Integrating optimization and reliability tools into the design of agricultural machines

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Abstract:
Agricultural machines should be designed to be optimal and reliable. Although, the deterministic design approach does not guarantee these requirements, it is traditionally applied in the design of agricultural machines. This is due to the difficulties to model the stochastic nature of the forces acting on agricultural machines, especially the forces acting on tillage machines. Therefore, the main objective of this work is to integrate the variability in tillage forces, derived from both the variability in tillage system parameters and the variations of soil failure patterns, into the reliability-based design optimization analysis. We aim to provide an alternative to the traditional deterministic design approach in order to get, at the same time, optimum and reliable agricultural machines.

Key words: variability, tillage forces, tillage machines, optimization, reliability analysis.

1 Introduction
In the design of agricultural machines, the soil heterogeneity and discontinuous structure should be taken into account. Given the difficulties related to the determination of soil failure patterns under mechanical loads and the calculation of the relevant forces, the design of agricultural machines is far from being a deterministic science. However, in the last six decades, researchers have worked to determine the patterns of soil failure under different operational conditions and propose models to estimate the acting forces. Immense efforts have been made in this area \cite{1}\cite{2}\cite{3}, but these models do not take into account the variability in tillage forces, where the relevant forces are calculated for assigned tillage system parameters.

The variability in tillage forces can be resulted by either the variability in tillage system parameters or the variations of soil failure patterns \cite{4}\cite{5}\cite{6}\cite{7}. The variability in tillage system parameters is caused by the variability in 1) soil engineering properties, 2) tool design parameters and 3) operational conditions. The variability in soil engineering properties reflects the heterogeneity and the discontinuity of agricultural soils as they are three-phase mediums composed of solid, liquid and gaseous particles \cite{8}\cite{9}. The variations in tool design parameters are due to the manufacturing processes, while the variations in operational conditions are due to the fact that these parameters are not completely controlled during tillage operation. The variability in soil failure patterns can be attributed to the variations of mechanical behavior of the soil under mechanical loads.

In a previous report, Abo Al-kheer et al. \cite{10} have achieved a successful modeling of the variability in tillage forces, taking into account, only the variability in tillage system parameters. In another work, the variability in tillage forces has been integrated into the reliability analysis \cite{11}. Furthermore, two reliability-based design optimization approaches, namely the reliability-index approach (RIA) and the sequential optimization and reliability assessment (SORA) approach have been applied in order to achieve reliable and optimum tillage machines \cite{12}. The drawback of these works is the negligence of the variations in soil failure patterns when modeling the variability in tillage forces. Therefore, the main objective of this work is to model the variability in
tillage forces caused by both the variability in tillage system parameters and the variation in soil failure patterns. And then, to integrate this variability into the reliability-based design optimization approach.

2 Materials and methods
2.1 Overview of the work
The variability in tillage forces can be generated by two main sources: the variability in tillage system parameters which has a global effect on tillage forces and the variability in soil failure patterns which has a local effect on tillage forces. Therefore, we call the tillage forces related to the tillage system parameters as global tillage forces and those related to the soil failure as local tillage forces. An overview of this work is illustrated in Figure 1. Firstly, the variability in global tillage forces derived from the variability in tillage system parameters are modelled and added into the variability in local tillage forces. Secondly, the variability in total tillage forces is integrated into the reliability-based design analysis in order to achieve optimum and reliable machines.

![FIG. 1- Overview of the work](image)

2.2 Modeling the variability in total tillage forces
We suppose that the total tillage force is the sum of two types of forces, namely the global tillage force and the local tillage force. Conventionally, a tillage force $P$ is determined by its horizontal $P_H$ and vertical $P_V$ components. Therefore, the total horizontal and vertical forces can be calculated, according the earlier assumption, by Equations (1) and (2).

$$P_H = P_{HG} + P_{HL}$$
$$P_V = P_{VG} + P_{VL}$$

where $P_{HG}$ is the global horizontal force in kN, $P_{HL}$ is the local horizontal force in kN, $P_{VG}$ is the global vertical force in kN and $P_{VL}$ is the local vertical force in kN.

The variability in the global tillage forces ($P_{HG}, P_{VG}$) can be modelled using the methodology proposed by Abo Al-kheer et al. [10]. This methodology is based on the estimation of tillage forces according to the McKyes-Ali model accounting for the variability in tillage system parameters. The variability in tillage system parameters was modelled by means of experimental observations.

