Numerical investigation of Buoyancy Driven Cavity with thin fine adjacent to the heated wall

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Abstract:
In the present study a numerical investigation is carried out using the SarahCFD code which is an in house CFD code based on finite volume method. The application case is unsteady natural convection inside a closed differentially heated cavity (L=1m, H=0.24) with a thin fin adjacent to the heated wall. For comparison purpose, three cases are considered. The first one is a classical differentially heated cavity, which will be used as reference case. The second and the third cases are mounted with a thin fin at the hot sidewall. The fin is respectively adiabatic and conducting. Only the early unsteady stage (up to 10 seconds) is presented in the present paper. Temporal development and spatial flow structures in the vicinity of the fin are presented and shows clearly the improvement of heat transfer rate for the conducting fin case.

Keywords: Natural convection, Heat transfer, Differentially heated cavity, thin fin

1 Introduction
Natural convection in differentially heated cavity is encountered in many environmental and industrial applications. Even with its simplest geometry, the configuration retains rich flow physics manifested by buoyant effect, recirculation and complicated instability phenomenon. In the literature, it is possible to find many experimental and numerical investigations on the 2D steady incompressible case applied at square cavity. Many studies are also dedicated to possible geometrical modifications in order to enhance or reduce the wall heat transfer. One of them is to use horizontal fin on the heated or cooled side wall (see e.g. Shi and Khodadadj, [1]; Tasnim and Collins, [2]; Bilgen, [3]). In frame of this case, the present paper presents the implementation of buoyant effect and transient simulation into a Finite Volume in house code. The 3D finite volume code resolves the incompressible Navier-Stokes equations. The heated cavity used here is the same described by Xu et al. [4], which has an aspect ratio of (A=H/L=0.24) as showed in Fig. 1. The horizontal walls are assumed to be adiabatic, while the vertical ones are isothermal. The hot wall (which is in the right side) is augmented with a fin in its middle.

2 Mathematical model and numerical method
The numerical procedure used to calculate the test case is based on a finite-volume approach for implicitly solving the incompressible unsteady Navier-Stokes and energy equations employing a cell-centred grid arrangement. The momentum-interpolation technique of Rhie and Chow [5] is used to prevent pressure-field oscillations and the pressure-velocity coupling is achieved using the SIMPLEC algorithm of Van Doormal and Raithby [6]. The resulting system of the algebraic difference equations is solved using the Strongly Implicit Procedure (SIP) of Stone [7]. The convection fluxes are approximated by a second-order bounded scheme, namely the MUSCL (Monotonic Upstream Scheme for Conservation Laws) of Van Leer [8].

1
3 Description of the test cases

Several computations are conducted with the following simulation parameters:

- The computational domain and boundary conditions are represented by Fig. 1, where the horizontal walls are adiabatic and vertical ones are isothermal. A constant temperature gradient is maintained between the two vertical sides.
- The hot wall (which is in the right side) is augmented with a fin in its middle. The normalized thickness and length of the fin are 1/120 and 1/6 respectively (normalized by the height of the cavity).
- All computations are done in frame of two-dimensional domain
- The fluid is assumed to be Newtonian and Incompressible (water)
- Buoyant effects are modeled via the Boussinesq approximation
- At t=0, the temperature of the whole domain is set at the reference temperature which is 298°C. Then a constant temperature is applied at both vertical sides (Cold temperature \( T_c = 296°C \) and hot temperature \( T_h = 300°C \))
- According to specialized literatures, the physics of such configuration is driven by the three governing non dimensional parameters: The Rayleigh number (Ra), the Prandtl number (Pr) and the aspect ratio (A).

\[
Ra = \frac{g(Th-Tc)H^3}{\nu \kappa} \quad Pr = \frac{\nu}{\kappa} \quad \text{and} \quad A = \frac{H}{L}
\]

(1)

Where

- \( g \) is the acceleration due to gravity
- \( \beta \) is the coefficient of thermal expansion
- \( \nu \) is the kinematic viscosity
- \( \kappa \) is the thermal conductivity
- \( T_c \) and \( T_h \) are the temperatures of the cold and hot sidewalls
- \( L \) is the width of the cavity
- \( H \) is the height of the cavity

- For all simulations the three governing parameters are fixed as follows:

\[
Ra = 1.3 \times 10^{10} \quad Pr = 5.78 \quad \text{and} \quad A = 0.24
\]

(2)

