Influence of the Fluid-Structure Interaction on the Modal Analysis, and on the Dynamics of Composite Monofin : Optimization of Propulsion

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Résumé :
Le but de ce travail est la détermination de l’efficacité propulsive d’une palme de nage. Pour cela, on utilise un modèle d’interaction fluide-structure prenant en compte le caractère anisotrope de chaque couche de la palme. Le problème couplé a été résolu à l’aide de la méthode des éléments finis.

Abstract :
The aim of this study is to determine the propulsive efficiency of a swimming fin. For this, we use a fluid-structure interaction model, taking into account the anisotropic character of each layer of the fin. The numerical solution of the coupled problem has been performed using finite element method.

Mots clefs : Fluid-Structure Interaction ; Modal Analysis ; Finite Element Method

1 Introduction
Dynamic analysis of aquatic locomotion is a fundamental parameter in the performance search. In the case of swimming with fins, the propulsive efficiency depends on several factors. Most models suggested aim to evaluate the dynamic performances, including drag and lift which are the two relevant parameters relevant to quantify the propulsive efficiency of a fin. Some proposed models are essentially discrete type [2], [3], while others, by being inspired by organs of propulsion of marine cetaceans, use continuous models [4], [5]. Most of these authors do not account for the highly coupled nature of the problem. In fact, for rate of stresses observed in actual swimming, the coupling between the fluid and the palm becomes stronger.

2 Mathematical model
In this work, we assume an amateur swimmer, where the scale of velocity \( U_0 \) is supposed to be very small compared to the compression wave velocity \( c_L \) in the fin. The displacement parameter \( \varepsilon = U_0/c_L \) allows to characterize the nature of the coupling problem considered in this work. In this case, we can show [1] that the adapted model is the inertial coupling. For the sake of the simplicity the problem is bidimensional (Figure 1) and the fin is immersed in a large pool. The fin is modeled by a multilayer linear elastic transverse anisotropic material. The different layers constituting the fin are denoted by \( \Omega_i \) and has the density \( \rho_i \). We denote by \( \mathbf{u}_i \) the displacement field in the fin and \( p \) the pressure field in the fluid. \( c_0 \) and \( \rho_0 \) denote the sound celerity and density of the fluid respectively. The longitudinal axis of the fin is denoted \( \mathbf{x} \). The force \( \mathbf{F} \) given in Eq. (1) is used to describe the motion of the fin. \( \theta_i \) denotes the orientation of fibers relative to the longitudinal axis \( \mathbf{x} \) on the fin and takes 0° or 90°. Here, each layer is made of either fiberglass or carbon fiber. Use of the ALE method is not essential in this study because the material is assumed linear. In the frame attached to the fin, the problem is to find \((\mathbf{u}_i, p)\) solutions :
types of calculations were carried out. The first is when the palm is plunged into the vaccum and
the surrounding fluid. In addition, frequencies can have accurate information in the dynamic behavior
of the system. For this, we looked at the modes of the fin in the vacuum and water. Indeed, to test the
quality of a fin, it is usual to search its quasi-static deformed shape and dynamic response in air. The
aim is to check if results of tests carried out of the water are strongly influenced by the presence of
the surrounding fluid. In addition, frequencies can have accurate information in the dynamic behavior
of the system. By introducing the spaces of test functions \( V = \{ \mathbf{v} \in H^1(\Omega_s), \mathbf{v} = 0 (\Gamma_0) \} \) and
\( \phi \in Q = H^1(\Omega_f) \), the variational formulation of Eq. (1) holds

\[
\begin{align*}
\int_{\Omega_s} \sigma(\mathbf{u}) : \varepsilon(\mathbf{v}) \, dx - \omega^2 \int_{\Omega_s} \rho \mathbf{u} \cdot \mathbf{v} \, dx + \int_{\Gamma} \rho v \cdot n \, d\Gamma &= 0, \\
\int_{\Omega_s} \sigma(\mathbf{u}) : \varepsilon(\mathbf{v}) \, dx &= \sum_{i=1}^{N_L} \int_{\Omega_i} \sigma(\mathbf{u}_i) : \varepsilon(\mathbf{v}_i) \, dx
\end{align*}
\]

(2)

where \( N_L \) is a number of layers. Using Lagrange finite elements, where \( \mathbf{u}_h \in P_2 \times P_2 \) and \( p_h \in P_1 \),
discretization of the weak formulation (2) induces a non-symmetrical system

\[
\begin{bmatrix}
K_s & B \\
\odot & \odot & \odot & \odot & \odot
\end{bmatrix} - \omega^2 \begin{bmatrix}
M_s & \odot & \odot & \odot & \odot \\
\odot & M_u & \odot & \odot & \odot \\
\odot & \odot & M_p & \odot & \odot \\
\odot & \odot & \odot & M_q & \odot \\
\odot & \odot & \odot & \odot & M_r
\end{bmatrix}
\begin{bmatrix}
\mathbf{U} \\
\mathbf{P}
\end{bmatrix} = 0
\]

