Dynamic modeling by XFEM of cracked 2D structures containing inclusion

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Abstract. This work deals static and dynamic modeling of cracked structures containing inclusions using the eXtended Finite Element Method (X-FEM). For this purpose, a computer code was developed in this study through on all the developments made, especially with the dynamic analysis, which constitutes the originality of this work compared to previous work. Several applications of validation and practical kind have been tested in this study demonstrate the effectiveness and robustness of the X-FEM and of the developed computer code for modeling such structures.

1. Introduction and background

In linear fracture mechanics, the Dynamic Stress Intensity Factor (DSIF) use to characterize the cracking of fragile and quasi-fragile structures under dynamic loadings. In literature we find many techniques to evaluate this parameter. Among which we mention the finite element method FEM [1], the finite difference method FDM [2], the boundary element method BEM [3] and the symmetric-Galerkin boundary element method SGBEM [4]. We note that the FEM is the most popular for its flexibility and efficiency. However, it requires a special treatment of discontinuities and singularities of fields due to the presence of the crack and the inclusion the subject of the present study. For this purpose, a new FEM approach named eXtended Finite Element Method (XFEM) has been developed by Belytschko and Black [6] in 1999. It consists to take into account the discontinuity at the crack edges, inclusion and the singularity at the crack tip by enrichment of neighboring nodes with new degrees of freedom via the new shape functions associated with elements containing these nodes. In 2004 a new enrichment function for inclusion has been included by Sukumar and Chopp [5], recently J - M PAIS, based on the eXtended Finite Element Method (XFEM), has treated this problem, but limited in static and fatigue loading[7]. the first who treated the dynamic problems by using XFEM, but without inclusion, is Belytschko and Chen [8] and Réthoré et al. [9], followed by the work of Grégoire [10]. The work of A.V. Phan et al. [4], treat this problem by another approach based to symmetric-Galerkin boundary element method SGBEM,which will be considered as validation of our approach.

In this context, this work consists in modeling the behavior of structures containing stationary cracks and inclusions, subjected to different types of dynamic loads (Heaviside step loading and triangular blast loading). The DSIF will be evaluated by XFEM using a global approach, based on the J integral. Also, in this work, we will test the effect of the position of the inclusion from the crack. The obtained results will be compared with the work of A.V. Phan et al. [4] using the SGBEM.

2. Review about XEFM

The XFEM introduces in the approximation of the displacement field three types of enrichments [6]:
- A discontinuous function $H$ (Heaviside function) that enriches the split nodes (Fig. 1):

$$H(x) = \begin{cases} +1 & \text{if } \phi(x) \geq 0 \\ -1 & \text{if } \phi(x) \leq 0 \end{cases}.$$  

Where $\phi$ is the level set function that determines the normal position of node $(x)$ from the crack.

- Four (04) singular functions for each tip node (Fig. 1):

$$F(x) = \sqrt{r} \{ \sin(\theta/2), \sin(\theta/2)\sin(\theta), \cos(\theta/2), \cos(\theta/2)\sin(\theta) \}.$$  

- One function associated with the interface nodes of the inclusion:

$$\nu(x) = \begin{cases} +1 & \text{if } \zeta(x) = 0 \\ 0 & \text{if not} \end{cases}.$$  

Where $\zeta$ is the level set function of the inclusion.

The approximate displacement fields are as follows:

$$u(x) \approx \sum_{i \in I_H} N_i(x)u_i + \sum_{i \in I_{\nu}} N_i(x)H(x)a_i + \sum_{i \in I_{F_k}} N_i(x)\left( \sum_{k=1}^{4} F_k(x)b_{i,k} \right) + \nu(x)a_c.$$  

In addition to traditional unknown $u_i$, we consider the unknowns $a_i, b_k$ and $a_c$ corresponding to the enrichment functions $H, F_k$ and $\nu$ respectively.

