Stability of a compressible open cavity flow

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Abstract:
In this paper, a global stability analysis is carried out in the case of a compressible open cavity flow. The parameters leading the cavity study are the following: the aspect ratio (length over depth) is set to 1, the Reynolds number based on the upstream velocity and the cavity length is equal to 7500. The Mach number will be varied between 0 and 0.9. We will particularly observe the interactions between the acoustic modes (quarter-wavelength mode) and the unstable Rossiter modes. Results will be compared to the conjecture of Block (1976) and East (1966).

Résumé:
Une étude de stabilité est réalisée sur une cavité ouverte en régime compressible. Les paramètres qui définissent la cavité sont les suivants : le rapport longueur sur profondeur est égal à 1, le nombre de Reynolds basé sur la vitesse amont et la longueur de la cavité est égale à Re=7500. Le nombre de Mach sera varié de 0 à 0.9. On étudiera en particulier les interactions entre les modes acoustiques (le mode 1/4 d’onde) et les modes d’instabilité de type Rossiter. Les résultats seront confrontés aux conjectures de Block 1976 et East 1966.

Mots clés : cavity flow ; global stability analysis ; compressibility

1 Introduction
Cavity flows are well known in aeronautics to be an important source of unsteady loads and noise which can be detrimental to the structure of a plane for example. To control this phenomenon, it still remains necessary to better understand the leading physical mechanisms. This paper is making a focus on the dynamics of a subsonic but compressible cavity flow. In this framework, two mechanisms are well established: (1) a feedback mechanism combining hydrodynamic instabilities and acoustics, and (2) a normal mode resonance mechanism. For mechanism (1), the vortex-acoustic resonance refers to the classical Rossiter’s [12] proposal where the synchronization of travelling vortices and acoustics leads to the semiempirical formulae (1a) for the tones frequencies. Bilanin & Covert [4] improved the Rossiter’s model adding the instabilities of the free shear layer in the feedback loop. Later, Block [3] introduced the length-to-depth ratio in the model by accounting for the shear layer vertical displacement and the acoustic pressure both at the leading and trailing edges of the cavity. This model is given in (1b).

Concerning the mechanism (2), Plumblee [11] formulated a pressure function which characterizes the acoustic response of the cavity whose maxima are reached at the frequencies of the cavity tones. East [7] expressed the locus of those maxima as a function of depth and obtained formulae (1c). Tam [13] then refined the theory of [11] and obtained a solution for the acoustic wave equation with a no flow condition. Finally, Tam & Block [14] have developed a precise mathematical model taking into account new parameters such as the shear layer thickness and the acoustic reflections from the bottom and upstream end walls of the cavity. In this last article, the authors compared the different above mentioned predictions in the range of low Mach numbers ($M \leq 0.4$). East [7] was the first to notice that the shear layer oscillations were amplified by acoustic coupling between mechanisms (1) and (2). Block [3] confirmed it considering the acoustic power level of the shear layer modes.
as a function of velocity. She pointed out that the velocity at which the acoustic amplitude (in dB) is maximum for lengthwise modes almost coincides with the velocity where the curves for lengthwise (1b) and depthwise (1c) modes intersect. Recent studies have also observed this particularity thanks to a global stability analysis [10].

\[
\begin{align*}
St_{Rossiter} &= \frac{U_\infty}{L} \left( \frac{n - \gamma}{M + \frac{1}{\kappa}} \right) \\
St_{Block} &= \frac{n}{k_r} + M \left( 1 + \frac{0.514}{L/D} \right) \\
St_D &= \frac{0.25}{M} \left( 1 + 0.65 \frac{L/D}{0.75} \right)
\end{align*}
\]

M denotes the freestream Mach number and L,D, the length and depth of the cavity. In (1a), \(n\) is a mode number, \(\kappa\) is the ratio of convection velocity of vortices to freestream velocity, and \(\gamma\) is a factor accounting for the lag time between the passage of a vortex and the emission of a sound pulse at the trailing edge of the cavity. In (1b), \(k_r\) is the real part of the wave number of the disturbance traveling downstream.

