Robust optimization of a truck timing gear cascade: numerical and experimental results

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Abstract:
The aim of this work is to present a robust optimization method for gear teeth corrections of a truck timing gear. Static Transmission Error (STE) fluctuations are the main noise sources radiated by gear systems. Their origins are the teeth deflections depending on the applied torque and the micro-geometrical tooth corrections and manufacturing errors. The teeth corrections are optimized in order to minimizing this excitation source for a wide operating torque range. The chosen optimization algorithm is the particle swarm method, a meta-heuristic well adapted to that kind of multi-parameters complex optimization. Several optimized solutions are generated and their robustness is tested towards manufacturing and assembling errors. Standard and optimized gears have been implemented on a real heat engine and acoustics measurements have been made. The results confirm an acoustic gain related to teeth correction optimization.

Keywords: gears, transmission error, acoustic measurements

1 Background of the study
The incoming acoustics norms in the field of transport being more and more severe and primary noise sources like power train and tire/roadway contact being already well studied, an effort is now made on secondary sources like the timing gears. Indeed, considering the high applied torques in a truck, the timing function is done by a geared system, displayed in FIG 1. It is designed with two helical gears cascades. The first cascade includes the camshaft gear, an idler gear and inner bullgear, that is to say two meshes. The second cascade includes the outer bullgear and the crankshaft gear. For geared system, the STE under load \cite{1} is one of the main noise sources. It corresponds to the difference between the actual position of the driven gear and its theoretical position for a very slow rotation velocity and for a given applied torque. Its characteristics depend on the instantaneous situations of the meshing tooth pairs. STE results from tooth deflections, tooth surface modifications and manufacturing errors. Under operating conditions, STE generates dynamic mesh force transmitted to shafts, bearings and finally to the crankcase. The vibratory state of the crankcase is the main source of the radiated noise \cite{2}.
To reduce the radiated noise, it is necessary to minimize STE fluctuations by introducing voluntary tooth micro-geometrical modifications. For this study, the selected optimization parameters for each gear pair are:
- the tip relief values $X$ of pinion and driven gear i.e. the amount of material remove on the teeth tip,
- the starting tip relief diameters $\Phi$ of pinion and driven gear,
- the added up crowning centered on the active tooth width $C_{\beta i,j}$.

The optimization of tooth modifications in simple mesh gear system for a given torque has been studied by many authors [3-5] but the approach for multi-mesh gear systems optimization is still unusual [6]. In this study, the first cascade with three helical gears has 8 parameters (2 by gear, and 1 by mesh) and the second cascade with two gears has therefore 5 parameters. Moreover the modifications made on teeth profile have to be satisfying for a wide torque range. This has to be done by an efficient method as the number of possible solutions is extremely important, due to the combinatory explosion phenomenon. The Particle Swarm Optimization (PSO) [7] has been chosen because it is particularly efficient as it is an order 0 meta-heuristic, i.e. it not necessary to evaluate first derivatives of the function.

Furthermore, robustness of the obtained solutions has to be studied. Indeed, dispersion of manufacturing errors generates a strong variability of the dynamic behavior and noise radiated from geared systems (sometimes up to 10 dB [8]). A statistical study of solutions permits to have a good overview of how the solution can be deteriorated when the manufacturing errors (dispersion over the optimization parameter values) and assembling errors (lead summed up and involute alignment deviations, respectively $f_{\phi}$ and $f_{\alpha}$) are considered.

2 Optimization problem description

This part describes the problem formulation. The fitness function definition, the chosen algorithm and the robustness study approach are detailed.

2.1 Static transmission error computation

The method for STE calculation retained is classical [9]. Equations describing contact between gears are solved for each meshing position, taking account of the elasto-static deformations and initial gaps between teeth surfaces. A validated computational code (CYLI) supplied by Renault Trucks has been used for this part.

