

Temperature fluctuations induced by frictional heating in isotropic turbulence

R. CHAHINE^a, W.J.T. BOS^a, A. PUSHKAREV^b, R. RUBINSTEIN^c

a. LMFA, CNRS, Ecole Centrale de Lyon, Université de Lyon, 69134 Ecully, France,
 robert.chahine@doctorant.ec-lyon.fr, wouter.bos@ec-lyon.fr

b. Institute of Theoretical and Applied Mechanics, Novosibirsk State University, Novosibirsk, Russia

c. Newport News, VA, USA

Résumé :

La dynamique et l'évolution des fluctuations de température induites par la dissipation visqueuse dans une turbulence isotrope sont étudiées utilisant des Simulations Numériques Directes. Les résultats de ces simulations sont en désaccord avec les prédictions de deux études théoriques récentes. Les résultats sont expliqués par des arguments phénoménologiques.

Abstract :

The dynamics and the evolution of temperature fluctuations generated by viscous dissipation in isotropic turbulence are studied using Direct Numerical Simulations. It is shown that the results are at odds with two recent theoretical studies on this subject. Phenomenological arguments are presented which explain the observed results.

Keywords : Turbulence, Temperature fluctuations, DNS

1 Introduction

The dynamics of a passive scalar in isotropic turbulence has been the subject of many recent studies in the field of fluid mechanics. Two recent studies treated the generation of temperature fluctuations by viscous dissipation, the wavenumber dependence of them and their Reynolds number scaling[1, 2]. Even though the models are based on different heat production terms, they predict the same scaling of the wavenumber spectrum of the heat fluctuations. In the present investigation, we will study this subject using Direct Numerical Simulations.

2 Basic equations

We consider the Navier-Stokes equations for an incompressible fluid,

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\frac{\nabla p}{\rho} + \nu \Delta \mathbf{u} + \mathbf{f} \quad (1)$$

where the last term is a negative viscosity forcing term $\mathbf{f} = -\nu^f \Delta \mathbf{u}$, p the pressure, ρ the density and ν the viscosity.

The evolution of the temperature field is given by an advection-diffusion equation with an additional term related to the heat generated by viscous friction.

$$\frac{\partial \Theta}{\partial t} + u_i \frac{\partial \Theta}{\partial x_i} = \alpha \frac{\partial^2 \Theta}{\partial x_i^2} + \underbrace{\frac{\nu}{c_p} \left(\frac{\partial u_i}{\partial x_j} \frac{\partial u_i}{\partial x_j} + \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i} \right)}_{\text{viscous friction term}}, \quad (2)$$

with α is the fluid diffusivity and c_p the specific heat.

Introducing the Reynolds decomposition $\Theta = \bar{\Theta} + \theta$ will lead to the following equation for the temperature fluctuations,

$$\frac{\partial \theta}{\partial t} + u_i \frac{\partial \theta}{\partial x_i} = \alpha \frac{\partial^2 \theta}{\partial x_i^2} + \frac{\nu}{c_p} \left(\frac{\partial u_i}{\partial x_j} \frac{\partial u_i}{\partial x_j} + \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i} \right) - \frac{\nu}{c_p} \overline{\frac{\partial u_i}{\partial x_j} \frac{\partial u_i}{\partial x_j}}. \quad (3)$$

3 Theoretical studies using closure theory

In recent studies De Marinis et al.[2] and Bos[1] have developed models relying on the eddy-damped quasi-normal Markovian (EDQNM) theory to investigate the dynamics of the temperature fluctuations induced by viscous friction.

Both models predict the occurrence of a spectral region with a $k^{1/3}$ slope in the temperature fluctuations spectrum $E_\theta(k)$. More precisely, the temperature fluctuations spectrum at high Reynolds numbers is (Fig.1),

$$E_\theta^{(1)}(k) \sim \left(\frac{\nu}{c_p} \right)^2 \epsilon^{2/3} k^{1/3} \quad (4)$$

where ϵ is the mean dissipation rate.

This scaling implies the following relations for the temperature fluctuations and the dissipation of heat fluctuations,

$$\overline{\theta^2}^{(1)} \sim \frac{\epsilon \nu}{c_p^2}, \quad \epsilon_\theta^{(1)} \sim \frac{\epsilon^{3/2} \nu^{1/2}}{c_p^2}. \quad (5)$$

