Modeling of simultaneous transfers of heat and mass in cavities with flat non-adiabatic walls

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Résumé:
La modélisation des phénomènes de transferts simultanés de chaleur et de masse, en convection naturelle, dans des cavités à parois non adiabatiques planes, a été effectuée. Elle a été basée sur un noyau central de fluide isotherme et stagnant dans ces cavités. Elle a été validée par les résultats expérimentaux, obtenus dans le cas d’un distillateur solaire de forme trapézoïdale et dont les températures de ses trois parois non adiabatiques sont différentes les unes des autres.
La résolution numérique du système d’équations a donné des résultats en bon accord avec ceux obtenus expérimentalement.

Abstract:
Modeling phenomena of heat and mass transfers, in natural convection, in cavities with non-adiabatic flat walls, was performed. It was based on a stagnant and isotherm fluid core in these cavities. It has been validated by the experimental results obtained in the case of a solar distiller trapezoidal with three non-adiabatic walls at the temperatures different from each other. The numerical resolution of equations system gave the results in good agreement with those obtained experimentally.

Keywords: Heat and mass transfer, natural convection, solar distillation, solar heating, solar drying.

Introduction

In engineering, mastery of heat and mass transfer phenomena is of utmost importance. This interest in improving yields in many areas, particularly, in the desalination of sea water, purification of brackish water, solar distillation, drying and greenhouses. In this regard, a wide variety of distillers, dryers, water heaters and solar greenhouses was developed.

The literature search shows that the theoretical studies ([1], [2], [3], [4], [6], [7], [8] and experimental [5], [8], [9]) undertaken in heat transfer and heat and mass transfer in the shaped cavities, trapezoidal, triangular, rectangular, slanted rectangular and chapel, concern the differentially heated cavities. Very few studies have been undertaken in the turbulent regime [10]. The numerical methods used are diverse: Finite differences, finite elements, finite volumes and the calculations codes. Although the theoretical results presented in many models agree fairly well with those of the undertaken experience, they are, however, specific to the configurations studied and the experimental conditions used.

In your opinion, controlling the heat and mass transfer phenomena in these cavities could effectively contribute to this goal. Also, to model, we relied the results of our experimental study [11] and the results of study of the structure of flows, laminar and steady, fluid in these cavities. A simulation model was then formulated. Validation was conducted with the experimental model. Indeed, the numerical solution of equations expressing the different transfers in the proposed model, together with the initial conditions identical to those of the experimental model gave results in good agreement with their experimental counterparts.
Nomenclature

\(A_j\) : Area of the wall j (m\(^2\)).

\(D(T_i)\): Mass diffusion coefficient at temperature Ti, (m\(^2\)/s).

\(C_i\): Heat capacity of the material medium i, (J/°K).

\(C_p\) : Specific heat at constant pressure, (J/Kg °K).

\(g\) : Gravitational acceleration (m/s\(^2\)).

\(h_{ij}\): Coefficient of heat exchange between moist air and surface j, (W/m\(^2\) °K).

\(h_{ij}^*\) : Coefficient of mass exchange between medium i and medium j (m/s).

\(h_i\): Radiance heat exchange coefficient between surfaces i and j (W/°K).

\(h_b\) : Heat transfer coefficient between the bottom of the tray and indoor air, (W/m\(^2\) °K).

\(h_e\) : Global exchange coefficient between glass and external environment, (W/m\(^2\) °K).

\(H_s\): Global incident solar flux, (W/m\(^2\)).

\(L_v(T_i)\): Latent heat of phase change of water at temperature Ti, (J/kg).

\(\bar{m}_i\) : Mass flow at wall i, (Kg/s).

\(q_{ij}\): Heat flow received by medium j from medium i, (W).

\(q_{ij}^*\) : Heat flow received by medium j from medium i through mass transfer, (W).

\(q_{ij}\): Heat flow received by medium j from medium I by infra red radiation (W).

\(\rho_i\) : Density of moist air at temperature Ti, (Kg/m\(^3\)).

\(T_i\) : Average temperature of medium i material, (°K).

\(u\) : Component of velocity along Ox, (m/s).

\(v\): Component of velocity along Oy, (m/s).

\(\alpha_i\) : Absorption coefficient for medium i.

\(\phi_i\) : Concentration of water vapor in air at temperature T, (mol/m\(^3\)).

\(\lambda\) : Thermal conductivity, (W/m °K).

\(\mu\) : Dynamic viscosity, (Pa.s).

