Multi-scale digital image correlation of strain localization

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Abstract :

In this study, we present a method to analyse experimentally strain localization using a multi-scale framework. A criteria for the onset of strain localization is identified for multi-scale methods. The strain field at the micro-scale are calculated by digital image correlation on samples with controlled artificial microstructure. Then, the Radon transform is used to determine the orientation of the strain localization band in the RVE. Thanks to the partition of unity, a displacement discontinuity is extracted which is going back to the macro-scale. Likewise we measured the macro behaviour of the sample with the image correlation and the X-FEM method to compare the displacement jumps found by both methods.

Keywords : Strain localization, Multi-scale analysis, X-FEM, Digital Image Correlation

1 Introduction

Multi-scale methods bring new perspectives to obtain detailed informations for structures with two (or more) scales in particular the methods based on the \textit{FE}^2 technique introduced by Feyel \cite{feyel1991}. The \textit{FE}^2 method consists in two nested finite element problems with one RVE at each integration points of the macro problem. The material properties at the macro-scale are obtained by homogenization from the material behaviour at the micro-scale. These methods are very efficient for small strain but in presence of large strain and failure, issues about the choice and the existence of the RVE appear. The definition hypothesis of the RVE are presented in Gitman in \cite{gitman1994}. This definition may not be reached when strongly non-linear process or failure occurs at the macro-scale.

Several methods accounting for strain localization at the macro-scale have been developed these last years. As Coenen et Al \cite{coenen1998}, some authors introduced cohesive laws at the macro-scale to represent non-linear failure at the micro-scale. The formation and the propagation of the localization is intertwined with the loss of material stability of the constitutive law. The geometrical non-linearities are also difficult to consider. Belytscko et Al in \cite{belytscko1998} separate the deformation corresponding to material failure from the bulk deformation to circumvent these difficulties and the orientation and magnitude of the discontinuities are passed to the macro-scale. On the contrary in this study, the objective is to characterise the formation of the localization from experiments by means of kinematic measurement at the micro scale. Experiments have been carried out on plates with a controlled artificial micro-structure (a network of micro-holes).

Imagery techniques like Radon transform described in \cite{galvin1996} can extract the geometrical support of the strain localization from the measured strain fields of each RVE. The proposed technique allows to follow the evolution of the support line of strain localization in the RVE. Using enriched shape functions, a displacement jump is then calculated from the micro-scale to describe the localization kinematics at the macro-scale. Then, a coarser mesh that does not account for the micro-structure allows for measuring a displacement jump at the macro-scale by means of the X-FEM and digital image correlation (DIC) as presented in \cite{marty2013}.
2 Model material

The experimental structure is a plate, 1 millimetre thick, with 10 by 10 holes placed at the corners of a regular grid. The hole diameter is 1 millimetre whereas the grid step is 2 millimetres. The geometry is presented in figure 1. The heterogeneous zone (with the holes) is centred in the plate to avoid the effects of free surfaces which are difficult to control. The bulk material is a stainless steel named 304L. It has an known elastic-plastic constitutive behaviour. For large strains, this non-linear behaviour combines with geometrical non-linearities and strain localization occurs within the controlled micro-structure. In this work, the total strain is assimilated to the Green-Lagrange strain due to large strains compared to the elastic strains. Moreover, the hypothesis of plane stress can be used because of the small thickness of the plate. The plate is loaded in tension along the horizontal axis as shown in figure 1. The displacement is prescribed with a rate of 0.2 mm per minute in the range of linear force displacement response and 2 mm per minute after. Digital images are obtained using a 2048x2048 pixels camera. These images are then processed to measure the displacement and strain fields using a finite element kinematics by DIC [2].

3 Micro-scale analysis

3.1 Criteria of localization band formation

To analyse the micro-scale displacement and strain fields, we construct a fine mesh considering the geometry of the micro-structure. By DIC, the measured displacement is computed for each image of the experiment. The strain field is then derived from the displacement using usual finite element formulas. The aim is to determine an experimental criteria for the onset of strain localization. Indeed, when the loading increases, the averaged strain is no longer uniform and localization occurs along a row of holes. Due to boundary effects (transition between the zone with the artificial micro-structure and the zone without micro-structure), the localization band is located along the right or left row of holes.