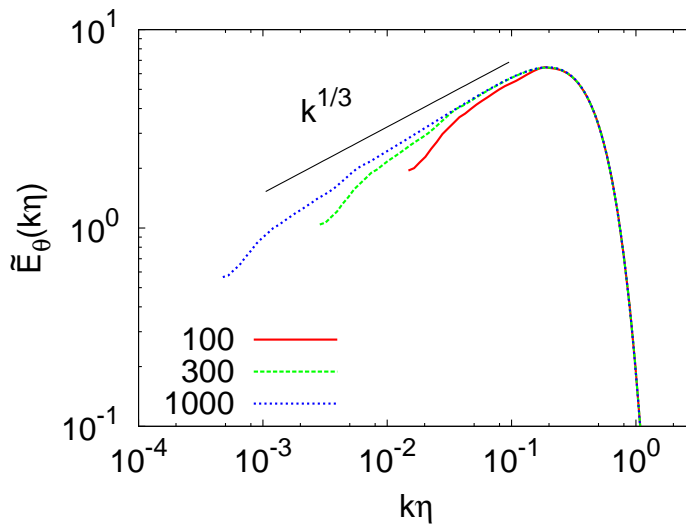


Figure 1: Normalized temperature spectrum in Kolmogorov units, for three different Taylor-scale Reynolds numbers. From reference [1]

4 Temperature fluctuations and their relation to dissipation rate fluctuations

We recently proposed a phenomenological theory [3], which relates the scalar spectrum to the spectrum of the fluctuations of the dissipation rate $E_\epsilon(k)$. Relying on the temperature fluctuations equation (3) and the following evolution equation of $E_\theta(k)$,

$$\frac{\partial E_\theta(k)}{\partial t} = T_\theta(k) - D_\theta(k) + P_\theta(k) \quad (6)$$

they derived the production term to be

$$P_\theta(k) \sim \frac{\tau(k)}{c_p^2} E_\epsilon(k) \quad (7)$$

where $\tau(k)$ is a correlation time. It is known that the scaling of $E_\epsilon(k)$ is not determined by small scale quantities only, as could be expected from Kolmogorov-like arguments. Yaglom proposed a model which fits the data of experiments on the dissipation rate qualitatively, taking into account the non-Gaussian character of its fluctuations [4, 5]. His model predicts the following dissipation spectrum:

$$E_\epsilon(k) \sim \epsilon^2 L(kL)^{-1+\mu}, \quad (8)$$

where L is a large-scale length, and μ is an intermittency parameter of order 1/3 (in reference [4] values are reported $0.3 < \mu < 0.5$). In a steady state, at high Reynolds number, a balance will be observed between $T_\theta(k)$ and $P_\theta(k)$ in the inertial range. The scalar transfer is assumed to be given by a Kovaznay-

type scalar transfer model [6]

$$T_\theta(k) \sim \frac{\partial}{\partial k} \left(E_\theta(k) E(k)^{1/2} k^{5/2} \right). \quad (9)$$

Substituting (8) in (7) and equating the resulting equation with (9) lead to the the temperature fluctuations spectrum

$$E_\theta^{(2)}(k) \sim \frac{\epsilon^{4/3} L^{2/3} k^{-5/3}}{c_p^2}, \quad (10)$$

where we assumed that $\tau(k) \sim \epsilon^{-1/3} k^{-2/3}$.

Considering the case of unity Prandtl number, $\nu = \alpha$, the variance of temperature fluctuations can be computed by integrating the previous spectrum between k_L and k_η , where $k_L \sim 1/L$ and $k_\eta \sim \epsilon^{1/4} \nu^{-3/4}$.

$$\overline{\theta^2}^{(2)} = \int_{k_L}^{k_\eta} E_\theta(k) dk \sim \frac{(\epsilon L)^{4/3}}{c_p^2}. \quad (11)$$

The destruction rate of temperature fluctuations is given by

$$\epsilon_\theta^{(2)} = \int_{k_L}^{k_\eta} 2\alpha k^2 E_\theta(k) dk \sim \frac{\epsilon^{5/3} L^{2/3}}{c_p^2}. \quad (12)$$

5 Numerical results

We solve the Navier-Stokes equations with a large scale forcing and the equation of the total temperature in a cubic three-dimensional periodic domain of size 2π , using a standard pseudo-spectral solver. The forcing is introduced by a negative viscosity, acting on the modes with wavenumbers smaller than 2.5. The initial temperature field is zero and the initial velocity field consists of random noise. All results are evaluated once a statistically steady state is reached where the velocity and temperature fluctuate with a constant variance.

