Identification à temps continu sur structure continue

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Abstract :

The purpose of this study is to show that it is possible to use continuous time identification method with a signal discretized either in space and in time domains. The partial differential equations, and particularly those governing continuous mechanical system behaviour, can be transformed into algebraic equations by using the well known properties of orthogonal functions. Before any identification, it is crucial to calculate data that are not directly available by the measurement. This calculation will be performed thanks to an expansion of the signals into an orthogonal basis. After this calculation, all the data calculated and recorded are expanded into a unique orthogonal basis. After this expansion step, the identification is performed through a very classical Least Square process. To validate the formalism, two different tests will be carried out: one for a bar in longitudinal motion and a second one with a bending beam. This study shows that it is possible to reconstruct quantities not available by a direct measurement and that identification is robust to noise. The proposed methodologies and formulations can be easily extended to other orthogonal functions in association with partial differential transformations.

Keywords: Continuous time identification, space domain, partial differential equation, Chebyshev,

1 Introduction

Continuous Time (CT) methods have been developed to allow identification with discrete-time signals. One of the main applications of this method is to transform the dynamic equations of the system behaviour into an algebraic equation set, generally in order to estimate directly mechanical parameters such as mass, stiffness, damping, etc. Traditionally, a discrete form of these equations is obtained by applying the Z transform on signals when linear assumptions are available. This leads to an equivalent recurrent equation when the signals are sampled at a constant rate and when the Zero Order Holder assumption is assumed. Over the last few decades, alternative methods have been proposed and gathered under the banner of Continuous Time Identification methods (see [1–4]). In these approaches, orthogonal functions are frequently used in an integral formulation of differential equations. Their main advantage is the transform of the integration of signals into a simpler integration of these functions by the use of a square matrix that depends on the chosen orthogonal functions. Therefore, the differential equations governing the behaviour of the system to be identified can be transformed into algebraic equations. In [5], the authors describe several applications that have been developed since the 1990s for identifying controlled systems and MDOF systems. From these results, extension of these approaches can be considered for any other classical behaviour like continuum mechanics. Partial differential equations can be discretized through space domain like differential equation through time domain. Moreover, Remond et al. [6] have dissociated expansion and identification steps leading to more attractive and improved results obtained with Chebyshev polynomials and to propose a more general identification methodology which can be easily extended to above mentioned classical behaviour in continuum mechanics. With the growing interest on sensor networks, it is now possible to record signals discretized in space domain. The purpose of this study is to show that it is possible to apply a continuous time identification method to discrete-space signals and partial differential equations.

In this paper, the general formalism is presented in the second section. A conventional continuous time identification method is first applied to a partial differential equation. Then, an extension of this method in space is derived. In the third section, two numerical benchmarks will be studied. The CT identification method and its extension are used to estimate a ratio between material and geometric quantities. The fourth section presents the result of numerical simulations with noisy data in order to investigate the robustness of the proposed approach.

2 Formalism

In the following, a methodology which permits the transformation of any partial differential equation onto an algebraic equation is described. Firstly, a general partial differential equation is presented. Secondly, the expansion of the signal in an orthogonal basis is considered. Then, all the information calculated in this expansion step is collected in order to obtain a time and space identification tool.

2.1 General partial differential equation

A partial differential equation is an equation involving functions and their partial derivatives such as the equation of motion of a beam. A general partial differential equation involving *h* variables will be taken as a support for this section.

$$\lambda_1 \frac{\partial^{\sigma_1} s}{\partial \chi_1^{\sigma_1}}(\chi_1,\chi_2,...,\chi_h) + \lambda_2 \frac{\partial^{\sigma_2} s}{\partial \chi_2^{\sigma_2}}(\chi_1,\chi_2,...,\chi_h) + ... + \lambda_h \frac{\partial^{\sigma_h} s}{\partial \chi_h^{\sigma_h}}(\chi_1,\chi_2,...,\chi_h) = 0 \quad (2.1)$$

($\lambda_1, \lambda_2,\ldots, \lambda_h$) are constant parameters, ($\sigma_1,\sigma_2,\ldots,\sigma_h$) are the order of differentiation with reference to the first to the h^{th} variable, respectively. The relationship between the displacements and the forces in main structures such as beams can be described using such differential or partial differential equations. In order to operate such a partial differential equation, it is crucial to measure or calculate the partial derivatives of the function *s*. The continuous time expansion is a good solution for this application.