DIC allows us to find the instant when the localization band occurs. The criteria used here is written in terms of plastic strain which is assimilated to the total strain because of the high ductility of the material. The analysis is performed independently in each unit cell of the structure. We assume that the localization bands appears when the standard deviation of the strain field within the unit cell reaches a critical threshold. Then, a localization band is considered in the concerned RVE. In literature, there exist mostly criteria of localization in energy like Dumstorff [5]. The criteria minimises the structure energy from the plastic strain.

The Green-Lagrange strain fields are presented in figure 2. On left, the strain field is uniform all over the plate. Each ligament between the holes supports the load. On the middle the strain begins to localize in the vertical row whether on the right or on the left. On the last picture, the strain localization occurred along the right row. To reveal the localization band, we use a criteria in terms of strain. As soon as the mean strain of the RVE in the loaded ligaments is superior to 30% then the RVE has a localization :

$$E_{crit} = 0.30$$

This averaging allows to avoid the effect of local strain fluctuations due to measurement noise.
3.2 Analysis tool for the localization band

Once the localization band appeared in the sample, the mesh of the representative volume element is extracted. Thanks to imagery techniques, in particular the Radon transform, we get the position and the inclination of the straight line along with the average strain is maximum.

The Radon transform is the integral of a field projection on the set of straight lines going inside the field. Each of these straight lines is defined by its angle $\theta$ and its intersection with the vertical axis $b$.

The integral expression is given for a field named $f$ by:

$$T_f = \int_{x} \int_{y} [f(x,y).(x \cos \theta + y \sin \theta + b) \, dx \, dy]$$

Because of large strains, the measure of strain that we adopt is the Green-Lagrange one. This Green-Lagrange strain is assimilated to the total strain. To obtain the localization position, we calculate the scalar strain in the meaning of Von Mises, written as:

$$E = \left[ \frac{1}{2} (u_{x,x} + \frac{1}{2} (u_{x,x}^2 + u_{y,y}^2))^2 + \ldots \right]^{1/2}$$

This strain field is represented in figure 3(a) for one RVE. To obtain the straight line maximising the strain, the Radon transform is normed by the Radon transform of an unity field in order to account for voids within the micro-structure. The result of the Radon norm is given in figure 3(b). On the represented RVE, the localization is inclined of $7^\circ$ from the vertical and is located near the center of the hole. In the next section, from this support of the strain localization, a displacement jump will be calculated on the RVE with X-FEM enriched shape functions.
Accounting of all RVEs on the row where strain localization occurs, the calculated position of the localization of each RVE is represented in figure 4. A good agreement between all the RVEs is obtained. However, angles from $-4^\circ$ to $7^\circ$ are calculated. This scattering is due to the effect of noise on the measured strain fields. This noise is even more important in the present case as the measured strains are as large as 80\%. However, the row of holes along which strain localization occurs is subjected to boundary effect as it is located at the transition between the micro-structured material and the bulk material. It is thus expected that gradient phenomena occur within this transition layer of unit cells.

![Figure 4: Visualisation of the straight lines in the row where localization is formed](image)

### 3.3 Extraction of the discontinuities

The RVEs containing a localization band are represented by quadrangles whose cinematic is enriched to enable to calculate displacement jump. Thanks to the partition of unity, discontinuous shape functions are introduced using the Heaviside function:

$$H = \begin{cases} 0 & \text{if } y < x\tan\theta + b \\ 1 & \text{if } y > x\tan\theta + b \end{cases}$$