5.1 Results on the scaling of the velocity and dissipation rate spectra

Taylor [7] has pointed out the independence of the dissipation rate of the viscosity at high Reynolds number. The following equation shows the dissipation rate dependence on the large scale quantities L and U , where U is the root-mean-square value of a velocity component:

$$\epsilon \sim \frac{U^3}{L}. \quad (13)$$

The normalized energy spectrum using quantities ϵ, L is:

$$\tilde{E}(k) = \frac{E(k)}{\epsilon^{2/3} L^{5/3}} \sim (kL)^{-5/3}. \quad (14)$$

Figure 2 shows that the data of the normalized energy spectra collapse at low values of k , which means

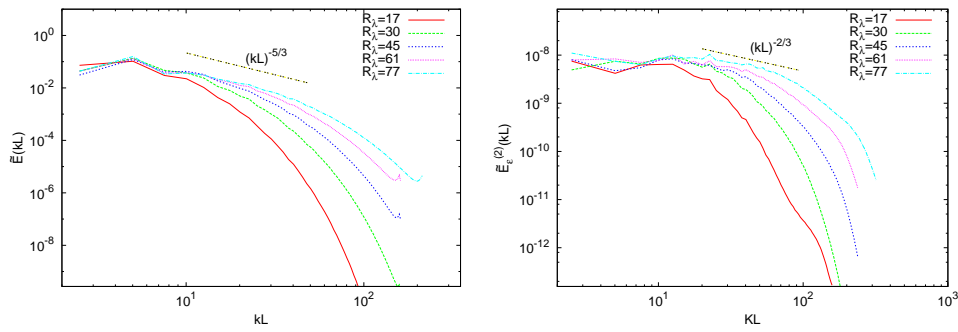


Figure 2: Spectra at different Reynolds numbers, normalized using the quantities ϵ, L . Left: kinetic energy spectra, Right: dissipation fluctuation spectra.

that the large scales are independent of the Reynolds number. Combining this hypothesis with equation (8), we can derive the normalized dissipation rate spectrum:

$$\tilde{E}_\epsilon(k) = \frac{E_\epsilon(k)}{\epsilon^2 L} \sim (kL)^{-1+\mu} \tag{15}$$

It is shown also in figure 2 that the data in the normalized dissipation spectra collapse well at large scales. We mention that the value of μ is considered here $1/3$. So these results confirm the sufficient accuracy of the model proposed by Yaglom to describe the Reynolds number dependency of the dissipation rate fluctuation spectrum.

5.2 Results on the scaling of the temperature fluctuations

In order to verify which of the previous theoretical models is correct, we will normalize the different scalar spectra as we did in the previous section. Normalizing the scaling (4) leads to,

$$\tilde{E}_\theta^{(1)}(k) = \frac{c_p^2 L^{1/3}}{\epsilon^{2/3} \nu^2} E_\theta^{(1)}(k) \sim (kL)^{1/3} \tag{16}$$

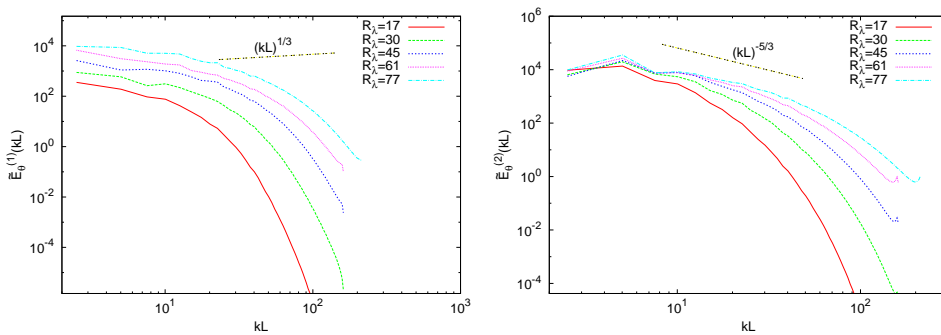


Figure 3: Temperature spectrum for different Reynolds numbers. Left: normalized using expression (16), Right: normalized using expression (17).

Using scaling (10), we have:

$$\tilde{E}_\theta^{(2)}(k) = \frac{c_p^2}{\epsilon^{4/3} L^{7/3}} E_\theta^{(2)}(k) \sim (kL)^{-5/3} \quad (17)$$

As figure 3 shows, the results are in far better agreement when using the second scaling (17), proving the accuracy of the model (10).

5.3 Reynolds number dependence of $\overline{\theta^2}$ and ϵ_θ

We normalize the different predictions (5) (11) and (12) in order to study their accuracy. The normalized dissipation and variance are then defined by

$$\tilde{\theta}^{2(1)} \sim \overline{\theta^2} \frac{c_p^2}{\epsilon \nu}, \quad \tilde{\epsilon}_\theta^{(1)} \sim \epsilon_\theta \frac{c_p^2}{\epsilon^{3/2} \nu^{1/2}}, \quad (18)$$

$$\tilde{\theta}^{2(2)} \sim \overline{\theta^2} \frac{c_p^2}{(\epsilon L)^{4/3}}, \quad \tilde{\epsilon}_\theta^{(2)} \sim \epsilon_\theta \frac{c_p^2}{\epsilon^{5/3} L^{2/3}}, \quad (19)$$

These quantities should become constant at large values of the Reynolds number, if the underlying assumptions are correct.

