Multibody modeling of non planar ball bearing

C. Bovet\textsuperscript{a,b}, J.M. Linares\textsuperscript{a}, L. Zamponi\textsuperscript{b}, E. Mermoz\textsuperscript{b}

\textsuperscript{a} Aix-Marseille Universit\'{e}, CNRS, ISM UMR 7287, 13288, Marseille cedex 09, France
\textsuperscript{b} Eurocopter, A\'{e}roport Internationale Marseille Provence, 13725 Marignane, France

Abstract:
This work presents the dynamic modeling of ball bearing which uses multibody dynamic formalism. Such formalism allows immediate integration of the model in helicopters main gear boxes simulations. Ball bearing is considered non lubricated in order to predict its behavior in case lubrication system failure. Rolling contacts are treated with the method proposed by Kalker \cite{10}. This approach is based on polynomial approximation of relative displacement on the contact ellipse. For low computational cost and without any spatial discretization, it gives a good estimation of tangential traction and creep. A numerical example of a ball bearing under thrust load is presented.

Mots clefs: multibody dynamic; ball bearing; rolling contact

1 Introduction

In helicopters, the mechanical transmission of power is achieved by the Main Gear Boxes (M.G.B). Rolling bearings are the most critical component in such mechanical system since it permits the relative rotation motion between shaft and housing and it carries high load and speed. Understanding of such components is essential. Thus local behavior of rolling element bearing has been widely studied. Notable works include quasi-static analysis \cite{7} and dynamic analysis with the well known computer code ADORE\textsuperscript{©} \cite{6}. In M.G.B, the power weight ratio must be as low as possible. Such design rule leads to more flexible shafts and housings. This increasing flexibility associated with the nonlinearity of rolling bearings, forbids only use of local analysis \cite{1}. The flexibility of the environment can not be neglected. For instance, misalignment induced by housing deformation may cause rolling bearings damage and premature failure \cite{14}.

Dynamic modeling of multibody system allows analysis, design, optimization and control of complex mechanical system. Joints are usually assumed to be ideal or simplified \cite{12} \cite{5}. Studies dealing with the use of real cylindrical or spherical joint in multibody simulation have been produced \cite{3}. A recent work proposes a model of planar deep groove ball bearing that can be used in planar multibody system \cite{13}.

The purpose of this study is to introduce a non planar dynamic model of non lubricated ball bearing with clearance that can be used in dynamic simulation of M.G.B. Non lubricated model is useful to predict behavior of rolling bearings in case of lubrication system failure. In such model, the description
of rolling contact is of prior importance. Number of simulated contacts becomes very important which could lead to long CPU time. Kalker’s approach [10] has been chosen since for low computational cost and without any spatial discretization, it gives accurate estimation of tangential traction and creep [2].

2 Problem formulation
2.1 Equation of motion of multibody system
Several formalisms are available in multibody dynamic. The present work uses the finite element approach [5]. A fundamental aspect in spatial multibody dynamic is the representation of large finite rotation. Most approaches use Euler angles or Euler parameters. The first method provides intuitive description of rotational motion but has the drawback to exhibit singularity. The second method avoids this singularity but needs one redundant variable. Their use require imposing additional non-linear constraint of normality. The present work employs Conformal Rotation Vector, this parametrization is free of any singularity and uses only three independent parameters [4]. Thus each body needs the use of six parameters. All generalized coordinates are collected in a column vector \( q \).

Bodies are usually interconnected with joints. These connections are expressed as algebraic equations. Holonomic constraints \( \Phi \) are formulated as implicit functions of the generalized coordinates and time\( (1) \). Non-holonomic constraints \( \Phi_{nh} \) encountered in this work are always linear in velocities and can be expressed in the form \( (2) \) where \( \dot{q} \) notation denotes for time derivative of \( q \).

\[
\Phi(q, t) = 0 \tag{1}
\]

\[
\Phi_{nh}(q, \dot{q}, t) = B_{nh}(q)\dot{q} + g_{nh}(t) \tag{2}
\]

