Effect of crack length and position on the impact response of plates by using the X-FEM

R. TIBERKAK\textsuperscript{a}, M. BACHÉNE\textsuperscript{b} and S. RECHAK\textsuperscript{c}

\textsuperscript{a.} Laboratoire ‘Structures’. Département de Mécanique, Université Saad Dahleb, BP 270 Soumaa, de BLIDA 09000, ALGERIE.
\textsuperscript{b.} Laboratoire de Mécanique physique et Modélisation Mathématique (LMP2M), Université de MEDEA 26000, ALGERIE.
\textsuperscript{c.} Laboratoire de Génie Mécanique et développement (LGMD), Ecole Nationale Polytechnique, BP 182 El-Harrach 16200, ALGER, ALGERIE.

Abstract:

In the present paper, isotropic plates containing cracks and subjected to impact loading are studied. By the present work, one can predict the eventual presence of discontinuities in plates by comparison of impact properties of cracked plate to the virgin ones. The extended finite element method (X-FEM) is used in the modeling of the impact problem in which the effects of shear deformation is included. In the numerical implementation, conventional finite element without any discontinuity is first carried out. Then enriched functions are added to nodal displacement field for element nodes containing cracks. The effects of crack length and crack position on contact force and on plate deflection are analyzed. The obtained results show that, the maximal contact force decreases while the deflection increases with the increase of the crack length $a$. The effect of crack position on the dynamic response is less pronounced when the crack is near by the fixed end.

Keywords: Dynamic response, Cracked plate, X-FEM, Impact loading, Low velocity.

1. Introduction

At microscopically level, cracks always exist in mechanical components. After hours of services, these micro cracks initiate and propagate to become effectively dangerous by decreasing the dynamic-mechanical properties of components. Numerical treatment of discontinuities generally induces extra difficulties during their implementation. The extended finite element method (X-FEM) applied to divers fracture mechanic problem has proved its efficiency where all numerical difficulties and all shortcomings associated with meshing of surface crack encountered with the classical finite element are alleviated. However, the finite element method is used as the building block in the extended finite element method, and hence much of the theoretical and numerical developments in finite elements can be readily extended and applied. For crack modeling, a discontinuous function and the two-dimensional asymptotic crack-tip displacement fields are added to the displacement-based finite element approximation. Partition of unity enrichment methods for discontinuities and near-tip crack fields were previously introduced by Belytschko and Black [1]. Moes et al. [2] proposed the generalized Heaviside function as a means to model the crack away from the crack tip, and developed simple rules for the introduction of the enrichment function at the crack-tip. Bachene et al [3] use the extended finite element method (X-FEM) to analyze vibration of cracked plates.

In the present study, enriched finite element method (X-FEM) is developed for the dynamic analysis of cracked plates subjected to impact loading. Different configurations related to crack parameters are examined, i.e; crack length and crack position. Contact force and maximum deflection of the plate are thus computed.

The paper is structured as follow. In the next section, the extended finite element method is presented. The mathematical model along with the X-FEM formulation is first presented. In section 3, development for the computation of the contact force and central deflection are described. Numerical
applications are presented and discussed in section 4. We finalize the paper by drawing some conclusions.

2. Mathematical model and X-FEM formulation

2.1. Displacement field

The mathematical model is based on Mindlin plates. Let us consider a Cartesian coordinate system in such a way that the xy-plane coincides with the midplane of the plate with sides $L$, $l$ and thickness $h$. The displacement field can be expressed as:

\[
\begin{align*}
    u(x, y, z) &= z\theta_x(x, y) \\
    v(x, y, z) &= z\theta_y(x, y) \\
    w(x, y, z) &= w_0(x, y)
\end{align*}
\]

(1)

Where $u$, $v$ and $w$ are the displacements in the $x$, $y$, $z$ directions, respectively. $w_0$ denotes the mid-plane displacement, and $\theta_x$ and $\theta_y$ are the normal rotations of the mid surface normal in the $xz$- and $yz$-planes, respectively.

2.2. X-FEM formulation

The X-FEM analysis consists first in the discretization of the plate by conventional finite element method omitting the presence of cracks. Then, enrichment functions are added to the nodal displacement of cracked elements in order to take into account the effect of discontinuity. Details of the enrichment can be found in [4-8]. In the dynamic analysis, we consider that the crack path is parallel to one side of the plate and suppose it to stop at the end of each element (Fig. 1).