In figure 4 we show the Reynolds number dependence of the previous normalized quantities. It is clear that for the second case $\tilde{\theta}^{2(2)}$ becomes independent of the Reynolds number. Also for the dissipation of the temperature fluctuations, the normalization $\tilde{\epsilon}_\theta^{(2)}$ is closer to a constant value than the first normalization.

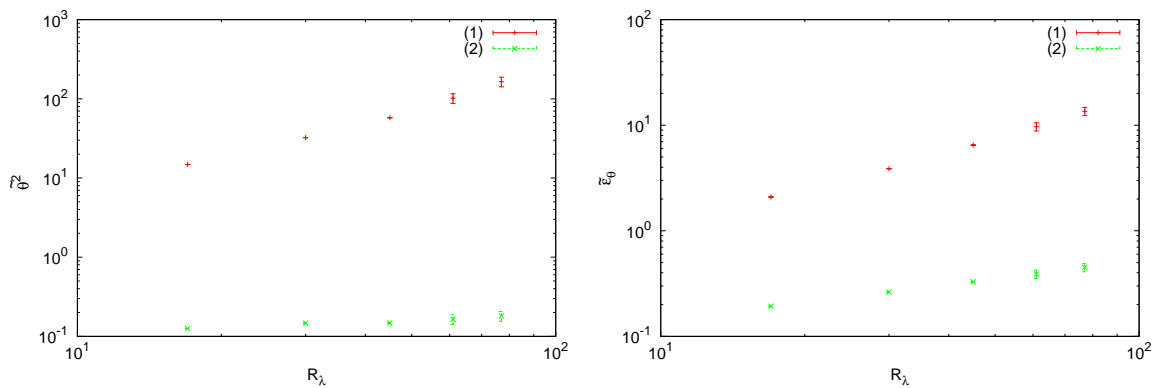


Figure 4: Reynolds number dependence of the normalized temperature variance and the normalized dissipation of temperature fluctuations, normalized using expressions (18) and (19).

6 Discussion

The intermittent character of the dissipation rate fluctuations affects dramatically the spectrum of the temperature fluctuations. The wavenumber dependence of the temperature spectrum is shown to be strongly correlated at large scales, whereas the Gaussian estimate and closure expressions of the Eddy-Damped Quasi-Normal Markovian type predict this spectrum to have a single correlation-length, of the order of the dissipation scale.

In figure 5 we present flow visualizations, where we show iso-vorticity surfaces and iso-temperature fluctuation surfaces. We notice that these surfaces are relatively smooth. This illustrates qualitatively that the temperature fluctuations induced by the fluctuations of viscous dissipations are correlated at large scales. We can speculate by comparing the two pictures that the positive fluctuations exist in the zones of high vorticity. A tentative explanation is that the correlation length of these temperature fluctuations corresponds to the length, rather than to the width of vortical structures. Further research is needed to elucidate this picture.

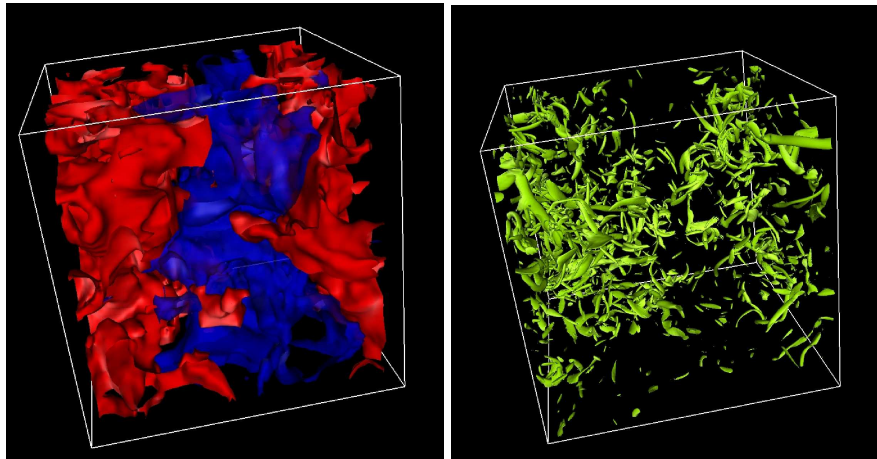


Figure 5: visualizations of (*left*) the temperature field (red isosurfaces correspond to positive heat fluctuations: $\theta/\theta_{rms} = 0.83$; (*right*) the vorticity field (iso-surfaces of the enstrophy for $\|\omega\|/\omega_{rms} = 2.77$). The Reynolds number is $R_\lambda = 77$. Visualizations by VAPOR [8],

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