The constrained dynamic problem is treated with the augmented lagrangian method. Non-holonomic constraints are considered using a dissipation function \( D \). Where \( p \) is a penalty coefficient and \( k \) is a scaling factor. \( \lambda \) is a set of Lagrange multipliers. The superscript \( T \) is transpose operator.

\[
D = \frac{1}{2} p\Phi_{nh}^T \Phi_{nh} + k\lambda^T \Phi_{nh} \tag{3}
\]

Dynamic equilibrium equations of constrained system are then obtained. \( M, B \) are respectively the mass matrix, the matrix of holonomic constraint gradient. The force vector \( g \) is the sum of external, internal and complementary inertia force. \( r \) is the dynamic residual vector.

\[
\begin{cases}
   r(q, \dot{q}, \ddot{q}, \lambda, \dot{\lambda}) = M\ddot{q} + B^T(p\Phi + k\lambda) + B_{nh}^T(p\Phi_{nh} + k\dot{\lambda}) - g(q, \dot{q}, t) = 0 \\
   \Phi_{nh}(q, \dot{q}, t) = 0 \\
   \Phi(q, t) = 0
\end{cases} \tag{4}
\]

This non-linear second order differential-algebraic system \( (4) \) is solved using HHT scheme [8]. As Newmark method, it allows direct resolution of second order differential system. It is unconditionally stable, second-order accurate and induces less numerical dissipation than damped Newmark algorithm. This scheme needs linearized form of the dynamic equilibrium \( (4) \). Knowing an approximate solution \( (q^*, \dot{q}^*, \ddot{q}^*, \lambda^*, \dot{\lambda}^*) \) at time \( t \) and considering a correction of the solution \( (\Delta q, \Delta \dot{q}, \Delta \ddot{q}, \Delta \lambda, \Delta \dot{\lambda}) \), the differentiation of \( (4) \) provides:

\[
\begin{bmatrix}
   M & 0 \\
   0 & 0
\end{bmatrix}
\begin{bmatrix}
   \Delta \ddot{q} \\
   \Delta \ddot{\lambda}
\end{bmatrix}
+ \begin{bmatrix}
   C_t & kB_{nh}^T \\
   kB & 0
\end{bmatrix}
\begin{bmatrix}
   \Delta \dot{q} \\
   \Delta \dot{\lambda}
\end{bmatrix}
+ \begin{bmatrix}
   K_t & kB^T \\
   kB & 0
\end{bmatrix}
\begin{bmatrix}
   \Delta q \\
   \Delta \lambda
\end{bmatrix}
= \begin{bmatrix}
   -r^* \\
   -\Phi^*
\end{bmatrix} + O(\Delta^2) \tag{5}
\]

Where \( K_t \) and \( C_t \) are the tangent stiffness and the tangent damping matrices. Finally, the nonlinear equation of motion is solved iteratively by Newton method.

\[
K_t = \frac{\partial r}{\partial q} \quad \text{and} \quad C_t = \frac{\partial r}{\partial \dot{q}} \tag{6}
\]
2.2 Dynamic ball bearing model description

Ball bearing consists of three parts: a number of rolling elements, the outer and the inner races and the cage. The interaction between the bearing elements constitutes the basic formulation of the dynamic model. In case of local computation, specific boundary conditions are applied on races. For global simulation, the races are linked with the shaft and the housing. In case of incompatible model e.g. rigid races and finite element shaft and housing, rigid link can be used.

![Diagram of Dynamic Ball Bearing Model](image)

**Figure 1 – Dynamic ball bearing model description**

### Boundary conditions

For a local study, the outer race is linked to a reference frame using hinge joint [5]. The reference frame could be an absolute frame or a fixed frame which consider housing misalignment due to housing flexibility. Such reference frame is derived from static FEM analysis of M.G.B.. The inner race is submitted to external load and is free to move to obtain dynamic equilibrium. The rotation velocity of the outer race is prescribed using non holonomic constraint even if it is possible to drive rotation angle in the hinge joint. This leads to a better numerical behavior. Then, both races have prescribed rotation velocities around their own rotation axis via non holonomic constraint of type:

\[
\Phi_{nh}(q, \dot{q}, t) = z^T \Omega(q, \dot{q}) - \omega_d(t) = 0
\]