- Computations are done with a two computational grids (100 x 50 and 500 x 200) and with time step of 0.01 sec.
- Only results for the first 10 seconds will be presented here.
- Three cases are considered:
  - The first one is considered as the reference case and is without fin
  - A second case is with an adiabatic fin
  - And the third one is with a conducting fin
4 Results and discussion

For such geometrical configuration, it is expected that the fin plays a critical role in transient natural convection flows. According to Xu et al. [9], following sudden heating and cooling, the transition of natural convection may be classified into three stages: an early stage, a transitional stage and a quasi-steady stage. In order to illustrate this phenomenon, Xu et al. [4] provides a figure (reported here as Fig.2) presenting the time series of the temperature at a specified point (2.075, 0.375) in the vertical boundary layer downstream of the fin. The figure shows the early transient flow marked by the LEE (Leading Edge Effect), which refers to an overshoot followed by a group of travelling waves, and reattachment of the thermal flow bypassing the fin. The transitional and the quasi-steady stages are clearly reported on the figure. Note that the present computations differ slightly from those done by Xu et al. [4]. (Not exactly the same Rayleigh and Prandtl numbers).

![FIG. 2](image_url) – Time series of temperature at P1 (2.075, 0.375) for conducting fin case, 
\[( \text{Ra} = 1.84 \times 10^9; \quad \text{Pr} = 6.69 \quad \text{and} \quad A = 0.24), \] 
Xu et al. [4]

Early transient flow (up to 10 sec):

Fig. 3 presents the isotherms for the first ten seconds corresponding to the early transient flow. For each time step reported, the figure shows isotherms for reference case, adiabatic fin and conducting fin. Twenty one contours from 296 to 300 are presented. The figures show the presence of an intrusion front underneath the top wall for all cases and a second intrusion front underneath the fin for the two last ones. A considerable difference behavior is clearly visible between the adiabatic and the conducting cases. For the conducting fin and since the top face of the fin is at hot temperature, one can see thermal plumes arising from that face. This phenomenon like Rayleigh-Benard instability is not occurring for the adiabatic fin, where the lower intrusion is blocked by the adiabatic side. The heat is accumulated underneath the fin which results in an increase of energy. Then this energy is released in form of convective plumes which by pass the fin and moves toward the vertical thermal boundary layer. This phenomenon is stronger with the conducting fin than it is with the adiabatic one.
FIG. 3 – Isotherms (21 contours from 296 to 300) at different time until 10sec (early transient stage), without fin, adiabatic fin and conducting fin (500 × 200)
A close view of dynamical and thermal fields for conducting fin is shown on Fig. 4.

![Diagram of dynamical and thermal fields](image)

FIG. 4 – Close up of velocities and temperature contours for conducting fin case (grid: 100x 50)

In order to describe the heat transfer through the cavity, the time series of the normal temperature gradient at the hot wall (excluding the fin surface) is plotted in Fig. 5. For the very early stage (t<1 sec), one can see that the gradient of temperature is very high. This is due to the initial conditions of great difference between the sidewall and the interior fluid. As the thermal boundary layer grows with time, the interior fluid temperature increases and the corresponding gradient decreases. Furthermore, instability oscillation is occurring for the two cases with the fin, while the case without the fin approaches the steady state. It also clear from the figure that heat transfer rate in the two cases with fin is slightly the same and much larger than without fin. Note here that we exclude the fin surface from this figure. If the fin surface was included as it is done by Xu et al. (2011), the case with conducting fin will be the case with larger heat transfer rate.

5 Conclusions
Natural convection in a differentially heated cavity with a fin on the hot sidewall is numerically investigated. The differential heating is suddenly applied on the two sidewalls. Computations are done for a reference case without fin and for two cases with adiabatic and conducting fin. The results show that natural convection and heat transfer is enhanced by the presence of the fin and if we take the heat transfer increase due to additional conducting surface of the fin, one can say that the case with conducting fin is the most favorable in frame of heat transfer enhancement. The early stage of the flow development (up to 10 first seconds) is described for the three cases. This investigation shows that heat transfer can be improved by adding a thin horizontal fin to the hot sidewall. In such situation, the flow near the finned sidewall is unstable. In case of conducting fin the heat transfer area is increased and consequently overall heat transfer enhancement is much better.
FIG. 5 – Time series of normal temperature gradient of the hot sidewall
(Excluding the fin surface area) (100 × 50)

References