The X-FEM approximation can be written as follows:

\[
[\Delta] = \sum_{i\epsilon I} [N_i][\delta_i] + \sum_{j\epsilon J} H(x)[N_j][\delta_j]
\]

(2)

As represented by Eq. (2), the X-FEM can be regarded as the superposition of the classical FEM (represented by the first term on the right hand side) and a discontinuous enrichment (represented by the second term on the right hand side).

\[
H(x) = \begin{cases} 
+1 \text{ on the top surface of the crack} & (y > 0) \\
-1 \text{ on the bottom surface of the crack} & (y < 0)
\end{cases}
\]

(3)

In Eq. (2), $N_i$ and $\delta_i$ are the classical shape function and the classical nodal displacement at node $i$, respectively. $I$ is the set of all nodal points of the plate, while $J$ represents the set of nodes of the elements located on the discontinuity. $K$ is the set of nodes enriched by near-tip enrichment functions.
\( \delta_j \) (\( j \in J \)) and \( \delta_k \) (\( k \in K \)) are the nodal degrees of freedom to be enriched. \( H(x) \) is a discontinuous function defined as the Heaviside function.

### 2.3. Elements of stiffness and mass matrices

In this section, the elements of stiffness matrix and mass matrix will be derived from the expression of the potential energy and the kinetic energy. The FEM formulation based on the Mindlin–Reissner plate theory taking into account the effects of shear deformation and rotatory inertia is used. Each node possesses three degrees of freedom \( w_i, \theta_{xx} \) and \( \theta_{yy} \). In conjunction with Eq. (2), the displacement field in the X-FEM formulation can be written as:

\[
\begin{bmatrix} w_0 \\ \theta_x \\ \theta_y \end{bmatrix} = \sum_i [N_i] \{ \delta_i \} + \sum_j H[N_j] \{ \delta_j^e \}
\]

Where \( \{ \delta_i \} = \{ w_{0i}, \theta_{x_i}, \theta_{y_i} \}^T \) and \( \{ \delta_j^e \} = \{ w_{0j}^e, \theta_{x_j}^e, \theta_{y_j}^e \}^T \)

The element stiffness matrix, expressed as follows:

\[
[K^e] = \begin{bmatrix} [K_{xx}^e] & [K_{xy}^e] \\ [K_{yx}^e] & [K_{yy}^e] \end{bmatrix} = \int_{A_e} \begin{bmatrix} (B_t)^T \{ C \} B_t \end{bmatrix} dA + \int_{A_e} \begin{bmatrix} (B_t)^T \{ C \} B_j \end{bmatrix} dA
\]

\( [K_{xx}^e] \) is the classical stiffness matrix, \( [K_{xy}^e] \) and \( [K_{yy}^e] \) are the coupling stiffness matrix and \( [K_{yx}^e] \) is the enriched stiffness matrix.

The element mass matrix:

\[
[M^e] = \begin{bmatrix} [M_{xx}^e] & [M_{xy}^e] \\ [M_{yx}^e] & [M_{yy}^e] \end{bmatrix} = \int_{A_e} \begin{bmatrix} [N_i] \{ p \} [N_i] \end{bmatrix} h dA + \int_{A_e} \begin{bmatrix} H[N_j] \{ p \} H[N_j] \end{bmatrix} h dA
\]

\( [M_{xx}^e] \) is the classical mass matrix, \( [M_{xy}^e] \) and \( [M_{yy}^e] \) are the coupling mass matrix and \( [M_{yx}^e] \) is the enriched mass matrix.

### 3. Contact force

In the following section, we present the mathematical development for the computation of contact force and central deflection. The dynamic equation of plate is given by the following [9-10]:

\[
[M]\ddot{u} + [K]u = [F]
\]

Where \([M]\) and \([K]\) are the plate mass matrix and stiffness matrix, respectively. \( \{u\} \) and \( \{\dot{u}\} \) are respectively the displacement and acceleration vector. \([F]\) is the equivalent of external load, which include the impact force. The dynamic equation of a rigid ball is given by the use of the Newton’s second law:

\[
m_i \ddot{w}_i = -F_c
\]

Where \( m_i \) is the mass of the ball (impactor) and \( F_c \) is the contact force.

Let us consider the contact between a spherical ball made of an isotropic material and a target isotropic plate. The contact is located at the centre of the plate. The contact force between the impactor and the plate is calculated using a modified non-linear Hertzian indentation law proposed by [9-10],

\[
F_c = k(w_i(t) - w_j(t))^{3/2}
\]
Where $k$ is a contact stiffness coefficient. Various analytical and experimental techniques for determination of the Hertzian contact constant for plate $k$ have been proposed in the literature [9-10]. $w_i(t)$ and $w_s(t)$ are the displacement of the impactor and the impacted point on the mid surface of the plate, respectively.

### 4. Numerical applications

In this section numerical examples of plates $(L, l)$ with pre-existing through crack parallel to $y$ (or $x$) axis and subjected to impact load are carried out. The crack fictitiously extends along $y$ axis. Non-dimensionalized parameters used in the text are given (Fig. 2): $\alpha = a/l$ (Ratio of crack length along $y$-axis to the plate length), $\beta = b/L$ (Ratio of crack position along $x$-axis to the plate length), $\gamma = c/l$ (Ratio of crack position along $y$-axis to the plate length).