Where \(\Omega\), \(z\) and \(\omega_d(t)\) are respectively the vector of material angular velocity, the axis of rotation of the race and the prescribed angular velocity. According to rotation parametrization, a matrix \(T\) links angular velocity of the body with rotation parameters and its derivatives. Identification with (2) gives:

\[
\begin{align*}
\Omega(q, \dot{q}) &= T(q) \dot{q} \\
B_{nh}(q) &= z^T T(q) \\
g_{nh}(t) &= -\omega_d(t)
\end{align*}
\]

### Internal interactions

Internal interactions are classified into two main parts, interaction between rolling elements and races or cage, and interaction between cage and race. Normal load \(F_n\) of point contacts between balls and races or cage are obtained with the classical Hertz theory of elastic contact (9). \(\delta\) is the contact deflection and \(k_h\) is the contact rigidity which depends on curvatures of contacting surfaces and material properties.

\[
F_n(q) = k_h(q) \delta(q)^{3/2}
\]

The computation of tractive forces \(F_T\) and moments is done using the Kalker’s rolling contact model [10] reminded hereafter. Only \(F_T\) depends to generalized coordinates derivatives. All geometric calculus are done analytically. They are not exposed here for brevity. Finally, contact force \(F\) is expressed in the form (10) where \(n\) is the normal vector.

\[
F(q, \dot{q}) = F_n(q) n(q) + F_T(q, \dot{q})
\]
Rolling contact problem

The description of rolling contact is of prior importance because net traction force defines time evolution of the ball bearing. Boussinesq has solved elastic contact between non conforming surfaces. Relative elastic displacement $u = (u, v, w)^T$, contact pressure $p_z$ and tangential traction $f = (q_x, q_y)^T$ are linked through integral equations. The local Coulomb model is used, let $\mu$ be the friction coefficient. In this work, the coupling between normal and tangential contact problem is neglected. It is exact if both bodies have the same elastic properties. Thus, in case of elliptical contact, pressure distribution and contact area $E$ are given by the Hertz theory of frictionless contact. Then, the tangential problem of contact is solved. In case of stationary rolling contact, assuming small strains and neglecting second order terms, the velocities of micro-slip between contacting points $s = (s_x, s_y)^T$ are given by:

$$s_x = c_x - \varphi \, y - \frac{\partial u}{\partial x} \quad \text{and} \quad s_y = c_y + \varphi \, x - \frac{\partial v}{\partial x}$$  \hspace{1cm} (11)

Where, $V_r$ is the rolling speed, $c = (c_x, c_y)$ and $\varphi$ are respectively creep and spin velocities dimensionalized by the rolling speed. Kalker’s approach makes use of the fact that in case of material elastic symmetry, the load displacement equation is valid [10]. If we assume a traction of the form (12), then the relative displacement inside the elliptical area $E$ are polynomials in $x$ and $y$ of the same degree. Polynomial coefficients $(a_{mn}, b_{mn})$ and $(d_{pq}, e_{pq})$ are linked linearly.

$$(u, v) = \sum_{m,n \geq 0} (a_{mn}, b_{mn}) \, x^m \, y^n \Leftrightarrow (q_x, q_y) = \frac{1}{\sqrt{1 - \left(\frac{a}{b}\right)^2 - \left(\frac{c}{b}\right)^2}} \sum_{p,q \geq 0} (d_{pq}, e_{pq}) \, x^p \, y^q$$  \hspace{1cm} (12)

Classical methods [9] assume a priori the stick and slip zones and often lead to a closed form for the traction and creep. Nevertheless, it breaks down when the spin motion becomes large. The difficulty of such problem lies in the different boundary conditions which have to be satisfied in slip and stick zone. Those areas are not known in advance. Kalker’s approach avoids this difficulty by combining boundary conditions. Let $T$ and $S$ be defined by (13). Then on the whole contact area $E$, the product $(TS)$ must be null. This problem is solved by minimizing the integral of $(TS)$ over $E$. Such approach blurs the distinction between stick and slip zones, which are now identified a posteriori.

