DOUBOCHINSKI’S REVERSE-PARAMETRICAL PENDULUM AND MOTOR

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Summary

The type of systems in which the excitation of motion periodic due to high-frequency energy sources is possible has been considered. Transfer of the energy of high-frequency source to the energy of low-frequency motions is achieved by the formation of combination frequencies.

Doubochinski’s reverse-parametrical pendulum in the common case is an oscillator, for example, a mechanical pendulum, making stable oscillations while interacting with periodical external forces. The discovery of this effect was made by brothers Danil and Yakov Doubochinski in 1969-1970.

Doubochinski’s reverse-parametrical pendulum (Fig.1) consists of two interacting components:

- A classical mechanical pendulum (or rotor) 1 having its own low frequency $\omega_0$ (for example, 0.5-1Hz), with a small ferromagnetic plate 2, rigidly fixed on its movable end;
- Unmovable resonance circuit LCR, placed under the equilibrium area of the pendulum trajectory 1-2 with minimal gap $\ell$ and fed by the alternative current of the fixed frequency $\Omega$ (the values of $\Omega$: $\Omega \leq \omega_0$, $\Omega > \omega_0$). Inductivity L is wound on the ferromagnetic core. The resonance characteristic of circuit LCR is determined by the position of the ferromagnetic plate 2, when the pendulum is in equilibrium ($X=0$).

Fig. 2 shows the experimental dependence between ponderomotive force $F$, generated by the magnetic field of solenoid L, and the pendulum position $X$. For example, in the case when the pendulum is moving from the left to the right to position $X=0$, the values of the ponderomotive force are positive and presented with sections $A_1$, $C_1$ and $B_1$; further, at $X>0$, the values of the ponderomotive force are negative and conditionally presented with sections $B_2$, $C_2$ and $A_2$. The resulting work of the ponderomotive forces acting on the pendulum is determined by the area hatched in the diagram. If the resulting ponderomotive force
compensates the dissipative energy losses of the mechanical part, then stable quasi-periodical pendulum oscillations will take place. While changing the alternative current amplitude feeding circuit LCR, there is observable changing (augmentation or diminution) of the pendulum oscillation amplitude. The rotation motion of the pendulum is characterized with analogous properties.

The pendulum (motor) is called reverse-parametrical because in comparison with the traditional classical understanding of the parametrical oscillations when the external periodical force influences one of the parameters of the mechanical part (changes, for example, the pendulum length), in our system, the pendulum using nonlinear characteristics of circuit LCR (in the considered case – the ferro-resonance properties), changes the parameter of liaison between them and forms the resulting ponderomotive force in analogy with the case shown in Fig. 2.

It was also found that if to substitute the ferrite plane 2, in the systems analogous to the one shown in Fig.1, for the closed circuit L1C1R1 (as it is shown in the literature [1-7]), and also to introduce additional liaisons between the two circuits (for example, the capacitive and dissipative coupling), then at modification of these systems there will be the following phenomena depending on the configuration of the coupling between them. The oscillators of different origin have a tendency to group in the stable dynamic structures characterized by the states of mechanical equilibrium, self-adaptation while changing the parameters of the force and environment, an opportunity for the frequency to tune automatically to the external force frequency, and many others.

1. Multiply-coupled oscillators and the grouping of resonators in stable dynamic structure

Classical theory considers three basic forms of coupling between coupled resonateurs and circuits: inductive coupling, resistive coupling, and capacitive coupling. Figures 3 illustrates the case of inductive coupling, in which a change in current in circuit A induces an emf in circuit B, and visa versa. The result of coupling two independent LCR circuits to each other in this manner, is to produce what classical theory describes as "a single resonator with two degrees of freedom." The resulting system has (in general) two proper frequencies, which (in general) are quite far from the proper frequencies of two uncoupled LCR circuits. In this sense we can say that the original oscillators have "disappeared" or melted down, they no longer exist as distinct entities within the combined system. (In their place two oscillation modes appear in the coupled circuit, each of which involves both LCR circuits.) The disappearance of the original resonators is underlined by the common electrical engineering practice, of replacing coupled resonators by equivalent circuits.