(3)

where \( \mathbf{U} \) and \( \mathbf{P} \) are the vectors of nodal values for \( \mathbf{u} \) and \( p \), respectively. The submatrices of Eq. (3) are defined by

\[
\begin{align*}
\mathbf{V}^T K_s \mathbf{U} &= \int_{\Omega_s} \sigma(\mathbf{u}) : \varepsilon(\mathbf{v}) \, dx; \\
\mathbf{V}^T M_s \mathbf{U} &= \int_{\Omega_s} \rho \mathbf{u} \cdot \mathbf{v} \, dx; \\
\mathbf{V}^T \mathbf{B} \mathbf{P} &= \int_{\Gamma} \rho v \cdot n \, d\Gamma \\
\Phi^T M_u \mathbf{U} &= \int_{\Gamma} \rho_0 \mathbf{u} \cdot n \phi \, d\Gamma; \\
\Phi^T M_p \mathbf{P} &= \int_{\Omega_s} \nabla p \cdot \nabla \phi \, dx
\end{align*}
\]

(4)

where \( \mathbf{V} \) and \( \Phi \) are the vectors of nodal values for \( \mathbf{v} \) and \( \phi \), respectively. \( M_u \) is the added mass matrix.

The non-symmetric system (3) was solved using the commercial software Comsol Multiphysics. Two
types of calculations were carried out. The first is when the palm is plunged into the vacuum and
the second into water. We give below the results for a model of up to five layers (\( N_L = 5 \)) and the
eigenfrequencies in vacuum and water. The fibers of each layer are arranged alternately along the two
directions orthogonal axis $x$ and $y$ of the mean plane of the fin. Tables 1 and 2 show that arrangement of layers has a strong influence on the natural frequencies, and the added mass decreases the natural frequencies.

<table>
<thead>
<tr>
<th>Oriented fibers</th>
<th>in a vacuum</th>
<th>in a water</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^\circ / 90^\circ$</td>
<td>12.62 76.02 205.33</td>
<td>1.71 46.06 103.15</td>
</tr>
<tr>
<td>$0^\circ / 90^\circ / 0^\circ$</td>
<td>29.38 122.99 265.07</td>
<td>3.52 21.72 55.17</td>
</tr>
<tr>
<td>$0^\circ / 90^\circ / 90^\circ$</td>
<td>22.68 90.16 192.04</td>
<td>2.42 14.56 37.35</td>
</tr>
<tr>
<td>$0^\circ / 90^\circ / 0^\circ / 90^\circ$</td>
<td>28.82 79.74 185.16</td>
<td>2.77 11.85 33.49</td>
</tr>
</tbody>
</table>

**Table 1** – Natural frequencies of the fin in the case of $0^\circ / 90^\circ$ and $90^\circ / 0^\circ / 90^\circ / 0^\circ$

<table>
<thead>
<tr>
<th>Oriented fibers</th>
<th>in a vacuum</th>
<th>in a water</th>
</tr>
</thead>
<tbody>
<tr>
<td>$90^\circ / 0^\circ$</td>
<td>12.61 73.51 158.46</td>
<td>1.71 13.30 39.45</td>
</tr>
<tr>
<td>$90^\circ / 0^\circ / 90^\circ$</td>
<td>9.52 51.39 105.50</td>
<td>1.14 8.74 23.87</td>
</tr>
<tr>
<td>$90^\circ / 0^\circ / 0^\circ / 90^\circ$</td>
<td>22.12 56.23 115.59</td>
<td>2.28 9.53 23.83</td>
</tr>
<tr>
<td>$90^\circ / 0^\circ / 90^\circ / 0^\circ / 90^\circ$</td>
<td>17.44 44.84 91.62</td>
<td>1.68 6.89 17.23</td>
</tr>
</tbody>
</table>

**Table 2** – Natural frequencies of the fin in the case of $90^\circ / 0^\circ / 90^\circ / 0^\circ / 90^\circ$

**Figure 2** – First mode of the fin and fluid pressure

4 The dynamic problem

The dynamic problem was conducted using the data proposed in [6]. For this, the fin is subjected to a combined translational and rotation motions. In this case, the quantity $\mathbf{F}$ introduced in the model problem (1) has the expression

$$\mathbf{F} = \begin{bmatrix} x \ddot{\omega}(t) + y \dot{\omega}(t) - h(t) \sin[\omega(t)] \\ y \ddot{\omega}(t) - x \dot{\omega}(t) - h(t) \cos[\omega(t)] \end{bmatrix}, \quad \omega(t) = \theta_0 \sin(2\pi ft); \quad h(t) = h_0 \sin(2\pi ft - \psi)$$