![Figure 2 Plate configuration](image)

A parametric study by varying various parameters such as: crack length and crack position in $x$ and $y$ directions, is performed. Contact force and central deflection are thus computed using a computer code developed for this purpose. The properties of the target plate and the impactor are:

- Plate properties: $E = 200$ GPa, $\nu_{12} = 0.3$, $\rho = 7800$ Kgm$^{-3}$.
- Impactor properties: $E = 200$ GPa, $\nu_{12} = 0.3$, $\rho = 7800$ Kgm$^{-3}$

#### 4.1 Effect of the crack length $\alpha$

An isotropic plate of 16 cm long, 10 cm wide, 8 mm thick and clamped at both ends ($x = 0$, $x = 16$ cm) is considered. The mechanical properties of the target plate and impactor are listed in section 4. The plate is subjected at its center to an impact force induced by a steel ball. The impact velocity is 10 m/s. The variation of contact force and central deflection versus time for different values of adimensional crack length $\alpha$ are illustrated in figures 3 and 4. From these figures, it is observed that the presence of cracks affects both contact force and central deflection. However, this effect is no more pronounced when adimensional crack length $\alpha \geq 0.6$.

![Figure 3 Contact force history](image)

**Figure 3** Contact force history for $\alpha = 0.0625$

![Figure 4 Central deflection history](image)

**Figure 4** Central deflection history for $\alpha = 0.0625$
4.2 Effect of the crack position

4.2.1 Effect of ratio crack position \( \tilde{b} \)

The variation of contact force and central deflection versus time for different values of position of the adimensional crack ratio along x-axis \( \tilde{b} \) is illustrated in figures 5 and 6. One can observe (figs. 7 and 8) that the presence of crack in the y-direction affects both maximum contact force and maximum central deflection. In terms of numerical values, maximum contact force is 14.259 \( KN \) and 15.177 \( KN \) for adimensional crack position \( \tilde{b} = 0.0625 \) and \( \tilde{b} = 0.125 \), respectively. For a better explanation of the presence of crack’s position on dynamic response of impact plates, we ought to plot the maximum contact force and central deflection as a function of adimensional crack position \( \tilde{b} \) and crack length \( \tilde{a} \). Figures 7 and 8 show the variation of the maximum contact force and central deflection as a function of adimensional crack position \( \tilde{b} \) and for different values of adimensional crack length \( \tilde{a} \). It observed that the maximum contact force increases with the decrease of \( \tilde{a} \). However, the maximum contact force becomes constant for values of \( \tilde{b} \) greater than 0.3 whatever is the value of \( \tilde{a} \). On the other side, the maximum central deflection increases with the increase of \( \tilde{a} \). This effect is no more pronounced for values of \( \tilde{b} \) greater than 0.125, meaning that the dynamic properties are more affected when the adimensional crack is closer to the impact point.

![Figure 5 Contact force history for \( \tilde{a} = 0.5 \)](image)

![Figure 6 Central deflection for \( \tilde{a} = 0.5 \)](image)

![Figure 7 Maximum contact force for different value of \( \tilde{b} \)](image)

![Figure 8 Maximum central deflection for different value of \( \tilde{b} \)](image)

4.4.2 Effect of ratio crack position \( \tilde{c} \)

In this section, the effect of position crack ratio along y-axis \( \tilde{c} \) on the dynamic parameters is conducted. In this numerical experiment both adimensional crack length \( \tilde{a} \) and position crack ratio \( \tilde{b} \) have fixed values (\( \tilde{a} = 0.4, \tilde{b} = 0.0625 \)). The impact is at the center of the plate and the impact velocity is \( 10m/s \). The geometrical and mechanical properties of the plate and the impactor are
similar to the one used in section 4. The effect of crack position \( \overline{c} \) along y-axis is presented in figures 9 and 10. From these figures, as far as dynamic response is concerned, both cases (a) and (c) give similar results because cracks are symmetric with respect to the impact point. On the other hand case (b) gives higher central deflection than the two other cases, meaning that the plate loses more rigidity when crack is closer to the impact point.

5. Conclusions

The effects of crack on the dynamic response of impacted cracked plate are studied. A mathematical model using the X-FEM in which transverse shear deformation is numerically implemented. A computer code is developed and various applications have been treated. A parametric study in which the effects of crack length and crack position have been conducted. From the present study some conclusions can be drawn:

(i) The contact force decreases with the increase of the crack length \( \overline{a} \) while central deflection increases.
(ii) The plate loses more rigidity when crack is closer to the impact point.
(iii) The effect of crack position on the dynamic response is less pronounced when the crack is near by the fixed end.

References