Similar results are obtained for resistive and capacitive coupling. It is emphasized in the classical treatment, that the coupling of resonators in this way can lead to circuits with much broader resonance bands and other properties which are used in the design of filters and other technical devices.

We, however, noted two very crucial limitations of this classical approach.
Firstly, the classical treatment never considered, in a systematic way, the effect of combinations of the three basic forms of coupling. Thus, in addition to inductive (L), resistive (R) and capacitive (C) couplings, we must consider LR, LC, RC and LCR couplings, each with its own characteristic possibilities. It is found that in the case of combined couplings, the properties of the coupled system can differ even much more strongly from those of the original, uncoupled resonators, than in the classical, singly-coupled case [7 - 10]. For example, in the region of one of its resonant frequencies an LR-coupled circuit can have a much higher effective Q-value, than either of the component resonators at their resonant frequencies [9, 10]. Moreover, the characteristics of the coupled circuit are can be extremely sensitive to changes in the coupling coefficients.

The equation system for multiple-coupled oscillators (Figure 4) can be written as

\[
\begin{align*}
L_{11}&\ddot{x}_1 + R_{11}\dot{x}_1 + C_{11}x_1 + L_{12}\ddot{x}_2 + R_{12}\dot{x}_2 + C_{12}x_2 = 0 \\
L_{21}&\ddot{x}_1 + R_{21}\dot{x}_1 + C_{21}x_1 + L_{22}\ddot{x}_2 + R_{22}\dot{x}_2 + C_{22}x_2 = 0,
\end{align*}
\]

where \(L_{ij}, R_{ij}, C_{ij}\) are self and mutual coefficients of inductive, resistive and capacitive couplings. In the general case, the zeros on right side must be replaced by functions \(F_1\) and \(F_2\), representing external periodic forces or sources included in the circuits, for example [1, 3]:

\[
\begin{align*}
L_{21}&\ddot{x}_1 + R_{21}\dot{x}_1 + C_{21}x_1 + L_{22}\ddot{x}_2 + R_{22}\dot{x}_2 + C_{22}x_2 = F_2(\omega_2 t),
\end{align*}
\]

where \(\omega_1, \omega_2\) are frequencies of external forces.

In the case where one of the oscillators is movable, the system of equations must be supplemented by the equation

\[
M\ddot{x} + H\dot{x} + \frac{\partial L_{12}}{\partial x}\dot{x}_1\dot{x}_2 = 0,
\]

where \(M\) and \(H\) - the mass and the friction coefficient of the movable oscillator, \(x\) - the coordinate offset one relative to the other oscillator (Fig. 5, for example).
Since the coupling coefficients enter into the expressions for the coefficients of the fourth degree algebraic equation determining the characteristic values of the differential equation system, it is not surprising that changes in their values can have a significant effect on the proper frequencies, damping and equivalent Q-values of the coupled system.

The physical importance and nearly unlimited possibilities of multiple couplings, first become clear, however, when we take into account an additional observation: Since the couplings between oscillators carry flows of energy, the physical components involved in those couplings, are subjected to mechanical forces, which in the case of oscillators that are free to move in space, lead to complicated self-organizing motions.

Suppose, for example, the inductive coupling of two LCR-circuits is achieved by placing the inductive elements of the two circuits (in the form of coils) parallel and near to each other, then the coils will experience a momentary mechanical force proportional to the product $J_1 \times J_2$ of the currents in the two loops (Figure 4). Assuming the currents are both sinusoidal oscillations with a common frequency $f$, it is easy to see that the net mechanical force, integrated over a single period, will be proportional to the cosine of the angular phase difference between the two currents.

In most cases of electrical and radio engineering, the coils or antennae involved in inductive couplings are essentially fixed relative to each other. As a result the mechanical forces generated between them, are ignored in classical analyses of the electrical behavior of the system. But now imagine, instead, that the resonators are able to move freely in space, as indicated in Figure 5. In that case the forces between the inductive elements produce relative acceleration between the resonators, causing them to change their positions. Any change in the relative position of the resonators, in turn, changes the value of the coefficient of mutual induction, and thereby also of the oscillatory characteristics of the coupled system, the phases and frequencies of the currents, and so forth.