2.2 Optimization fitness function establishment

The criterion retained to estimate one STE fluctuations is the peak-to-peak amplitude (STE$_{pp}$). Considering that the modifications made have to reduce the STE$_{pp}$ for a given $[T_{min}, T_{max}]$ torques range, the fitness
function $f$ is defined as the integral of $STE_{pp}$ over this torques range approximated by a 3-points Gaussian quadrature:

$$
f = \int_{T_{\text{min}}}^{T_{\text{max}}} p(T)STE_{pp}(T)dT \approx \frac{1}{2} \sum_{i=1}^{3} a_i STE_{pp}(T_i)$$  

(1)

$p(T)$ corresponds to the torque distribution function. It is assumed uniform, i.e. $p(T) = \frac{1}{T_{\text{max}} - T_{\text{min}}}$.

$a_i$ and $T_i$ are respectively the Gaussian weighting coefficients and the Gaussian points locations given in [10] for three Gauss points.

For the first three-gears-cascade, the multi-objectives aspect is simply handled by considering:

$$f_{84/73/56} = \frac{1}{2} \sum_{i=1}^{3} a_i \left(STE_{pp,83/73}(T_i) + STE_{pp,73/56}(T_i) \right)$$  

(2)

### 2.3 Particle Swarm Optimization

The method is based on a stigmergic behavior of a population, being in constant communication and exchanging information about their location in a given space to determine the best location according to what is being searched. In this case, some informant particles are considered, which are located in an initial and random position in a hyper-space built according to the different optimization parameters. The best location researched is thus the combination of parameters which ensures the minimum value of the fitness function defined earlier. At each step and for each particle $i$, a new speed $V_i(t)$ and so a new position $p_i(t)$ is reevaluated considering:

- the current particle velocity $V_i(t-1)$,
- its current position $p_i(t-1)$,
- its best position $p_{i,(\text{ind})}$,
- the best position of neighbors $p_{\text{glob}}$.

The algorithm working can be summarized to the system of equations (3).

$$ \begin{cases} 
V_i(t) = \varphi_0 V_i(t-1) + \varphi_1 A_1 [p_{i,(\text{ind})} - p_i(t-1)] + \varphi_2 A_2 [p_{\text{glob}} - p_i(t-1)] \\
p(t) = p_i(t-1) + V_i(t) 
\end{cases} $$  

(3)

$A_1$ and $A_2$ are random vectors of numbers between 0 and 1 and the coefficients $\varphi_i$ are taken following Clerc and Trelea [11-12] works: $\varphi_0 = 0.729$ and $\varphi_1 = \varphi_2 = 1.494$. These random numbers provide a stochastic aspect to the research of optimal solutions, avoiding the particles to get stuck in local minima by diversifying the solutions. On the other hand, the best position coefficients allow focusing researches around promising area. Moreover, this meta-heuristic is an order 0 algorithm which does not require the calculation of the first derivative of the fitness function. For all these reasons, the Particle Swarm algorithm is well-suited for this problem. Repeated use of the method for different initial conditions provides several optimized solutions.

### 2.4 Robustness statistical study

Let’s say that a solution $S_0$ is determined by the PSO. The robustness study is done by a Monte-Carlo method, i.e. 10000 others solutions are computed, chosen randomly in an hyperspace centered on the optimized solution parameters values, limited by the tolerances interval of each considered manufacturing error. These 10000 results allow the establishment of the density probability function of each selected optimized solution. They also allow us to compute statistical variables such as mean value and standard deviation. For statistically independent variables, the theoretical convergence on these values is proportional to $n^{-1/2}$, where $n$ is the number of solutions computed. The industrial request is to consider that the distribution functions of all parameters and errors should be taken uniform. The convergence has been tested
and confirmed this convergence law. Therefore the number of samples for a Monte-Carlo simulation has been set to 10000 ensuring an error less than 1%.

FIG 2 shows an example of probability density functions for some of the optimized solutions found with the PSO algorithm without manufacturing errors. It illustrates how the final optimized solution is selected. The solution S1 has the smallest mean and maximum values. Despite the fitness value for the optimized solution S2 is the lowest, the solution S1 is obviously the best choice to make considering the deterioration capability of the solution, because S1 probability density function has the smallest mean and maximum value.