3. Interaction integral method for DSIF computation

There are several methods to evaluate the DSIF. In this work, we use the method of the $J$ integral by using the interaction integral (Fig. 2). Because its global character, this latter is the most stable technique.
This method introduced by Sih et al [11], combines with the actual field an auxiliary field satisfying
the boundary conditions of the problem [11]. In this case, The J integral is given as follows:

\[ J = J_{act} + J_{aux} + M. \]  

(5)

Where \( J_{act} \), \( J_{aux} \) are the J integrals in the actual and auxiliary fields, respectively, and \( M \) is the
interaction integral that we are interested in, defined by :

\[ M = \int_{A} \left[ \sigma_{y} \frac{\partial w_{aux}}{\partial x_{1}} + \sigma_{y} \frac{\partial u_{aux}}{\partial x_{1}} - W^{M} \delta_{ij} \right] \frac{\delta q}{\partial x_{y}} d\Gamma = \frac{2}{E} (\pi \int_{\Omega} K_{I}^{aux} + K_{II}^{aux}). \]  

(6)

With \( W^{M} = (\pi \int_{\Omega} K_{l}^{aux} + K_{II}^{aux})/2 \) is the strain energy of interaction and \( E' = E \) in plane stress and
\( E' = E / (1 - \nu^{2}) \) in plane strain. Therefore, the stress intensity factor in mode I and II take the form:

\[ K = \frac{E'}{2M}. \]  

(7)

We take \( K_{I}^{aux} = 1, K_{II}^{aux} = 0 \) in mode I and \( K_{I}^{aux} = 0, K_{II}^{aux} = 1 \) in mode II. The computing procedure
of M is based on the Gauss points within the elements of J domain area A (see Fig 2).

4. Validation problem

We consider a plate of size \( 2w \times 2h = 30 \text{mm} \times 40 \text{mm} \) containing an internal crack of length \( 2l = 4.8 \text{mm} \) and an inclusion of diameter \( d = 4 \text{mm} \) as shown in Fig. 3. The plate is subjected to a
uniaxial tension \( \sigma(t) \) in form of a Heaviside step load or a triangular blast load with \( t1 = 2 \mu s \) and
\( t2 = 8 \mu s \). The inclusion is eccentrically positioned relative to the crack center as shown Fig.3. The
material properties for the plate and the inclusion are respectively given as : \( E = 260 \text{ GPa} \) and 640
GPa, \( \nu = 0.08 \) and 0.01, and \( \rho = 3,220 \text{ kg/m}^3 \) and 3,515 kg/m3. The DSIFs evaluate at crack tip A,
and normalized with respect to the SIF of a similar situation in infinite plate under a uniaxial
tension \( \sigma_{0} \), without inclusion. The normalized DSIF for this problem are defined as

\[ \overline{K}_{I} = \frac{K_{I}}{\sigma_{0}\sqrt{\pi a}} \quad \overline{K}_{II} = \frac{K_{II}}{\sigma_{0}\sqrt{\pi a}} \]  

(8)
Fig. 3 The validation problem: a) Cracked plate with inclusion, b) Different types of loads.

![Fig. 3](image)

Fig. 4 The plate under Heaviside step loading with different positions of inclusion; (a) \( e = 3d/4 \), (b) \( e = d/2 \), (c) \( e = d/4 \).

![Fig. 4](image)

Fig. 5 The plate under triangular blast loading with different positions of inclusion; (a) \( e = 3d/4 \), (b) \( e = d/2 \), (c) \( e = d/4 \).

![Fig. 5](image)

Figure 4 and 5 can be shown, there is an acceptable correlation between the obtained results and those of Phan et al. [4] using SGBEM, for different position of the inclusion as well as for KI and KII.

We can note here for DSIF KII, our results are close to zero more than those obtained by Phan et al. [4].

Compared to the Heaviside step loading, the triangular blast loading increase more the negative value of KI and decrease positive peaks. This shows that, the Heaviside step loading is more
dangerous than the triangular blast loading. The quality of the obtained results demonstrates well the effectiveness of this computer code.

Conclusion

This study presents a computational procedure to evaluate the DSIF for stationary cracks in plate within inclusion using XFEM. The correlation of the obtained results with the literature for several treated configurations demonstrates the effectiveness and the robustness of this procedure. As perspectives this approach can be extended to problems of Multi inclusions, Multi cracks, different form of the inclusion and dynamic crack propagation.

References