\(\rho\) : Density of moist air at temperature T, (Kg/m\(^3\)).

\(\tau_i\) : Total solar radiation absorption coefficient for medium i.

Subscript:
- \(e\): Refers to outside cavity.
- \(i or j = 1\): Refers to glass.
- \(i or j = 2\): Refers to water in the tank
- \(i or j = 3\): Refers to absorbent tank.
- \(i or j = 4\): Refers to moist air inside the cavity
- \(i or j = 5\): Refers to aluminum plate
- \(i or j = 6\): Refers to atmospheric air outside the cavity
- \(r\): Refers to radiance

1 Flow study
1.1 Experimental results

Experimental results Fig.3-4, obtained on a trapezoidal distiller Fig.1-2, showed a phase of operation at substantially constant mass flow (evaporation and condensation) Fig.4. Interpretations lead to the hypothetical core fluid of substantially uniform temperature. To confirm this hypothesis of the isothermal core, theoretical studies on the structures of flows, with initial conditions identical to those
of the experimental model, were performed on theoretical cavities of straight sections (a), (b) (c), (d), (e) and (f).

Legend:
1- Glass; 2- Towelling; 3- Flexible rubber hose; 4- Water valve; 5- Test tube; 6- In air pipe tank; 7- the condensed water recovery out pipe; 8- thermocouples support; 9- Distiller lower door; 10- Wedges; 11- Pivoting platform; 12- Support; 13- Pyranometer support; 14- Sprinkler pipe; 15- thermocouples welds; 16- Thermocouples thread; 17- Upper part of the distiller; 18- Rubber seal; 19- Ground.

Fig. 1: Experimental setup

Fig. 2: Straight section of experimental setup

Fig. 3: Temperature vs. Time

Fig. 4: Amount of condensed water vs. Time

(a) (b) (c) (d) (e) (f)
1.2- Flow study
1.2.1 – Introduction

The flow of fluid is studied in laminar and steady régime. The dimensions and geometry of the configuration (a) are identical to those of the experimental model Fig.2. The dimensions of the glass and the water strip, in the configurations (b) and (f) are also the same as the experimental model. In addition, they was characterized by the ratio \(a / b\). As to the dimensions and type of cavity of configurations (c), (d) and (e), they was characterized by the ratio \(L / e\) and their inclination \(\alpha\).

1.2.2 - Basic Hypotheses

The assumptions are: The flow of humid air in the cavities is assumed to be two-dimensional. The air within the cavities is saturated with vapor. Its pressure \(P\) is the atmospheric. The thermal conductivity \(\lambda\), the specific heat \(C_p\) and the dynamic viscosity \(\mu\) are taken equal to those of saturated moist air at normal atmospheric pressure and at temperature corresponding to time instant \(t_1\), Fig. 4; their values are, respectively, 0.025 (W/m K), 1006 (J/kg K) and \(1.9 \cdot 10^{-5}\) (Pa·s). Due to the low moist air flow velocities (natural convection), we can neglect viscous friction and assume that the air is incompressible; we assume also the Newtonian fluid and obeys the Boussinesq approximation. Heat transfer by infrared radiation between walls is neglected due to the low temperature difference. The diffusion coefficient of vapor in the air is that of water in air at atmospheric pressure and temperature corresponding to time instant \(t_1\), Fig. 4. Its value is \(0.22 \cdot 10^{-4}\) (m²/s).

1.2.3 - Equations

Governing equations are the following:

- Momentum equations.
  \[
  u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\mu}{\rho} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)
  \]
  \[
  u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\mu}{\rho} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + g\beta(T - T_c)
  \]

- Continuity equation.
  \[
  \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
  \]

- Heat transport equation.
  \[
  u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)
  \]

- Mass transport equation.
  \[
  u \frac{\partial \phi}{\partial x} + v \frac{\partial \phi}{\partial y} = D \left( \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \right)
  \]

1.2.4 Boundary conditions

Interest is paid to the structure of the flow at time instant \(t_1\) within the stationary region, Fig.4. The temperatures of non-adiabatic walls are 308.6 (°K) for glass, 322.6 (°K) for water and 294.2 (°K) for aluminum wall. These temperatures are taken as boundary conditions for the non-adiabatic walls.
The concentrations are calculated [2] using the temperatures of the non-adiabatic walls. Their values are $\phi_1 = 2.24$ (mol/m$^3$) for glass, $\phi_2 = 4.99$ (mol/m$^3$) for water and $\phi_3 = 1.02$ (mol/m$^3$) for the aluminum surface.

