}^{\max} \quad \{6\} \\
Q_{l,i}^{\min} & \leq Q_{l,i}^{k} \leq Q_{l,i}^{\max} \quad \{7\} \\
S_{ij}^{k} & \leq S_{ij}^{\max} \quad \{8\} \\
z_{C,i} & \in [0; 1] \quad \{9\}
\end{align*}
\]

Where \( P_{g,i}^{k} \) and \( P_{l,i}^{k} \) are the active power produced and consumed at node \( i \) \((i=1,2,\ldots,n)\) and for sample \( k \) \((k=1,\ldots,N)\), respectively. \( Q_{g,i}^{k} \) and \( Q_{l,i}^{k} \) are the reactive power produced and consumed at node \( i \) and for sample \( k \), respectively. \( P_{\text{unc},g,i}^{k} \) and \( P_{\text{unc},l,i}^{k} \) are the active power produced and consumed by the uncertainties respectively. \( Q_{\text{unc},g,i}^{k} \) and \( Q_{\text{unc},l,i}^{k} \) are the reactive power produced and consumed by the uncertainties, respectively. \( V_{i}^{k} \) is the voltage at node \( i \) and corresponding to sample \( k \). \( \theta_{ij}^{k} \) is the angle difference between nodes \( i \) and \( j \), and \( S_{ij}^{k} \) is the apparent power flowing on line \( ij \). \( G_{ij}^{k} \) and \( B_{ij}^{k} \) are the susceptance and the conductance of the line \( ij \), respectively. Finally, \( Q_{C,i}^{k} \) is the reactive power produced by the reactive power compensation. The variables allowing controlling the voltages in the grid are the generation units voltages and the reactive power compensation. Transformer taps are not included in this model but could be considered with appropriate modifications [Lopez, 2015]. This stochastic MINLP is thus composed of \( 2^nN \) equality constraints (Eq. 2 and 3), \( 2^nN \) inequality constraints on the voltage (Eq. 4), \( 2^nN_q \) inequality constraints for each generator (Eq. 5 and 6 where \( N_q \) is the number of generators), \( 2^nN \) inequality constraints on the reactive power compensation (Eq. 7), \( 2^nN_q \) inequality constraints on the line flows (Eq. 8 where \( N_q \) is the number of lines) and \( n \) binary constraints on the location of the reactive power compensation. The size of the problem can become huge if a large grid (\( n \)) and a large number of samples (\( N \)) are considered.

2.3. Approximation method: Benders Decomposition

Since the RPP problem is a MINLP, approximation methods are used to decompose the problem into feasible problems. The particular method used in this paper is the Benders decomposition. While this method is not exact in non-convex problems, the successful application of Benders decomposition for power system optimization in (Fang, 2015), (Geoffrion, 1972), (Nasri et al, 2015)
and (Wang et al, 2015) shows that the approximation provides good results. The problem is divided in one master problem and several subproblems which can be defined as follows:

- The master problem aims at finding the optimal location and maximum size of the reactive power support equipment (1).
- Eq. (11) represents the possible feasibility cuts that are added to the master problem if a subproblem is not feasible.
- Eq. (12) forces the installation variables and the maximum reactive power compensation to be equal to the values found by the master problem.

The idea of the decomposition is to check the feasibility of the subproblems by looking at the optimality of the solution found with the master problem. The master problem finds the optimal $z_{C,i}$ and $Q_{C,i}^{\text{max}}$ and sends them to the subproblems which assess the feasibility of these solutions for the different samples. If some subproblems end with an objective higher than zero, i.e. there are some violations of the voltage limits, then a feasibility cut is added to the master problem using the Lagrange multipliers of equations (19) and (20).