In a practical point of view, the mean value is the first criterion to consider. If the mean values are of the same order of magnitude (up to a difference of 15%), the solution with the smallest maximum value of the fitness function should be retained.

![FIG 2: Probability density functions for the standard solution and three selected optimized solutions for the three-gears-cascade](image)

An additional observation can be done. The fitness function value for the optimized solution is not the minimum value found on FIG 2. That can be easily explained by the complexity of the fitness function and by the fact that the chosen algorithm does not ensure to find a global optimum. Moreover, the tolerance ranges of the optimization parameters are wide.

3 Results – Expected static transmission errors and actual ones

After considering the STE\textsubscript{pp} and its robustness, optimized solutions have been retained for the first three-gears-cascade and for the second two-gears mesh. The evolution of the STE\textsubscript{pp} is calculated as a function of the applied torque for the standard and the optimized sets of gears. Some measurements have been done to determine the actual teeth topologies, allowing the confrontation of the recommendations made and the tooth modifications get. This permits to underline the robustness study pertinence, especially in this study where the (confidential) tolerance intervals are of the same order of magnitude as the tooth modifications themselves.

3.1 Theoretical versus actual tooth topologies

The topology measurements permit to compare the recommended tooth modifications to the actual performed ones, for both standard (corrected but not optimized) and optimized tooth modifications. The whole data are presented in FIG 3 for the crankshaft gear. FIG 4 displays only the discrepancies between theoretical and actual tooth surfaces for the all the studied gear. The results plotted are the mean value of all teeth topologies for a given gear. The analysis of these topologies leads to the following observations:

- There are relatively important discrepancies between theoretical and actual teeth topologies.
- The largest discrepancies correspond to the idler and bull inner gears for optimized solutions.
- There are manufacturing errors which cannot easily be translated in terms of tip relieves and crowning; the exact topology is used in CYLI to compare STE.
FIG 3: Comparisons of theoretical and actual teeth mean topologies of the crankshaft gear for standard and optimized solutions (same scale on each graph)

FIG 4: Comparisons of theoretical and actual teeth mean topologies of the studied gears for standard and optimized solutions (same scale on each graph)

FIG 5 displays the peak-to-peak transmission errors for the three considered meshes. Two results are particularly relevant. The deterioration of theoretical configuration is coherent with the discrepancies presented in FIG 4: the Idler Gear / Bull Inner Gear mesh for the optimized tooth modifications is the mesh with the worst discrepancies. Indeed the corresponding $\text{STE}_{\text{pp}}$ is worse than the standard actual $\text{STE}_{\text{pp}}$. For the other meshes, the curve indexed 2 and 4 shall be compared. The robust optimization done is thus efficient as the $\text{STE}_{\text{pp}}$ is lower for optimized solutions for the whole torque range.

FIG 5: Comparisons of theoretical and actual $\text{STE}_{\text{pp}}$ as a function of the applied torque for the three meshes

### 3.2 Acoustic benefits of the optimization

Both standard and optimized gears sets have been mounted on a thermal engine and the corresponding radiated noise has been measured for an operating torque higher than $T_{\text{max}}$. The complete timing system had to be considered (e.g. the oil pump pinion is necessary for the engine oil supply). Results are plotted in FIG 6.
Nevertheless, the measurements show at least 1 dB of total power reduction for the whole range of operating rotation speeds. This result is satisfying given that the levels (confidential) are initially not high, 5 of the 10 gears of the cascade have not yet been optimized and the operating torque is outside the range considered for the optimization.

![Diagram](FIG 6: Total power in function of the engine rotation speed)

### 4 Conclusions - Acknowledgment

A robust optimization based on the swarm particle algorithm and coupled with a statistical approach of the robustness through Monte-Carlo simulation has been presented. Considering the severe dispersion associated with gears manufacturing errors, this method shows the benefits if this approach by reducing by at least 1 dB the total power level. This result is satisfying given that only 5 among 10 gears have been optimized and that the initial STE were not high. This total power level reduction is thus precious considering the incoming noise norms.

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### References