What we have said about inductive coupling, holds also true for resistive and capacitive forms of coupling. The "feedback" between mechanical motion and electrical oscillations, via variations in the coefficients of coupling and of the relative phases of oscillations in the interacting circuits, opens up the possibility of...
emergence of new forms of combined electro-mechanical oscillations and self-regulating, self-organizing behavior, which have no equivalent in classical treatments of coupled oscillating systems.

Theoretical and experimental investigations [3,7,8], demonstrated that the simultaneous presence of *more than one dynamic form of coupling* -- for example, combined resistive and inductive couplings -- radically transforms the behavior of the coupled system, and leads under certain conditions to a pronounced tendency for coupled oscillating systems to group together in stable formations.

Figure 6 depicts the simplest type of experimental demonstration, in schematic form. Systems $S_1$ and $S_2$ are LCR circuits, where $S_1$ is provided with a sinusoidal voltage source $E$ of frequency $\omega$, and $S_2$ operates as a passive resonator. The two circuits are coupled to each other by inductive, capacitive and resistive couplings. Assume further that $S_1$ is fixed, and $S_2$ is free to move with respect to it along the $x$-axis. Under certain general assumptions on the dependence of the coefficients of coupling on position, each value of the frequency $\omega$ corresponds to a certain definite separation distance between $S_2$ and $S_1$, at which the net mechanical forces, associated with the couplings between them, become zero. When the frequency $\omega$ is changed, $S_2$ moves to occupy the corresponding, new position of stable equilibrium. In general, the equilibrium position is a complicated, piecewise continuous function of the frequency, undergoing discontinuous jumps at certain critical values of $\omega$.

In a modified form of this experiment, $S_1$ and $S_2$ are passive resonators, coupled with each other by inductive, resistive and capacitive couplings, and interacting with a third, oscillating system -- for example a solenoid with a periodic voltage source, or a field of electromagnetic radiation (Figure 7).

It can easily be demonstrated that *phase fluctuations* are essential to the mechanism of self-organization of coupled resonators and to the maintenance of stable constellations formed by them. Moreover, under certain conditions it is possible to excite undamped stable oscillations of the resonators around their equilibrium positions, in which the frequency of the spatial oscillation can differ by one or two orders of magnitude from that of the electrical oscillations in the circuits.
In the general case, with more than two oscillators and resonators freely moving in space, and coupled by various combinations of inductive, resistive and capacitive couplings, extremely complicated motions are possible, with "phase changes" at critical values of the frequencies of the energy source or sources included among the oscillators. Taking into account both the electrical oscillations and the oscillations of position among the interacting circuits, already three interacting LCR circuits are sufficient to generate a vast spectrum of frequencies and an enormous array of oscillatory modes.

2. Basic features of the phenomenon of grouping of multiply-coupled resonators:
1. The tendency of oscillators, under certain conditions, to group together into stable constellations.
2. The existence of three basic types of mutual couplings between oscillators: inductive, capacitive and resistive.
3. The fluctuational character of stable dynamic equilibrium characterizing the quasi-stationary regimes of oscillation.
4. The presence of a phase-based mechanism of self-regulation, capable of compensating for perturbations in internal and external conditions through changes in the coupling coefficients between the oscillators.
5. Self-adjustment of the spatial configuration of the resonators, permitting the multiply-coupled system to maintain its stability in response to changes in the parameters of external forces and internal dynamics.
6. The emergence of a new system, constituted by the interacting oscillators in stable formation, in which each oscillator makes use its own independent motion to maintain its "individuality", while at the same time participating in the whole coupled system.
7. Significant improvement in the Q-value and other assimilative characteristics of the coupled system of oscillators, compared to those of the component systems, as a result of the "nonlinear arithmetic" of coupling.
8. The above-mentioned characteristics are determined by the partial characteristics of the resonators as well as by the character of the couplings between them. There are systems, for example, in which the capacitive coupling varies as a function of relative position of the resonators, while the coefficients of dissipative and inductive coupling remain fixed. In others, two of the couplings might vary while the third remains fixed, and so on.

References