2.4. Stochastic formulation

As opposed to the robust formulation of RPP which is detailed in the previous section, a stochastic form of the problem can be formulated. The idea is to make a tradeoff between the installation costs of reactive power compensation and the costs of a lack of reactive power compensation. If the reactive management is not sufficient, the voltages on the grid could reach unacceptable values and some loads would be shed to maintain adequate voltage profiles. The reactive power compensation costs is thus in competition with the expected values of the load shedding (LS) that results from critical scenarios:

$$\text{Min} \sum_{i=1}^{N} z_{C,i} (C_{f,i} + C_{o,i} Q_{c,i}^{\text{max}}) + C_{LS} E(\text{LS})$$

The load shedding term $LS^k$ appears directly in the left-hand side term of the load flow equations (2) and (3) because it represents a decrease of the terms $P_{l,i}^k$ and/or $Q_{l,i}^k$ due to the part of the load (active or reactive) which is curtailed. The feasibility cuts provided by the Benders decomposition are replaced by optimality cuts (Geoffrion, 1972)

3. Sampling methods for stochastic problems

Stochastic problems need a model of the uncertain parameters to be solved.Pdf are a correct model for planning problems since the time frame is quite long and an approximation of the stochastic behavior of uncertainties is required. As the scenario approach is used in this paper, samples are generated from these probability distributions which model the different uncertain parameters such as the wind power produced by wind farms and the loads. Multiple methods are possible for samples generation, the most-used one being Monte Carlo (MC) sampling. In MC, random samples are drawn from (possibly correlated) pdf’s. While this method is easy to implement, a large number of samples is necessary to take low probability cases into account and to get a sufficient statistical accuracy on the results. Indeed, critical samples with low probability but high impact on the solution could be located in the tails of the input pdf’s and would require a very large sample set to be considered. Methods for variance reduction can be used to implement smarter ways to generate samples and identify critical ones. The objective of efficient sampling is thus twice: reducing the number of samples and conserving the most critical ones to reduce the variance of the results.
Several methods aim at reducing the number of samples and concentrating on the most critical ones. The first type of methods are variance reduction techniques such as importance sampling and stratified sampling which concentrate the samples in chosen zones of the pdf’s (Homem-de-Mello et al, 2014). The cross entropy method (Fonteneau-Belmudes et al, 2011) is an iterative importance sampling that allows finding the appropriate importance function to find critical zones. A second type of methods include the scenario reduction methods (Bruninx et al, 2014 and Gröwe-Kuska et al, 2003) and clustering techniques (Feng et al, 2016) which do not require any predefined division of the sample space but reduce the number of samples after their random generation. Scenario reduction methods use a probability-distance metric to distinguish and classify the samples. Samples which are found to have a close behavior are grouped. These methods allow reducing the number of samples while keeping the same variance as the original large sample set.

3.1. Stratified Sampling

Stratified sampling method consists in dividing the pdf in several zones in order to homogeneously go through the sample set and take a large number of samples into account. An example of stratified sampling for a two uncertainties case is shown in Figure 1. Classical stratification includes the Latin Hypercube Sampling (Shields et al, 2015) which divides each uncertainty pdf in equal probability zones before taking random samples inside each zone. More advanced techniques such as Latin Hyperrectangle sampling (Mease et al, 2006) or Partial Stratified Sampling (Shields et al, 2015) use non equal probability zones by partitioning the space in non-equal zones. Stratified sampling is interesting if there is an a priori knowledge of the zones that could contain critical samples. This information is often not available for electrical grids and methods to find the most interesting zones have to be developed. Improved stratified sampling methods were already studied by (Shields et al, 2015) which developed special techniques to iteratively divide the sample space in smaller zones in order to get a homogeneous covering of the latter. Another technique called Targeted Stratified Sampling (Shields et al, 2016) aims at finding the limit between the secure zone and the non-secure zone in the sample set i.e. samples leading to critical situations and samples leading to non-critical ones. The division of the space between zones is performed according to a performance criterion or performance function.