with $\theta_0 = 40^\circ$; $\psi = \pi / 2$; $h_0 = 1c$; $f = 0.225$ [Hz] and $c = 0.7$ is the chord of the profile, that is to say, the length of the fin. The phase $\psi$ is introduced to model the muscle dissymmetry. To avoid a resonant frequency, the excitation frequency is taken far enough from the first natural frequency of the coupled system. The hydrodynamic parameters that seem most relevant are the total force $\mathbf{R} (= \int_\Gamma \sigma(\mathbf{u}) \mathbf{n} d\Gamma)$
exerted on the fin during the movement phase. The two components of \( \mathbf{R} \) are respectively the drag \((D)\) and lift \((L)\) of the fin. The quantity \( T = -D \) is called thrust. Different types of layers exist in the manufacture of fins. Throughout the model the thickness of the fin is fixed in advance. We use the same physical characteristics as in the case of modal analysis. Using the same notation as before, the variational formulation of boundary value problem (1) is then written

\[
\frac{d^2}{dt^2} \int_{\Omega_s} \rho \mathbf{u} \cdot \mathbf{v} dx + \int_{\Omega_s} \sigma(\mathbf{u}) : \varepsilon(\mathbf{v}) dx + \int_{\Gamma} p \mathbf{v} \cdot \mathbf{n} d\Gamma = - \int_{\Omega_s} \rho \mathbf{F} \cdot \mathbf{v} dx \\
\frac{d^2}{dt^2} \left( \int_{\Omega_s} \rho \frac{\partial \mathbf{u}}{\partial t} dx + \int_{\Gamma} \rho_0 \mathbf{u} \cdot \mathbf{n} d\Gamma \right) + \int_{\Omega_s} \nabla p \cdot \nabla \phi dx = \int_{\Gamma} \rho_0 \mathbf{F} \cdot \nabla \phi dx
\]

\[(5)\]

In this section, we use a particular kinematics proposed in [6]-[7], even if our models are not exactly similar. Indeed, the kinematics will allow us in future to develop a new experimental protocol for measuring various hydrodynamic parameters of a fin. As the model problem (1) is linear, it is interesting to see the different contributions of each elementary movement in the dynamic response of the fin.

4.1 Dynamic response: translational motion

The rotation \( \omega(t) \) is canceled and the movement is then sinusoidal along the direction \( y \). The Figure 3 shows that the two-layer model seems to give a greater thrust than the other models. This is consistent with the results of modal analysis, where this is the first model that has the lowest frequency. This type of movement is not interesting for the propulsive efficiency. Indeed, it leads to a zero mean propulsive efficiency. On the other hand, we see a greater amplitude for thrust, compared to the lift.

![Figure 3 – Thrust and lift of fin in the case of translation motion](image)

(a) \( 0^\circ/90^\circ/0^\circ/90^\circ/0^\circ \)  
(b) \( 90^\circ/0^\circ/90^\circ/0^\circ/90^\circ \)  
(c) \( 0^\circ/90^\circ/0^\circ/90^\circ/0^\circ \)  
(d) \( 90^\circ/0^\circ/90^\circ/0^\circ/90^\circ \)
4.2 Dynamic response : rotation motion

The function $h(t)$ is canceled and the movement will be a sinusoidal rotation around foot. According to the Figure 4, the two-layer model always gives a greater thrust than the other models. But by eliminating this model in the response curves, we can see that the five-layer model gives the best performance, as shown in the last figure. The three layer model gives a better lift compared to other models. Thus, this type of movement provides a propulsive efficiency rather interesting. This phenomenon is also well observed in the movement of marine mammals. On the other hand, such movement can be interesting if you want to stay stationary at one position.

4.3 Dynamic response : combined rotational-translational motion

In order to have a reasonable performance of the system, we must combine both translational and rotation motion, and take the full expression of the excitation force $\mathbf{F}$. According to the Figure 5, the two-layer model always gives a greater thrust than the other models. In general, the three-layer model seems to give a better compromise. Indeed, its thrust remains positive all the time, while its lift is negative value and has less importance than other models. It is possible that by varying some physical parameters, we can significantly reduce hydrodynamic quantities, such as the moment and lift.

5 Conclusions

The above results allow us to draw some conclusions :

– The presence of layers provides some flexibility as indicated by the results of modal analysis. The first mode is flexural type, which justifies the use of models proposed in [2].

– Fins with anisotropic material structures allow to implement a technique of layers parameterization to improve performance. It is quite possible now to bring special attention to the structure of the layers, and types of constituent materials thereof.
It should be noted the sensitivity of the dynamic behavior of the model with respect to the materials used, and the boundary conditions for the fluid domain. Indeed, the presence or absence of rigid walls alter significantly the natural modes of the coupled system. Thus, the dynamic behavior of a swimmer depends on the localization in the pool where it is at the given moment. To obtain a better thrust, the fin have to be elastic and sought at least in rotation. The amplitude of the vertical translation must be controlled to avoid a too high lift, in order to remain at a constant depth. The use of multilayer fins allows to control an excessive variation of lift.

Références