A unified theory linking the normal mode resonance mechanism to the feedback mechanism is still lacking. An attempt is made in this paper by using a global stability approach. In the first part, global stability theory is developed. Then the flow cases and the numerics are described. Finally, stability results for bidimensional and tridimensional perturbations are presented.

## 2 Base flow and Global stability analysis

The fluid motion is governed by the unsteady compressible Navier Stokes equations formulated as

\[
\mathcal{B}(\mathbf{q}) \partial_t (\mathbf{q}) = \mathcal{R}(\mathbf{q})
\]

where \(\mathbf{q}\) represents the aerodynamic field \((\rho, \mathbf{u}, T)^T\), the pressure being eliminated using the perfect gas state equation (see (3)). \(\mathcal{R}(\mathbf{q})\) and \(\mathcal{B}\) are differential operators defined in (3) and (4), and \(\mathbf{u} = (u, v, w)^T\) stands for the velocity field in a Cartesian coordinate system.

\[
\mathcal{R}(\mathbf{q}) = - \left( \begin{array}{c}
\rho \nabla \cdot \mathbf{u} + \mathbf{u} \cdot \nabla \rho,
\rho \nabla \mathbf{u} \cdot \mathbf{u} + \frac{1}{\gamma M^2} \nabla p - \frac{1}{\rho C} \nabla \cdot \tau(u),
\rho \mathbf{u} \cdot \nabla T + p \nabla \cdot \mathbf{u} - \gamma (\gamma - 1) \frac{M^2}{C^2} \tau(u) : \mathbf{d}(\mathbf{u}) - \frac{\gamma}{RT \rho C} \nabla^2 T,
p - \rho T
\end{array} \right)
\]

\[
\mathcal{B}(\mathbf{q}) = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & \rho I & 0 & 0 \\
0 & 0 & \rho & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

The system of equations (3) has been made nondimensional using the length of the cavity \(L\), and the upstream quantities \(U_\infty, \rho_\infty, \text{ and } T_\infty\) as respective velocity, density and temperature scales.

In the present paper, the aerodynamic field \(\mathbf{q}\) is decomposed into a bidimensional steady base flow \(\mathbf{q}_0 = (\rho_0, u_0, v_0, T_0)^T\) and a three-dimensional disturbance \(\mathbf{q}_1 = (\rho_1, u_1, v_1, w_1, T_1)^T\) of infinitesimal amplitude \(\epsilon (\mathbf{q} = \mathbf{q}_0 + \epsilon \mathbf{q}_1)\). \(\mathbf{q}_0\) is solution of the steady form of the nonlinear system (2).

\[
\mathcal{R}(\mathbf{q}_0) = 0
\]

After having substituted \(\mathbf{q}\) in (2) and retaining only terms of order \(\epsilon\), the following linearized equations are obtained:
\[ B(q_0) \partial_t q_1 = A(q_0) q_1 \]  

where \( A = \frac{\partial \mathcal{R}}{\partial q} \) designates the linear Navier Stokes differential operator. Since the base flow is homogeneous in the transverse direction \( z \), the perturbation is sought under the form of normal modes

\[ q_1(x, y, z, t) = \hat{q}_1(x, y)e^{(\sigma + i\omega)t + i\beta z} + c.c. \]  

where the real numbers \( \sigma \) and \( \omega \) represent respectively the temporal growth rate and the frequency of the global mode \( \hat{q}_1 \), \( \beta \) is the real transverse wave number and \( c.c. \) stands for the complex conjugate. Substituting (7) in (6) leads to a generalized eigenvalue problem for \( \sigma + i\omega \) and \( \hat{q}_1 \):

\[ (\sigma + i\omega)B(q_0)\hat{q}_1 = A(q_0)\hat{q}_1 \]  

### 3 Flow configuration

In this study, we aim at applying the theoretical notions developed in the previous section to the case of a square cavity \( (\frac{L}{D} = 1) \) in a compressible regime. We chose a Reynolds number of 7500 and different Mach numbers in the range \([0, 0.4]\).