2.2 Expansion of the signal

First, each discretized signal can be expanded in an orthogonal basis, reducing considerably the amount of data needed for the calculation. In order to calculate this expansion following a unique direction χ_i, we can first fix all the variables χ_j except the variable χ_i. This expansion can be written by the following equation:

$$s(\chi_1,...,\chi_i,...,\chi_h) = \sum_{k=1}^{k=M} S_i(k)\phi_k(\chi_i) = \langle S_i \rangle_M \{\phi(\chi_i)\}_M \quad (2.2)$$

where $<S_i>_M$ are expansion constants,$\{\phi(\chi_i)\}_M$ is the regressor and the (χ_j) variables are fixed except for j=i. For a given variable χ_i, we can describe the signal as an expansion of M orthogonal functions ϕ. Equation (2.2) can be rewritten for *n* different positions of the variable χ_i=(χ_{i1}, χ_{i2},..., χ_{in}), in order to calculate the constants $<S_i>$:

$$\begin{Bmatrix} s(\chi_{i1}) \\ \vdots \\ s(\chi_{in}) \end{Bmatrix} = \langle S_{i1} \quad ... \quad S_{iM} \rangle \begin{bmatrix} \phi_1(\chi_{i1}) & ... & \phi_1(\chi_{in}) \\ \vdots & \ddots & \vdots \\ \phi_M(\chi_{i1}) & \dots & \phi_M(\chi_{in}) \end{bmatrix} = \langle S_i \rangle_M \left[\phi(\chi_i)\right]_{M\times n}\Big|_{\chi_{j\neq i}\ fixed} \quad (2.3)$$

The final notation in (2.3) will be used in the following as a convention.

2.3 Calculation of the partial derivatives

A second advantage is that using this expansion it is easy to estimate the derivative of the signal, as the orthogonal functions ϕ and their derivatives are known. Then, the vector $<S_i>_M$ estimated with (2.3) is the only information needed to perform this calculation:

$$\left\{\frac{\partial^{\sigma_i} s}{\partial \chi_i^{\sigma_i}}(\chi_i)\right\}_n = \langle S_i \rangle_M \left[D\right]^{\sigma_i}_{M\times M} \left[\phi(\chi_i)\right]_{M\times n}\Big|_{\chi_j \neq \chi_i\ fixed} \quad (2.4)$$

with $<S_i>_M$ the constants given by the expansion process, [D] the derivative matrix related to the orthogonal functions.

2.4 Multi-directional identification tool

The two previous steps show that the only data needed for the estimation of the partial derivatives of a function is the function itself at different "locations" in the considered *h* directions: [χ_1=(χ_{11}, …, χ_{1n}), …, χ_h=(χ_{h1}, …, χ_{hn})]. In order to reduce equation (2.1) onto an algebraic equation, an expansion in a single direction (and a single orthogonal basis) can be performed. Let consider one direction noted χ_C for this reduction. All the partial derivatives calculated with equation (2.4) can be considered as input data.

Furthermore, the iteration of the section 2.3 allows the calculation of a given partial derivative at different locations. All the data needed can be rearranged as follow:

$$\left\{ \frac{\partial^{\sigma_i} s}{\partial \chi_i^{\sigma_i}}(\chi_C) \right\}_n = \left\{ \frac{\partial^{\sigma_i} s}{\partial \chi_i^{\sigma_i}}(\chi_{C1}) \quad \cdots \quad \frac{\partial^{\sigma_i} s}{\partial \chi_i^{\sigma_i}}(\chi_{Cn}) \right\}_n^T \Bigg|_{\chi_i \neq \chi_C \text{ fixed}} \tag{2.5}$$

This calculated input data is directly expanded into an orthogonal basis of variable χ_C:

$$\left\{ \frac{\partial^{\sigma_i} s}{\partial \chi_i^{\sigma_i}}(\chi_C) \right\}_n = \left\langle S_{\sigma_i,C} \right\rangle_M [\phi(\chi_C)]_{M\times n} \tag{2.6}$$

The σ_i partial derivatives of *s* were calculated in step 2.3. The equation (2.1) can be transformed into the following algebraic equation:

$$\left\langle S_{\sigma_1,C} \right\rangle_M + {}^{\lambda_2}\!/_{\lambda_1} \left\langle S_{\sigma_2,C} \right\rangle_M + \ldots + {}^{\lambda_h}\!/_{\lambda_1} \left\langle S_{\sigma_h,C} \right\rangle_M = 0 \tag{2.7}$$

From this algebraic equation we can deduce the *(h-1)* constants (λ_2/λ_1, …, λ_h/λ_1).