![Figure 1: Stratified sampling for 2 uncertainties](image)

3.2. Modified Stratified sampling

In this paper, the objective is to develop an iterative stratified sampling technique in order to locate the most critical samples in the sample set. Most critical samples represent situation of loads and wind power generation that could negatively impact the power flows on the grid and lead to an increase of the reactive power support. These extreme situations would lead to a more expensive reinforcement of the reactive power compensation and have to be considered in order to get a more realistic approximation of the complete sample set. This iterative process starts from an initial stratified sampling and uses information from the first solutions to improve the sampling process for the next iteration. Figure 2 shows the different steps of the iterative process. As it is presented, the first step consists of the initial sampling in order to get an initial sample set. From this set, a selection criterion is computed and a certain number of samples are selected according to it. This criterion can be the value of the Lagrange multipliers on certain constraints or the value of the load shedding for the different samples. Once the critical samples have been identified, the critical zones are deduced from them and new samples are generated inside them. This process is iteratively repeated until the variation of reactive power compensation between two consecutive iterations is lower than a given value. An example of selection criterion is presented in equation 16 where the idea is to sum the Lagrange multipliers of the voltage constraints (lower and upper bounds) on each node \(i\) for each sample and to define a corresponding criterion. \(\lambda^u_{ik} (\text{resp. } \lambda^l_{ik})\) corresponds to the Lagrange multipliers of the upper (resp. lower) bound of the voltage constraint on node \(i\) and for sample \(k\). The values of \(IC^k\) are then sorted for each sample.
and the samples leading to the largest values are considered as the most critical ones because their voltages constraints are the
tightest ones. The zones containing these critical samples are identified and used in the next step for the refinement process as
shown in Figure 3. The critical zones are divided according to the procedure of Figure 2 and new samples are generated inside them.
The RPP problem is then solved with this newly defined sample set and the process is repeated until an ending criteria is reached.

\[
IC^k = \sum_{i=1}^{N} \lambda_{i}^{N_k} + \lambda_{i}^{N_{k-1}} \quad (17)
\]

![Figure 2. Steps of the iterative process](image.png)

![Figure 3. Refinement process](image.png)

3.3. Improved stratified sampling

An important aspect of the iterative stratified sampling is the definition of the initial stratified sampling which leads the process towards
an accurate solution. Indeed the initial sampling has to be dense and homogeneous enough to sufficiently cover the sample space
at first but it has also to be limited to avoid an excessive size of the sample set. A classical stratified sampling is first used in this
paper where the sample space is divided in square zones as Figure 2 illustrates. While this solution is convenient for a relatively
small number of uncertainties (i.e. dimensions), it becomes intractable for multi-dimensional cases. Another type of initial sampling
is based on the assumption that the most critical cases are in the tails of the distribution. The zones are thus defined as illustrated
for two uncertainties in Figure 4 where a large zone is placed at the middle of the zone where the most probable and normally less
critical samples are located. Smaller square zones are defined around this first central zone to get closer to the tails of each
distribution that contain the critical samples. The idea is to find the zones leading to the largest value of the selection criterion and to
increase the number of samples in these zones. This kind of stratified sampling should allow reducing the sample size and the
variance on the results.
4. Test case and application

4.1. Description of the test case

The methodology developed in this paper is assessed on a 9-node system which is adapted from (Anderson et al, 2003) and described in Figure 5. The deterministic loads at nodes 5, 6 and 8 are replaced by stochastic loads and a wind farm is added at node 5. Uncertainty sources are characterized by uniform pdf's since a robust optimization problem is considered and the objective is only to find the most critical samples. The uncertainties, located at nodes 5, 6 and 8, are the active and reactive power produced or consumed at these nodes where a constant power factor is assumed in order to deal with only 3 uncertainties. Table 1 presents the minimum and maximum power consumed (positive values) by stochastic loads (nodes 5, 6 and 8) or produced (negative values) by the wind farm at node 5. Line data can be found in (Anderson et al, 2003) and reactive power compensation can be located at each node of the system. The problem is solved with the solvers IPOPT (for NLP) and CBC (for MILP) and is implemented in Matlab with the toolbox OPTI.