The geometry is described in Figure (1), where \( \Gamma_{in}, \Gamma_{out}, \Gamma_{ext} \) represent respectively the inlet, outlet and upper boundaries and \( \Gamma_a \) and \( \Gamma_w \) are the lower boundaries. The dimensions used are \( l_1 sp = 200, l_2 sp = 250 \) and \( h sp = 183 \). In [2], where an incompressible cavity flow is studied, a confined geometry is considered because there is no acoustics. The governing equations are solved using the following boundary conditions:

\[ u = (1, 0, 0)^T, \rho, T = 1 \text{ on } \Gamma_{in} \cup \Gamma_{out} \cup \Gamma_{ext} \]  

\[ u = 0, \text{ on } \Gamma_w \]  

\[ \partial_y u = 0, v = 0, \partial_y w = 0 \text{ on } \Gamma_a \]  

On the lower wall, a slipping condition is set for \( \Gamma_a \) and a no-slip one for \( \Gamma_w \). This last condition will generate a boundary layer of moderate thickness at the upstream edge.

To avoid possible reflections of acoustic waves in the mesh, two strategies are applied. The first one uses the theory developed by [6]. We add a linear damping coefficient, \( \sigma(x) = \alpha x \), designing thereby an absorbing layer all around the cavity, outside the white area of Figure 1. The real number \( \alpha \) represents the damping rate over a distance \( L \) controlled by a constant \( \xi = -\frac{\alpha L^2}{20} \). In our study, \( \xi = 4 \).

The second strategy is a numerical dissipation imposed by a progressive grid stretching from the cavity to the limits of the domain.

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**Figure 1** – Schematic of the computational domain – the white area is the physical domain of interest, padded into a sponge zone, the grey shaded area.
4 Numerical method

A finite-element method is used to resolve the governing equations (8). Their variational form is spatially discretized using an unstructured mesh composed of triangular elements. The unknowns \((u, \rho, T)\) are spatially discretized using P2-P1 Taylor-hood elements with 6-node P2 elements for the velocity components and 3-node P1 elements for the density and the temperature. The mesh and the sparse matrices resulting from the variational formulation of (8) are generated with the software FreeFem ++ (http://www.freefem.org). All matrix inversions are performed by use of a multifrontal sparse LU solver (MUMPS, see [1]).

To check the convergence of our simulations, different calculations were made using different spatial discretizations: (P2,P2,P2,P2), (P2,P2,P1,P1) and (P1b,P1b, P1,P1). The maximum divergence observed between the eigenvalues is about \(10^{-2}\).

4.1 Base flow

A Newton iteration method is applied to solve the steady Navier-Stokes equations (5), increasing little-by-little the Reynolds number until reaching a value of 7500.

4.2 Stability calculations

Once the base flow is computed, the generalized eigenvalue problem (8) is solved thanks to the "Implicit Restarted Arnoldi method" of the ARPACK library using a shift and invert strategy ([8]). Complex shift parameters have been investigated to obtain the spectrum along the imaginary axis of the \((\sigma, St)\) plane, \(St\) being the adimensional value of the pulsation \(\omega\) of the mode.

5 Results and Discussion

5.1 2D simulations

In this section, we look at bidimensional perturbations, the transverse wave number \(\beta\) is set to zero. Each stability analysis was made by only changing the Mach number. The case \(M = 0\) has been computed with an incompressible version of the set of equations (3).

\[
\begin{align*}
&\text{Figure 2 – (left) } St - M \text{ spectrum for flow over a square cavity at } Re=7500: \text{ the symbols correspond to the unstable modes provided by the 2D stability analysis results, coloured by the temporal growth rate } \sigma. \text{ The thin lines and the thick line indicate the Block formulae (1b), and the East formulae (1c) respectively – (right) Global spectrum representing for each Strouhal the successive modes obtained for Mach numbers within the range } 0 \leq M \leq M_{lim}, \text{ where } M_{lim} \text{ corresponds to the values indicated by arrows in the left figure.}
\end{align*}
\]