The initial conditions are those of the beginning of the experiment:
- Concerning the temperatures: 300.2 (°K) for glass, 294.2 (°K) for water, 294.2 (°K) for the aluminum plate and 294.2 (°K) for moist air.
- The concentration is determined at ambient temperature. Its value is $\phi_0 = 1.02$ (mol/m$^3$).
- The velocities are zero at the walls (no slip condition) and at the initial instant.

1.2.5 Numerical resolution

The system of equations is solved by the finite element method. The computer code COMSOL Multiphysics available at CDER (Center for Renewable Energy Research) is used.

1.2.6-Results and comments

1.2.6.1 Velocity field

![Velocity field plots](image-url)
Comment

The fig.5-10 show that far from the walls, the velocities are low compared to those at the walls. Thus, we can, far from the walls, assuming in first approximation, that the fluid is stagnant core.

- In the case of configurations (b) and (f), the study showed that beyond certain limits ratios (a / b), the deformation of the velocity field undermines the hypothesis of stagnant core. The configurations limitations reports (b) and (f) are obviously different.

- In the case of configurations (c), (d) and (e), the study showed that the hypothesis of stagnant core is dependent on the ratio (L / e) and (α). Thus, the ratio (L / e) and (α) must meet: 4 < L / e ≤ 11.3 (approximately), for configuration (c) and L / e = 5 and 30 ° ≤ α ≤ 60 for configurations (d) and (f). (The study was limited to the L / e = 5, widely used in the industry).

1.2.6.2 Temperature field

Comment

Far from the walls and recirculation zones, the Fig.11-15 show that the isotherms are substantially equidistant and parallals. Therefore, the heat transfer in these areas, which correspond to the stagnant cores, is by conduction.
1.2.6.3 Concentration Field

Comment

Far from the walls and the recirculation zones, Fig.16-18 show that the isoconcentration lines are relatively more distant from one another, from center of the core to its periphery where they are substantially equidistant and parallels. This reflects a mass transfer mode by diffusion inside the core with a low penetration of the mass in the core.

Conclusion

The studies showed the existence of a few moving core of fluid in the cavities which assume, in the first approximation, as stagnant. Inside the core, the heat and mass transfer is achieved by the diffusion mode only.

3 Modeling

3.1 Formulation of the theoretical model

The proposed model is based on global balances incorporating a stagnant core of fluid in the cavities. Although the studies has shown a thermal stratification in the stagnant core of fluid, the model assumes its temperature uniform. The heat and mass transfer coefficients are deduced from correlated relations of the mean Nusselt and Sherwood numbers proposed in the experimental part [11].

3.2 Validation of the theoretical model

To validate our model, we applied the experimental model, summarized in Fig.19.

Legend

Heat transfer:
- By convection and conduction.
- By mass transfer.
- By infra red radiation.

Temperatures:
- $T_1$: Glass, $T_2$: water strip, $T_3$: Absorbing receptacle, $T_4$: Stagnant zone, $T_5$: Aluminium flate, $T_6$: Ambient environment, $T_7$: Cooling water.

Fig.19
3.2.1 Basic assumptions

The basic assumptions are: The temperatures $T_1$, $T_2$, $T_3$, $T_4$, $T_5$ and $T_7$ are assumed uniform, the side walls in the direction of the length of the experimental setup are assumed adiabatic, the heat losses at the absorbent tray are assumed to be its base, the air inside the stagnant core is assumed saturated in vapor, its pressure is equal to atmospheric pressure; It does not absorb and does not exchange heat by infra red radiation with the walls of the cavity, its heat capacity is assumed to be negligible.

3.2.2 Equations

The system of equations simulating the operation of the distiller is composed of four differential equations, translating the heat balance at the glass pane, at the water strip, at the absorbent tray and at the stagnant core.

Considering the above hypotheses, the heat and mass balance can be written as:

- At the glass pane
  \[ C_1 \frac{dT_1}{dt} = \alpha_1 H_s - Q_{e1} + q_{41} + q_{r21} - q_{r15} \]  
  With: $Q_{e1} = h_{e1} A_1 (T_1 - T_6)$, $q_{41} = h_{41} A_1 (T_4 - T_1)$, $q_{r21} = h_{r21} (T_2 - T_1)$ and $q_{r15} = h_{r15} (T_1 - T_5)$.