<table>
<thead>
<tr>
<th>Active power</th>
<th>Node 5</th>
<th>Node 6</th>
<th>Node 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min</td>
<td>-200 MW</td>
<td>100 MW</td>
<td>100 MW</td>
</tr>
<tr>
<td>Max</td>
<td>200 MW</td>
<td>170 MW</td>
<td>200 MW</td>
</tr>
</tbody>
</table>

Table 1. Uncertainties power data

4.2. Iterative stratified method for a robust RPP problem

The first assessment of the method is the comparison of the iterative process with MC simulations for the robust RPP problem. The decomposed RPP (with Benders decomposition) is first solved several times with a classical MC approach using different numbers...
of samples from 100 to 10000 and the location and size of reactive power support equipment are compared. Figures 6 shows the sample space and the location of the different samples for the iterative process with red samples being the initial 125 samples and the blue ones being the samples generated during the following iterations. Each axis of Figure 6 corresponds to the cumulative distribution function (cdf) of one uncertain variable, i.e. the active power production or consumption at nodes 5, 6 and 8. It is visible that the samples are highly concentrated in certain zones of the uncertain space that corresponds to the most critical zones, i.e. the zone corresponding to high loads for the three stochastic loads (right upper corner of fig. 6) and the zone with high load at nodes 6 and 8 but high wind farm production at node 5 (left upper corner of fig. 6). The concentration of blue points shows in some zones the samples leading to voltages which are close to their bounds and require the largest values of reactive power compensation to avoid voltage limits violations. These zones contain the largest part of the variance of the sample set.

Several numbers of samples (100, 1000, 5000 and 10000) were generated to solve the stochastic RPP with conventional MCS method. The simulations were performed 3 times for each number of random samples. The resulting mean and variance of the maximum reactive power installed on node 6 are presented in Figure 7 where data are given in per unit and an interpolation is used to bind the four points. The results show that a larger sample set increases the required reactive power compensation because more extreme samples are considered. The variance of the result decreases as more samples are used to solve the RPP problem. The results of the iterative process with 4 iterations are presented in Table 2. An initial stratified sampling is used with 125 zones and one sample per zone. Every iteration leads to a larger number of smaller critical zones in which new samples are generated. The simulation using 10000 samples show that the mean of required maximum reactive power is about 50.46 MVar while the iterative procedure gives 53.23 MVar. The method developed in this paper allows designing a robust reactive power planning by identifying the most critical samples in a limited number of iterations. The computational time is reduced from 27366 s with MCS and 10000 samples to 2870 s with the iterative process using 4 iterations with 200 samples per iteration. With fewer samples, the iterative process succeeds in identifying the most impacting samples in the sample space and in finding the reactive power compensation which is required for these extreme samples. The mean and variance of the iterative process are computed after four uses of this process for 125 initial samples and 1000 initial samples. The results are given in Table 3 and clearly show that a denser initial sampling leads to more precise results.

Communication 6D /1
4.3. Iterative stratified method for a stochastic RPP problem

First, the stochastic RPP is solved with MC using 10000 samples and the results are compared with the robust approach. The results show that there is a tradeoff between the expected load shedding and the values of the reactive power compensation which is lower than in the robust case. For extreme samples, a certain amount of load shedding is allowed to reduce the price of reactive power compensation. It should be highlighted that the relative prices between the load shedding and the reactive power compensation have to be carefully chosen because they have a strong effect on the tradeoff. Secondly, the improved stratified sampling is used to solve the stochastic RPP and the results are compared with those of MC. With the improved stratified sampling, 100 samples are used in the central zone and 600 samples per successive zone. Four successive zones are defined in this problem and the total number of samples is 2500. Table 4 contains the total cost, the expected load shedding and the reactive power compensation for the robust case and the stochastic case with MC and the improved stratified sampling. The total cost (TC) regroups the costs of the reactive power compensation and the cost of the expected load shedding as shown in equation 17, where the latter is assumed to be 8 times larger than the reactive power compensation costs. The fixed installation costs are not considered since they are identical for the three simulations (reactive power compensation is installed at node 6). As presented in Table 4, the stochastic approach allows reducing the necessary reactive power compensation by allowing some load shedding in extreme cases while the robust case covers every situations. The distribution of the load shedding for both MC and improved stratified sampling are presented in Figures 8 and 9 respectively. The improved stratified sampling seems to well approximate the MC method while reducing the number of samples. This stratified sampling has still to be combined with the iterative process in order to give more precise results.