Figure 2 shows the \(St - M\) and the \(St - \sigma\) plots of the stability results. The symbols represent the eigenvalues of (8). Let’s remind that the flow is unstable if the growth rate \(\sigma\) is positive. On the first Figure, two sets of curves are represented. The thick line represents the East’s formulae (1c) symbolizing the acoustic resonance
and the thin lines represent Block’s formula (1b) symbolizing the shear-layer oscillations, taking $k_r = 0.61$. The second Figure shows the global spectrum. Different levels of frequencies of the eigenvalues are identified by means of different colors. For sake of clarity, only the first successive eigenvalues (until the arrows on the left Figure). Indeed, we can see on the left Figure that after the arrows the evolution of the eigenvalues is no more monotone. In the left Figure, the location of the eigenvalues with respect to the curves mentioned above is striking. This is the main result of this paper. We first notice that at low Strouhal number, the eigenvalues almost follow the equation (1b). Concerning the equation (1c), our results confirm the East and Block conjecture : we see that to the highest local growth rates (the encircled symbols) are reached when equation (1c) intercepts equation (1b). This is in accordance with the results of [3] mentioned in section (1), supposing that the most amplified mode in term of growth rate is the most “noisy” in term of acoustic amplitude.

Generally speaking, increasing the Mach number stabilizes the Kelvin-Helmholtz instabilities (see [9]). In the case of a cavity, all the unstable modes present in the spectrum are shear layer modes but the compressibility effects look more complex : increasing the Mach number first destabilizes the flow, then reaches a maximum and then stabilizes it. At very low Mach number, the shear layer instabilities are weak compared with acoustics. Then, the first increasing part is due to the acoustic resonance occurring when curves (1c) and (1b) cross each other leading to a maximum growth rate. In the second part, the traditional stabilizing mechanism dominates leading to a stabilization of the flow when increasing the Mach number.

### 5.2 3D stability analysis

In this section, the complete tridimensional form of the global modes (7) is sought. The transverse wavenumber $\beta$ is therefore a parameter of the stability analysis.

![Figure 3 – 3D stability analysis](image)

**Figure 3 – 3D stability analysis.** (left) $\sigma - \lambda/D$ spectrum (right) Strouhal – $\lambda/D$ spectrum. The symbols are results of 3D global stability analysis for $M = 0.3$ (circle), $M = 0.5$ (gradient), $M = 0.7$ (triangle) and $M = 0.9$ (square).

In the Figure (3), two plots are represented. $\lambda$ denotes the spanwise wavelength, linked to $\beta$ by $\beta = \frac{2\pi}{\lambda}$. Various simulations were carried out at different Mach numbers ($M=0.3, 0.5, 0.7$ and $0.9$). Looking at the spectra, no compressible effect is seen, i.e. there is almost no difference between the different Mach numbers simulations. Concerning the growth-rate evolution with $\lambda$ shown in Figure 3 (left), we first observe a destabilization reaching a peak at $\frac{\lambda}{D} \approx 0.25$. The structure of the most unstable modes is centrifugal, as referred in [5]. After that, increasing $\lambda$ totally stabilizes the flow at $\frac{\lambda}{D} \approx 1$.

The same growth rate shape is observed in Brèz [5], who performed many tests for different cavity geometries, Mach numbers and Reynolds numbers. The case $M=0.3$ and $Re=6960$ is the closest to our’s. However, we observe that the growth-rate is ten times lower than that of [5]. A reason could be the difference in the Reynolds numbers.

Note that computations (not shown here) proved that increasing $\beta$ stabilizes the unstable flow obtained by a stability analysis with bidimensional perturbations.
6 Conclusions
In this paper, a global stability analysis has been carried out for bidimensional and three-dimensional perturbations. This new tool confirmed existing works ([7], [3] and [14]) : a maximum growth rate is reached when equation (1c) and (1b) intersect, underlining an interaction between the normal mode mechanism and the feedback mechanism. The novelty here, lies in the fact that both mechanisms are simultaneously accounted for, offering a unified model of flow-induced cavity tones, considering the temporal growth rate as the main parameter of the study.

Références