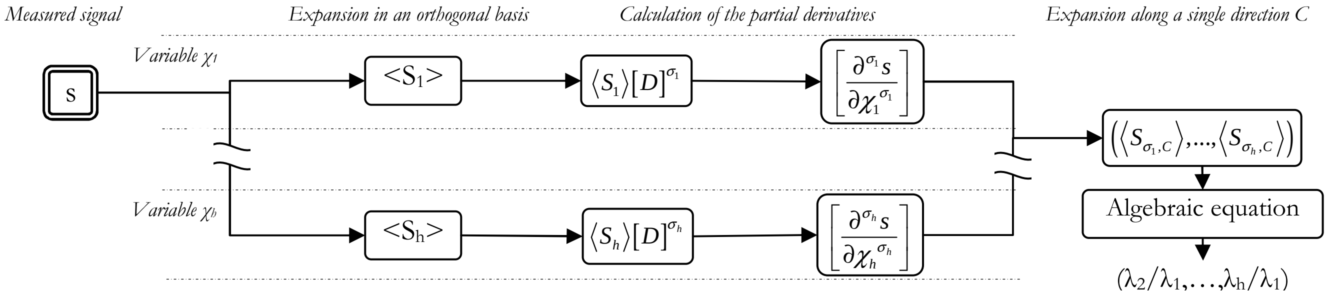

FIG. 1 : Summary of the multi-directional identification tool

3 Test cases

The structure studied in this paper is a cantilever beam clamped at one edge (x=0) and free at the other boundary (x=L). The first case is concerned with longitudinal motion *u* of this beam which can be described with an analytic solution. For the second test, the transverse displacement *v* corresponding to the flexural motion of the beam is calculated at different positions noted x_i and at different time t_j, with a Finite Element model. Such models are based on a discretization of the beam which gives directly and precisely displacements v at nodes of the discrete geometry.

3.1 Partial differential equations

The two different examples studied in this paper involve different levels of differentiation and non dispersive and dispersive waves. Therefore these two beams are good examples in order to estimate the ability of this method to estimate partial derivatives and to solve parameter identification problems.

The classical partial differential equation of a bar in longitudinal motion without any external excitation involves the partial derivatives of the displacement *u*. Similarly the Euler-Bernouilli equation, defining the behaviour of a bending beam without external excitation, can be expressed as the sum of the partial derivatives of the displacement *v*.

$$\rho S \frac{\partial^2 u}{\partial t^2} = ES \frac{\partial^2 u}{\partial x^2} \ ; \ \rho S \frac{\partial^2 v}{\partial t^2} = -EI \frac{\partial^4 v}{\partial x^4} \tag{3.1}$$

where ρ is the density, *E* the Young Modulus of the material, *S* the cross-section area and *I* the flexural inertia of the beam.

3.2 Application of the multi-directional identification methodology

In order to apply the methodology presented before to these two test cases, we will choose the directions: χ_1=t, the time and χ_2=x the position along the beam.

3.2.1 Estimation of time derivatives

Firstly, the expansion along the time is expressed as in (2.3). As this step is common to both examples, this expression will be derived only with the longitudinal displacement *u*.

$$\left\{u(t)\right\}_n = \left\langle U_t \right\rangle_M \left[\phi(t)\right]_{n\times M}\Big|_{x\ fixed} \tag{3.2}$$

where ϕ is a function of an orthogonal basis of dimension *M*, and $<U_t>$ constants for a given position x_i and for different time *t*. For the bending motion, this expression can be extended by replacing *u* by *v*, $<U_t>$ by $<V_t>$. Using (2.4) and (3.2) the acceleration at a considered position *x* for different time *t* can be estimated by:

$$\left\{\frac{\partial^2 u}{\partial t^2}(t)\right\}_n = \left\langle U_t \right\rangle_M \left[D\right]^2_{M\times M} \left[\phi(t)\right]_{n\times M}\Big|_{x\ fixed} \tag{3.3}$$

with $<U_t>$ constants calculated with (3.2), [D] the matrix of derivation and [ϕ] the values of the *M* orthogonal functions at time locations $t=(t_1, \ldots, t_n)$. This first step permits the calculation of the acceleration of *u* and *v* at different time *t* and can be repeated at different location *x*.