(4)

where the equation $y = x\tan\theta + b$ is the line supporting strain localization inside the RVE. This Heaviside function has values in the $[0, 1]$ interval, 0 on the ‘left’ of the localization and 1 on the ‘right’. Discontinuous shape functions are equal to:

$$H_i = N_i(H - H(x_i)), \quad \forall i \in 1...4$$

(5)

where $x_1, x_2, x_3, x_4$ are the coordinates of the four nodes of the RVE element and $N_1, N_2, N_3, N_4$ the classical shape functions. Then, we exploit the so-called ‘shifted’ enriched functions proposed by Zi [9]. Both parts of the displacement, the continuous one and the discontinuous ones are calculated with the least squares method. To obtain the continuous displacement, only the nodes of the contour of the RVE mesh are used. For the discontinuous displacement, the remaining nodes are used. The classical and the enriched shape functions are gathered in the matrices $L_{cont}$ et $L_{dis}$ for the nodes concerned. They are written as:

$$\begin{align*}
L_{cont} &= [N_1 \ N_2 \ N_3 \ N_4] \\
L_{dis} &= [N_1 \ N_2 \ N_3 \ N_4 \ H_1 \ H_2 \ H_3 \ H_4],
\end{align*}$$

(6)

Finally, the calculated displacement is regathering on the contour and the remaining nodes of the RVE. This is presented in figure 5 where the displacement normal to the axis of the localization is shown.

### 4 Macro-scale analysis

#### 4.1 X-FEM digital image correlation

To analyse the structure at the macro-scale, one element of the DIC mesh is used to represent each RVE of the micro-structure. Indeed, knowing the final position of the localization we can enrich the
elements cut by the localization using X-FEM. In figure 6 are respectively shown the DIC error fields of micro and macro DIC for a mean strain equal to 25%. The macro X-DIC accounts the holes of the plate with the mesh and the DIC error is equal to 0 inside the holes. The maximums of the DIC error are located around the holes where the highest strains are obtained and where imperfection of the image texture can deteriorate the matching the images. The mean DIC error over the mesh is almost equal to 10% of the dynamic range of the image. For the micro correlation the error is concentrated in the loaded ligaments. For the micro DIC analysis, the mean error is lower than for the macro correlation (4% of the dynamic range of the image). Indeed, the finer mesh accounting for the micro-structure geometry allows for capturing local strain concentration around the holes whereas the macro DIC produce an homogenized strain field. Consequently, the DIC error obtained from the macro DIC are higher. The displacement field obtained by the macro DIC is shown in figure 7.

Figure 5: Position of RVE nodes calculated (magenta) and origin of nodes position (blue).

Figure 6: Error field for respectively the micro and macro image correlation in percent of the dynamic range of the image.

The line of elements where failure occurs is enriched with the X-FEM and the displacement jump is measured. In figure 7, the displacement field is plotted for a mean strain of the plate of 25%. One can observe the displacement discontinuity that is measured using the enriched macro DIC kinematics.

4.2 Comparison of micro-macro results

Thanks to DIC at both scales and different tools, the mean displacement jumps extracted from the analyses and they are compared. In figure 7, both displacement jumps in pixels are plotted as a function of the mean strain of the plate for mean strain levels from 5% to 17%. The measurement uncertainty for the displacement jump is about 0.1 pixel for the macro DIC. These uncertainty is difficult to quantify for the micro DIC analysis but it might be slightly higher and probably about 0.5 pixel. Thus, to uncertainty measurement, a good agreement is obtained for the mean displacement jumps extracted from the micro displacement and directly measured using a X-FEM description of the macro displacement.
5 Conclusions

The aim of this work is to extract discontinuities at a macro-scale from strain localization at a micro-scale in multi-scale framework. Indeed, from the fields calculated by DIC, we use a criteria for the onset of strain localization. Thanks to imagery techniques and enriched finite element, we extract a displacement jump that is compared to the jump measured directly by DIC at the macro-scale. A good agreement is obtained between the two estimates of the discontinuity. It can thus be concluded that the averaging operator extracting the displacement discontinuity for the micro displacement gives results that are consistent with directly measurement at the macro-scale even for very large strains and diffuse non-linear processes. Future works will consist in analysing the stress fields in the RVE and interface traction along the discontinuity with a similar framework. Then scale transition algorithms developed for non-linear behaviour and failure could be validated.

References