The local tillage forces ($P_{HG}, P_{VG}$) have been observed in many works in the literature but there are no available models can be used to estimate these forces [3][6]. However, the majority of reports are attributing these forces to nearly the same parameters contributing to the global tillage forces [13]. Therefore, we assume that the local tillage force components can be estimated as a percentage of the global tillage force components as shown in Equations (3) and (4).


\[ P_{HL} = \tau \cdot P_{HG} \] (3)

\[ P_{VL} = \tau \cdot P_{VG} \] (4)

where \( \tau \) is the percentage of the local tillage force to the global tillage force. The high values of \( \tau \) corresponding to a brittle soil failure and the little values of \( \tau \) corresponding to a flow soil failure. In other words, the values of the local tillage forces \( (P_{HL}, P_{VL}) \) are important for the brittle soil failure since the force cyclic pattern is much more pronounced that with flow failure, while the values of these forces are nearly zero when the soil failure is of flow type [14].

The linear correlation between the global and local tillage forces may not be accurate for all soil texture types and all operational conditions. Thus, more work should be done to investigate the relationship between the global and local tillage forces. Based on the earlier assumptions, the total tillage forces can be represented by only the global tillage forces. This leads to simplify the calculation procedures and to reduce the computational time.

### 2.3 Reliability-based design optimization

Reliability-based design optimization (RBDO) approaches have been proposed to overcome the drawbacks of deterministic design optimization (DDO) methods, by quantifying the reliability of performance or risk of failure in probabilistic terms and include these terms directly in design optimization as reliability constraints. The reliability constraints are the key constraints in the RBDO problem, as they require a considerable computation effort and this reveals the classical problems of efficiency, accuracy and stability. During the last few years, a variety of different formulations have been developed to overcome the numerical difficulties and to improve both the efficiency and accuracy. Accordingly, one can distinguish between three different approaches [15], namely the two-level approach, the mono-level approach and the decoupled approach.

Abo Al-kheer et al. [12] have coded and tested two reliability-based design optimization approaches, namely the reliability-index approach (RIA) and the sequential optimization and reliability assessment (SORA) approach, to solve the RBDO problem for agricultural machines. They reported that the SORA method is more efficient and reduces the computational time, comparing with the RIA method. Therefore, this method can be adopted to solve the RBDO problem for agricultural machines.

The SORA method employs a single-loop strategy with a series of cycles of deterministic optimization and reliability assessment. In each cycle, optimization and reliability assessment are decoupled from each other; the reliability assessment is only conducted after the deterministic optimization to verify constraint feasibility under uncertainty. The key to this method is to shift the boundaries of violated constraints to the feasible direction based on the reliability information obtained in the previous cycle. The design is quickly improved from cycle to cycle and the computational efficiency is improved significantly [16]. The RBDO problem can be written, according to this method, as

\[
\begin{align*}
\text{Minimize} \quad & f(\{d\}^k) \\
\text{Subject to} \quad & G_i(\{d\}^k - \delta_i^{k-1}, \{\bar{y}\}_i^{k-1}) \geq 0 \quad i = 1, ..., m \\
& h_j(\{d\}^k) \leq 0 \quad j = m + 1, ..., M
\end{align*}
\] (5)

where \( f \) is the objective function, \( \{d\} \) is the vector of design variables, \( k \) is the current cycle, \( G_i \) is the \( i \)th performance function, \( h_j \) is the \( j \)th deterministic constraint, \( m \) is the number of performance functions and \( M \) is the total number of constraints, \( \{\bar{y}\}_i^{k-1} \) is the vector of minimum performance target point (MPTP) in physical space with respect to \( i \)th limit state obtained in the previous cycle \( k - 1 \) and \( \delta_i^{k-1} \) is the shift parameter, given as

\[
\begin{align*}
\{\bar{y}\}_i^{k-1} &= T(\{u^*\}^T) \\
\delta_i^{k-1} &= \{d\}^{k-1} - \{\bar{y}\}_i^{k-1}
\end{align*}
\] (6)
where \( \{d\}^{k-1} \) is the vector of design variables, \( \{y\}^{k-1} \) is the MPTP in the physical space calculated by the probabilistic transformation \( T(\cdot) \) of \( \{u^*\}^T \) which is obtained by solving the inverse reliability problem presented in Equation (7).

\[
\text{Minimize} \quad G_i(\{u\}) \\
\text{Subject to} \quad ||\{u\}|| = \beta_i^T
\]

(7)

where \( \beta_i^T \) is \( i \)th target reliability index corresponding to the \( i \)th limit state \( G_i \). The use of the inverse reliability analysis instead of a full reliability analysis leads to time reduction and efficient strategy because the feasible region is identified with respect to the MPTP.

### 3 Numerical application

Abo Al-Kheer et al. [10] found that the variability in the global horizontal and vertical forces on the shank of a chisel plough shown in Figure 2, followed lognormal distributions. The distribution parameters of these forces were \( \mu_{PHG} = 0.872 \), \( \xi_{PHG} = 0.449 \), \( \mu_{PVG} = 0.004 \) and \( \xi_{PVG} = 0.447 \), where \( \mu \) and \( \xi \) are the scale and shape parameters of a lognormal distribution, respectively. The correlation coefficient between \( P_{HG} \) and \( P_{VG} \) was found to be \( \rho(P_{HG}, P_{VG}) = 0.93 \).