- At the water strip
  \[ C_2 \frac{dT_2}{dt} = \tau_2 H_s + q_{32} - q_{24} - q_{r21} - q_{r25} \]  
  With: $q_{32} = h_{32} A_2 (T_3 - T_2)$, $q_{24} = h_{24} A_2 (T_2 - T_4)$, $q_{r24} = h_{r24} A_2 L_w (T_2) (\rho_4 - \rho_2)$, $q_{r25} = h_{r25} (T_1 - T_5)$.

- At the absorbing receptacle
  \[ C_3 \frac{dT_3}{dt} = \tau_3 H_s - q_{32} - q_b \]  
  With: $q_b = h_b A_3 (T_3 - T_4)$.

- At the aluminum plate
  \[ C_5 \frac{dT_s}{dt} = \tau_5 H_s + q_{45} + q_{r15} + q_{r25} - Q_{e5} \]  
  With: $q_{45} = h_{45} A_5 (T_4 - T_5)$, $q_{r45} = h_{r45} A_5 L_w (T_5) (\rho_5 - \rho_4)$, $q_{r15} = h_{r15} (T_1 - T_5)$, $q_{r25} = h_{r25} (T_1 - T_5)$ and $Q_{e5} = h_{e5} A_5 (T_5 - T_7)$.

- At the stagnant zone
  * Heat balance
    \[ 0 = q_{24} + q_{r24} - q_{41} - q_{r41} - q_{45} - q_{r45} \]  
  * Mass Balance (the vaporized water flow mass) $\dot{m}_2 = \frac{q_{24}}{L_w (T_2)}$ and those of condensed water on the glass pane and on the aluminum plate $\dot{m}_4 = \frac{q_{41}}{L_w (T_4)}$ and $\dot{m}_5 = \frac{q_{45}}{L_w (T_5)}$, respectively. Then the mass balance at the stagnant zone becomes
\[
\frac{q^*_{24}}{L_w(T_2)} = \frac{q^*_{41}}{L_w(T_1)} + \frac{q^*_{45}}{L_w(T_5)}
\] 

(6)

### 3.2.3 Coefficients in the equations system

The resolution of the previous equations requires knowledge of different factors. For text shortening, we give the coefficients and how they were determined: bibliography, experimentally or bibliography and calculations: \( h_{e1} \) [12], \( h_{r1j} \) [13], \( h_{32} \) [14], \( (h_{41}, h^*_{41}, h_{45}, h^*_{45}, h_{24}, h^*_{24}) \) [11], \( h_b \) [Exp], \( h_{e5} \) [Exp], \( (\alpha_1, \alpha_2, \alpha_3) \) [bibliography and calculations].

### 4 Numerical resolution

The governing equations consist of 4 first order differential equations (1-4) and 2 algebraic equations (5) and (6). The fourth order Runge Kutta method is used.

### 5 Results and comments

**Results:** Figs (20-25) depict the results along with the corresponding experimental ones.

**Comment**

The Figs.20-25 show the numerical results are in very good agreement with the experimental ones. Indeed, a maximum deviation of less than 1.5 (K) for the temperature of the window pane, 3 (K) for the water temperature, 2 (K) for the receptacle temperature, 0.5 (K) for the temperature of aluminum plate, 170 (gr) for the production of the glass pane (which constitutes about the quarter of the total production) and 150 (gr) for the aluminum plate are observed. However, concerning the glass pane, Fig. 24 although the form of the curve is physically acceptable, relatively higher gaps are observed, as compared with the other curves. These differences are due, in the authors’ opinion, to the hypothesis of uniform temperature in the stagnant zone and the glass pane. Indeed, the experimental results...
(temperature records over three levels of the glass pane) show that the temperature is not uniform; it decreases from bottom to top. For the overall output of condensed water (glass+aluminum plate), the difference is less than 7%.

**Conclusion**

The model we have developed seems reliable because its application to experimental model proved successful. This model, based on an isothermal and stagnant core fluid within the cavity is independent of the geometric shape of the latter. It is therefore applicable to any closed cavity, operating at low temperatures. Due correlated relationships of the mean Nusselt and Sherwood numbers that we proposed [11], only the cavities formed with flat non-adiabatic walls are concerned. This model can then be used with interest in many applications: Evaluation of heat loss by heat transfer (panels and solar water heaters) or simultaneous transfer of heat and mass (distillation, drying, solar greenhouse), among others. Nevertheless, we believe it can be refined, especially, on the side of the stagnant zone where it was assumed his uniform temperature.

**References**