3.2.2 Estimation of space derivative

The *M* coefficients for the expansion in space are given by:

$$\left\{u(x)\right\}_q = \left\langle U_x \right\rangle_M \left[\phi(x)\right]_{q\times M}\Big|_{t\ fixed} \tag{3.4}$$

ϕ are functions of an orthogonal basis of dimension M, and U_x constants for a given time *t* and different locations *x*. Using (2.4) and (3.4), we are able to express the second derivative of *u* in space. For the beam in bending motion, the main change is the following: instead of estimating the second partial derivative, the fourth partial derivative in space is needed in order to estimate (3.1).

$$\left\{\frac{\partial^2 u}{\partial x^2}(x)\right\}_q = \left\langle U_x \right\rangle_M \left[D\right]^2_{M\times M} \left[\phi(x)\right]_{q\times M}\Big|_{t\ fixed} \quad ; \quad \left\{\frac{\partial^4 v}{\partial x^4}(x)\right\}_q = \left\langle V_x \right\rangle_M \left[D\right]^4_{M\times M} \left[\phi(x)\right]_{q\times M} \tag{3.5}$$

where $<U_x>$ are constants calculated in (3.4), [D] the matrix of derivation and [ϕ] the values of the *M* orthogonal functions at $x=(x_1., \ldots. , x_q)$.

3.2.3 Expansion in a single direction

In order to reduce these partial differential equations (3.1) to algebraic equation, it is necessary to express both partial derivatives in the same direction. For this case study, the space direction will be selected: $\chi_C = x$.

If the expansion in time is chosen, the information of only one sensor is used for the identification. With an expansion in space, the information of multiple sensors is averaged by a least square method, as explained in the following section. Using (3.3), we can calculate for all x_i and at a given time $t=t_j$ the acceleration. Then it is possible to calculate for all x the acceleration on the orthogonal basis as follow.

$$\left\{\frac{\partial^2 u}{\partial t^2}(x)\right\}_N = \left\langle U_{\sigma_t=2,x} \right\rangle \left[\phi(x)\right]\Big|_{t=t_j} \tag{3.6}$$

with $\left\langle U_{\sigma_t=2,x} \right\rangle$ the coefficients of the acceleration at time t_j expanded along x. For the calculation of the acceleration in bending motion, the previous expression is reused replacing only u by v, $\left\langle U_{\sigma_t=2,x} \right\rangle$ by $\left\langle V_{\sigma_t=2,x} \right\rangle$.

3.3 Results of the continuous identification process

Previous derivations show a priori the potential of this method: firstly it is easy to calculate the derivatives of a discretized signal; secondly a partial differential equation can be reduced to an algebraic equation. An example of application of these two advantages will be discussed in the following sections.

3.3.1 Partial differential equation reduction

Then, using (3.1), (3.5) and (3.6) , we can calculate ρ/E in a Least Square sense. Similarly, using(3.1), (3.5) and (3.6) , we can estimate $\rho S/EI$ by using Least Square algorithm, leading to next formula.

$$\rho/_E = -\frac{\langle U_x \rangle [D]^2}{\langle U_{\sigma_t=2,x} \rangle} \quad ; \quad \rho S/_{EI} = -\frac{\langle V_x \rangle [D]^4}{\langle V_{\sigma_t=2,x} \rangle} \tag{3.7}$$

4 SIMULATIONS

For the calculations, a Finite Element model is used as a reference offering estimation of displacement in a discrete manner. Two beams, with cross-section of 1.2 10^{-4}m² and flexural inertia of 9 10^{-11} m^4, density of 3900 kg/m^3 and Young Modulus of 300 10^9 N/m² are studied. Each beam is respectively 0.2m, and 0.4m long. A discretization with 100 elements has been used for the calculation. Additive and multiplicative noise are mixed to the signals with a signal to noise ratio of 26 dB. The performances of proposed method are evaluated by a Monte Carlo simulation with 1000 realizations. For this investigation, the response of the two beams at different wavelengths is studied.

4.1 Simulation parameters

Before any identification, it is necessary to determine the optimal numbers of orthogonal functions M, of sensors q located along x dimension and time samples m. Equations (3.1) make the classical assumption that the time and space responses are completely uncoupled. Therefore, the frequency responses in time and space are strongly related to each other. High frequency response presents small time period and small wavelengths. For this exploratory research, we will chose the same number of samples and sensors, leading to m=q. The m samples will be concentrated into two periods of the signal.

In order to calculate the optimal expansion, the number and positions of samples and sensors will be chosen as the Gaussian points of Chebyshev functions, which ensure the convergence for any continuous function that satisfies a Dini-Lipschitz condition [9]. The size of the orthogonal basis will be chosen as M=16 in regard to the low frequency response of the structure. As results, we study the ratio between the estimated parameter via the CT identification method (respectively ρ/E and the axial force for the longitudinal bar, and ρS/EI and the bending moment for the bending beam) and the theoretical parameter values.