$$T = \left\| \frac{f}{\mu p_z} - \frac{s}{\|s\|} \right\| \quad \text{and} \quad S = \|s\|$$  \hspace{1cm} (13)

\begin{figure}[h]
  \centering
  \includegraphics[width=\textwidth]{figure2}
  \caption{(a) Net traction comparison for increasing longitudinal (b) Distribution of $\|f\|/\mu p_z$ for a ball rolling on a plane with longitudinal creep $c_s R/\mu a = 0.8$}
  \textbf{FIGURE 2 – Rolling contact model results}
\end{figure}

This method gives net traction forces and power dissipation of rolling contact. Kalker’s linear creep theory is compared to an exact approach by Chevalier [2]. Linear creep theory is limited to infinitesimal
creep and derived from load displacement equation [10]. Figure 2(a) compares net traction between rolling contact models and experimental data. The test case is a ball rolling on a plane with increasing longitudinal creep. Strip theory [9] gives also good results but it can not be used when spin motion is present. Classical traction distribution induced by longitudinal creep is shown in Figure 2(b), the stick zone is close to the leading edge (right) whereas slip zone is close to the trailing edge (left).

3 Results and discussion

This section presents a local study of a ball bearing which dimensions are summarized in Figure 3. Outer race is fixed rigidly with the housing. A thrust load of 1500 $N$ is applied on the inner race. The rotation velocity of the inner race is 6000 rev/min. Estimated friction coefficient between balls and races is 0.08. Quasi-Static simulation provides initial values of generalized coordinates and initial velocities are obtained assuming pure rolling at contact points. Because of approximate initial values, the ball bearing behavior exhibits a stabilization period (see Figure 4). In order to facilitate convergence, a simplified contact model is used during this period. This simplified model is a regularization of coulomb law for traction force and spin moment.

![Figure 3 – Geometry and dimensions of the ball bearing](image)

Only results of the first ball are shown since each ball gets the same behavior. Figure 4(a) shows time evolution of adimensionalized creep velocities and net traction. Switching between simplified model and Kalker’s model produces, essentially, transverse creep. It was expected since in this model, spin motion and transverse tangential force are not uncoupled anymore. Even when creep velocity tends to zero, net traction are not null because spin motion produces transverse tangential force. This is coherent with experimental observations [9]. As can be seen in Figure 4(b), the simplified model overestimates spin moment. The system exhibits oscillating behavior which amplitude slightly decreases over time because of numerical damping and friction. The oscillating behavior is mainly due to Hertz contact theory. This contact force model is equivalent to a nonlinear spring which rigidity depends on geometry and material properties. Balls are compressed between two nonlinear springs. Therefore, the system is prone to oscillating behavior. In order to limit this phenomenon, a possibility is to use dissipative contact force models. A study of some of relevant compliant contact force models for multibody systems dynamics can be found in [11].

Mean power dissipation of the rolling contact between ball and inner ring is 74 $mW$; outer contact 3 $mW$. Greater dissipation of inner contact was expected since inertial force tends to unload inner contact. Thus it is more disposed to get relative motion.

4 Conclusions

This work presented the dynamic modeling of non planar ball bearing. It combined multibody dynamic formalism and realistic rolling contact model. Ball bearing was considered non lubricated in order to predict its behavior in case lubrication system failure. Normal contact forces on rolling elements were computed using Hertz contact theory. Kalker’s nonlinear creep theory was used to solve tangential rolling contact problems between balls and races. Without spatial discretization, it gave accurate estimation of net traction, moment, creep and power dissipation of rolling contacts. Ball bearing dynamic
behavior was integrated using HTT scheme and boundary conditions were imposed with augmented lagrangian method. A numerical example of a ball bearing under thrust load was used to show its dynamic behavior and rolling contact characteristics. The use of realistic joint in multibody dynamic simulation becomes essential especially for flexible mechanisms. The model presented is immediately usable in multibody dynamic simulation. It is the starting point for integration of dynamic behavior of helicopters main gear boxes.

![Graph](image)

(a) Net traction and creep velocities of ball race contacts  
(b) Spin moment and velocity of ball race contacts

**Figure 4 – Ball race contact time evolution**

**Références**