Based on the assumptions proposed in Section 2.2, the variability in the local horizontal and vertical forces should have lognormal distributions with the following distribution parameters \( \mu_{P_{HL}} = \ln(\tau) + \mu_{P_{HG}}, \xi_{P_{HL}} = \xi_{P_{HG}}, \mu_{P_{VL}} = \ln(\tau) + \mu_{P_{VG}}, \text{ and } \xi_{P_{VL}} = \xi_{P_{VG}} \). In this work, \( \tau \) was selected to be equal to 0.2 for brittle failure. Therefore, the variability in the total horizontal and vertical forces follow lognormal distributions with the following distribution parameters \( \mu_{P_H} = 1.054, \xi_{P_H} = 0.449, \mu_{P_V} = 0.186 \) and \( \xi_{P_V} = 0.447 \). The correlation coefficient between \( P_H \) and \( P_V \) is equal to \( \rho(P_H, P_V) = 0.93 \). Probability distribution parameters of local, global and total tillage forces for the studding shank are summarized in Table 1.

<table>
<thead>
<tr>
<th>Force type</th>
<th>Distribution type</th>
<th>Probability distribution parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Local horizontal force, ( P_{HL} )</td>
<td>Log-normal</td>
<td>( \mu_{P_{HL}} = -0.737, \xi_{P_{HL}} = 0.449 )</td>
</tr>
<tr>
<td>Local vertical force, ( P_{VL} )</td>
<td>Log-normal</td>
<td>( \mu_{P_{VL}} = -1.605, \xi_{P_{VL}} = 0.447 )</td>
</tr>
<tr>
<td>Global horizontal force, ( P_{GL} )</td>
<td>Log-normal</td>
<td>( \mu_{P_{HG}} = 0.872, \xi_{P_{HL}} = 0.449 )</td>
</tr>
<tr>
<td>Global vertical force, ( P_{VG} )</td>
<td>Log-normal</td>
<td>( \mu_{P_{VG}} = 0.004, \xi_{P_{HL}} = 0.447 )</td>
</tr>
<tr>
<td>Total horizontal force, ( P_H )</td>
<td>Log-normal</td>
<td>( \mu_{P_H} = 1.054, \xi_{P_{HL}} = 0.449 )</td>
</tr>
<tr>
<td>Total vertical force, ( P_V )</td>
<td>Log-normal</td>
<td>( \mu_{P_V} = 0.186, \xi_{P_V} = 0.447 )</td>
</tr>
</tbody>
</table>

**TAB. 1-** Probabilistic characteristics of soil tillage forces
The SORA method was implemented in MATLAB program (Mathworks INC. 2008) for finding the minimum volume of the shank chisel plough presented in Figure 3. The limit state function of the studied shank was calculated using the finite element model CALFEM [17] and the optimization problem was solved by the optimization toolbox based on the SQP algorithm. The tolerance of convergence criteria was fixed to \(10^{-3}\) for the absolute changes in design variables, the relative changes in the objective function and for the constraint verification. For this methods, \(b\) and \(h\) were considered as design variables and their initial values were \(32 \text{ mm}\) and \(58 \text{ mm}\), respectively. The deterministic constraints based on the design variables were assumed to be \(b \geq 20\) and \(b \geq 0.4h\). The reliability level was calculated by solving the inverse reliability problem for a target reliability index equals to \(\beta^T = 3\).

The SORA method is converged to the minimum objective function of \(f^* = 2.223 \times 10^6 \text{ mm}^3\) (Figure 4) corresponding to the optimal point \((d)^* = [27.663 \text{ mm}, 69.158 \text{ mm}]\).

In an earlier report, Abo Al-Kheer et al. [12] found that the minimum objective function which respects a target reliability index of \(\beta^T = 3\), was \(f^* = 1.967 \times 10^6 \text{ mm}^3\) when considering only the variability in tillage forces derived from the variability in tillage system parameters. Compared to the results of this work, we observe that the variability in tillage forces resulted by the variations of soil failure patterns has a significant effect on the objective function. The augmentation of the objective function is of 11.5\%. Therefore, both the variability in local and global tillage forces should be taken into account in the reliability-based design analysis in order to achieve optimum and reliable tillage machines.
4 Conclusions

This work was aimed to integrate the variability in total tillage forces, resulted by both the variability in tillage system parameters and the variation in soil failure patterns, into the reliability-based design optimization analysis in order to get optimum and reliable tillage machines. The obtained results show that the variability in tillage forces caused by the variations of soil failure patterns has a significant effect on the objective function. It can be concluded that the variability in local and global tillage forces should be taken into account in the reliability-based design analysis. However, local tillage forces were calculated as percentages of global tillage forces. Thus, more experimental investigations should be done to determine the relationship between the global and local tillage forces.

References