\[ TC = (c_{o,i}Q_{ci}^{max}) + c_{LS}E(LS^k) \]  \hspace{1cm} (18)

<table>
<thead>
<tr>
<th>Iter. 1</th>
<th>Iter. 2</th>
<th>Iter. 3</th>
<th>Iter. 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum reactive power (MVar)</td>
<td>25.28</td>
<td>31.94</td>
<td>46.33</td>
</tr>
<tr>
<td>Number of samples per critical zone</td>
<td>1</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>Number of critical zones</td>
<td>125</td>
<td>20</td>
<td>40</td>
</tr>
</tbody>
</table>

Table 2. Results of the iterative process

<table>
<thead>
<tr>
<th>125 initial samples</th>
<th>1000 initial samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean of maximum reactive power (MVar)</td>
<td>53.39</td>
</tr>
<tr>
<td>Variance of maximum reactive power</td>
<td>2.12e-4</td>
</tr>
</tbody>
</table>

Table 3. Comparison of mean and variances of the iterative process for two different initial sampling

<table>
<thead>
<tr>
<th>Robust RPP (MCS)</th>
<th>Stochastic RPP (MCS)</th>
<th>Stochastic RPP (improved stratified sampling)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reactive power compensation ( (Q_{ci}^{max}) ) in MVar</td>
<td>53.15</td>
<td>20.01</td>
</tr>
<tr>
<td>Expected load shedding ( (E(LS^k)) ) in MVar</td>
<td>0</td>
<td>0.32</td>
</tr>
<tr>
<td>Total costs (€)</td>
<td>53.15</td>
<td>22.57</td>
</tr>
</tbody>
</table>

Table 4. Comparison of the robust and stochastic approaches

5. Conclusion and perspectives

In this paper, the objective was to develop a stochastic OPF for the RPP problem considering intermittent power generation and loads. The original MINLP formulation of RPP is approached by a Benders decomposition to make the problem easier to solve. An iterative sampling process is proposed to identify the most critical samples and is compared to the classical MC. Results show that
the iterative process identifies the samples leading to the highest voltage violations and allows a robust design of the reactive support equipment. The MC method requires a much larger sample set to find the same amount of reactive power corresponding to the extreme samples. The main weaknesses of this method are the definition of the initial sampling, which has to be large enough to sufficiently cover the sample domain, and the computational time. A stochastic formulation of the problem is then proposed and compared to the robust formulation. As expected, the stochastic formulation allows reducing the required reactive power compensation while tolerating some load shedding in some critical cases. The tradeoff is largely impacted by the definition of the cost of load shedding and the cost of reactive power compensation. The improved stratified sampling is also compared to MC in the stochastic formulation and shows good results. This method has still to prove that an effective reduction of the variance is reached compared to MC and could also be combined with the iterative process to improve the initial sampling. The main remaining challenge is the extension of the methodology to real-size electrical grids with thousands of nodes. Indeed, such a large MINLP is not computationally tractable and a decomposition is required. As the size increases, the number of samples must be drastically limited for computational purposes while, in the same time, a larger grid requires a larger sample set due to the increasing number of uncertainties. While stratified sampling turns out to be interesting to find critical samples, its efficiency could be limited by the size of the problem (mainly due to the need to define an efficient initial sampling). Samples reduction techniques should be furthermore analyzed to reduce the dimension of the stochastic problem.

![Empirical CDF](image1)

**Figure 8.** Cumulative distribution function of the load shedding for MC method and 10000 samples

![Empirical CDF](image2)

**Figure 9.** Cumulative distribution function of the load shedding for the improved stratified sampling (2500 samples)

### Acknowledgment

This project has been subsidized by the Brussels-Capital region – Innoviris. It is performed in partnership with Tractebel Engineering.

### Bibliography


X. Fang, F. Li, Y. Wei, R. Azim, Y. Xu, “Reactive power planning under high penetration of wind energy using Benders decomposition”, IET Generation, Transmission and Distribution, 2015, pp. 1835-1844