4.2 Estimation of the material/ geometric quantities ratio

The error obtained with and without noise for a bar in longitudinal motion and a bending beam is represented in FIG. 2. Even if the size of the orthogonal basis is arbitrarily chosen and fixed (M=16), the expansion is sufficiently well calculated to obtain accurate results. Without noise, the ratio between the calculated and theoretical parameter is equal to one. The evaluated and theoretical parameters are strictly equal at different wavelengths. For the simulation with noise, the wavelength in other words the excitation frequency becomes a crucial tuning parameter for an optimal identification. For the bar in longitudinal motion, the identification results of the two tested beams (0.2 and 0.4m long) are similar in regard to the wavelength. Indeed, for each beam, the best identification results are obtained for k=1.

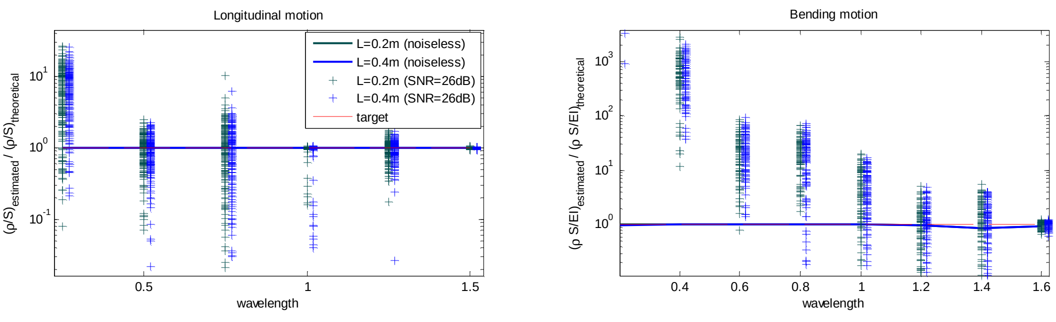

FIG. 2 : estimated over theoretical parameter, with and without noise, for a beam 0.2 and 0.4m long, and for a bar in longitudinal motion and a beam in bending motion

For the beam in bending motion, the identification begins to be accurate for k>1. The best results are obtained for k=1.5. The ratio for all the realizations are concentrated around 1, expect for a few singular points. In order to calculate the parameter as accurate as possible, it will be interesting to estimate the mean value with a large number of realizations, eliminating the realizations which appear erroneous. The results of this calculation are presented in TABLE 1.

	Theoretical	Mean value	Mean value without the erroneous realizations
Longitudinal motion ρ/E (s²/m²) (k=1)	L= 0.2m : 1.3 10^{-8} L=0.4m : 1.3 10^{-8}	L= 0.2m : 1.2662 10^{-8} L=0.4m : 1.2668 10^{-8}	L= 0.2m : 1.2982 10^{-8} (99.8%) L=0.4m : 1.2963 10^{-8} (99.7%)
Bending motion $\rho S/EI$ (s²/m^4) (k=1.5)	L= 0.2m : 2.0222 10^{-2} L=0.4m : 2.0222 10^{-2}	L= 0.2m : 2.22 10^{-2} L=0.4m : 2.4 10^{-2}	L= 0.2m : 1.73 10^{-2} (85.6%) L=0.4m : 1.72 10^{-2} (86.5%)

TABLE 1 : theoretical and estimated parameter with SNR=26dB

These results show that the identified parameter can be estimated accurately with noise, for the bar in longitudinal motion (error less than 1%) and a bending beam (error less than 14%). The difference between these two examples could be explained -in the case of the bending beam- by the estimation of the fourth derivative, which enhances the noise effect and the dispersive waves, of exponential form, which are hard to describe with the Chebyshev functions.

5 Conclusion

This paper considers a single challenge: the ability to calculate accurately geometric and material parameters. The proposed identification method is based on a classical Continuous Time identification theory and uses the partial differential equation of motion of continuous structures. The approximated derivation operator has been detailed both in the time and the space domains. The proposed process is clearly segmented into a signal expansion step and an identification step performed through a classical Least Square method. The main advantage of such a method is the high integration capability in structures, for various applications as self checking or Structural Health Monitoring.

Improvements of the proposed method can be investigated with the determination of a precise basis size and also as Garnier has proposed in [10]. As the least square method suffers from bias, it is possible to introduce instrumental variable method in order to improve identification step.

This exploratory research offers perspectives as a tool for identification on continuous structures towards the main direction of monitoring such structures in the framework of sensor networks.

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