The Optimum Level for Wall Curtailment in Wall-Frame Structures to Resist Lateral Loads

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Abstract:
Wall-Frame Structures have been widely used to resist lateral loads, when a wall-frame structure is loaded laterally the upper part of shear wall takes negative role in resist this loads because of the difference in the free deflected forms of the wall and the frames, the optimum level for curtailment the shear wall has been discussed and the curves which specify this level and which proposed by (Nollet and Stafford Smith, 1993) have been corrected. The most important results in this research are: the interruption of the shear walls at the optimum level for which the top deflection of the curtailed wall-frame is a minimum eliminates the reverse force and this level of curtailment always lies between the point of inflection and zero wall shear in the corresponding full-height wall structure.

Introduction:
Wall-Frame Structures have been widely used to resist lateral loads for buildings up to 60 stories (Stafford Smith and Coull 1991). When a wall-frame structure is loaded laterally [Fig.1], the different free deflected forms of the wall and the frames cause them to interact horizontally through the floor slabs: consequently, the individual distributions of lateral loading on the wall and the frame may be very different from the distribution of the external loading.

Referring to the distribution of bending moment for the full-height-wall structure [Fig. 2(b)]; the wall moment in the region above the point of inflection, where \( \frac{d^2y}{dx^2} = 0 \), is opposite in sense to the external load moment, while the moment in the frame (which is carried mainly by axial forces in the columns) is actually greater than the external load moment. Therefore, if the wall were curtailed anywhere in the region above the point of inflection, the moment carried by the frame would be reduced to become equal to the external moment.
Similarly, for the distribution of the shear force in the full-height-wall structure [Fig. 2(c)], the shear in the wall above the point of zero shear, where \( \frac{d^2y}{dx^2} = 0 \), is opposite in sense to the external load shear, while the shear in the frame exceeds the external shear. Therefore, if the wall were curtailed anywhere in that uppermost region, the shear in the frame would be reduced to become equal to the external load shear.

Fig.2 (a) Typical deflection diagram of laterally loaded wall-frame structure; (b) Typical moment diagrams for components wall-frame structure; (c) Typical shear diagrams for components wall-frame structure

An inspection of [Figs. 2(b) and 2(c)] shows that, if the wall were curtailed between the points of zero shear and inflection, the shear in the frame above the curtailment level would be increased while the moment in the frame above that level would be reduced. In order to investigate the effects of curtailment in more detail, and to determine the optimum level of curtailment, a mathematical solution for a continuum model of the curtailed structure is displayed.

**Continuum analysis of curtailed Wall-Frame Structure:**

In wall-frame structures that are plan-symmetrical about the axis of loading, and therefore do not twist, the high in-plane stiffness of the floor slabs causes the lateral deflections of the walls and frames to be effectively identical. Consequently, the structure can be represented by a planar model. Under these conditions a mathematical solution for a continuous medium, or "continuum," model of the curtailed wall-frame structure has been developed by (Nollet and Stafford Smith, 1993) as follows:

A curtailed wall-frame structure of total height \( H \), subjected to a uniformly distributed horizontal load, \( w \) [Fig. 3(a)], can be considered as the superposition of two substructures [Fig. 3(b)]:

- The lower substructure, substructure 1, is a wall-frame structure of height \( H_1 \), subjected to the external uniformly distributed lateral load, \( w \), a top concentrated shear force, \( S_1 \), and a top moment, \( M_1 \), where \( S_1 \) and \( M_1 \) = the accumulated shear and moment from the upper region acting on the lower region.
The upper substructure, substructure 2, is a moment-resisting frame of height $H_2$, subjected to only the external uniformly distributed load $w$. Therefore, the lateral deflection at any height $x$ of the whole structure is:

For $x \leq H_1$, \[ y(x) = y_1(x) \]
And, for $x > H_1$, \[ y(x) = y_1(H_1) + y_2(x_2) \]
Where $y_1(x)$: The total lateral deflection at the height $x$ of the Substructure1 $y_2(x_2)$: The lateral deflection at the height $x_2$ from the base of the Substructure2. These values can be found by solution the equations which govern the continuum model of the curtailed wall-frame structure (Nollet and Stafford Smith, 1993)

**Solution for Substructure 1**

For uniformly distributed lateral load, $w$, the lateral deflection at any height $x$ of the Substructure1 is:

$$y_W(x) = \frac{WH_1^4}{EI} \left( \frac{k^2 - 1}{k^2} \left( \frac{1}{4} \left( \frac{x}{H_1} \right)^2 - \frac{1}{6} \left( \frac{x}{H_1} \right)^3 + \frac{1}{24} \left( \frac{x}{H_1} \right)^4 \right) + \frac{1}{k^2} \left[ \frac{2}{(H_1/H_2)^2} \left( \frac{x}{H_1} - \frac{x}{H_2} \right)^2 \right] \right)$$

$$+ \frac{WH_1^4}{EI} \left( \frac{1}{k^2} \left[ \frac{\cosh kx - 1}{(kH_2)^4 \cosh kH_1} \right] - \frac{\sinh kx}{(kH_2)^3} \right)$$

For the top concentrated shear force, $S_1$, the lateral deflection at any height $x$ of the Substructure1 is:
For the top moment, \( M_1 \), the lateral deflection at any height \( x \) of the Substructure 1 is:

\[
y_M(x) = \frac{MH_1^2}{EI} \left( \frac{x}{H_1} \right)^2 - \frac{(\cosh k_0 x - 1)}{(k_0 H_1)^2 \cosh k_0 H_1}
\]

Therefore the total lateral deflection at any height \( x \) of the Substructure 1 can be expressed by

\[
y_1(x) = y_W(x) + y_S(x) + y_M(x)
\]

Where: \( \xi_1 = \frac{H_1}{H} \).

\[\alpha^2 = \frac{(GA)}{EI}; \text{where} \ (GA) \text{ is the total racking shear rigidity of the set of frames.} \]

The racking shear rigidity of a single uniform frame \( i \) is given by (Stafford Smith et al. 1981):

\[\frac{(GA)_i}{EI} = \frac{12E}{h\left(\frac{1}{l_i^c} + \frac{1}{l_b^c}\right)}\]

In which \( E = \) the modulus of elasticity; \( h = \) the story height; \( \sum I_c = \) the sum of the inertias of the columns in a story of frame \( i \); \( l_b = \) the inertia of a beam and \( l \) its span, \( \sum l_b \) being summed for all the beams in a single floor of frame \( i \).

\[EI = EI_c + EI_w; \ I_c = \text{the inertias of the columns,} \ I_w = \text{the inertias of the walls}
\]

\[k^2 = \frac{EI + E\Sigma A_c^2}{E\Sigma A_c^2}; \Sigma A_c^2 = \text{Second moment of area of the column sectional areas about their common center of area.}\]

\[\text{Solution for Substructure 2}\]

For uniformly distributed lateral load, \( w \), the lateral deflection at any height \( x_2 \) from the base of the Substructure 2 is:

\[
y_2(x_2) = \frac{WH_2^4}{EI} \left[ \frac{(x_2 - \frac{H_2}{2})^2}{2(x_2 H_2)^2} \right] \left( 1 - \xi_1 \right)^2 + \frac{WH_2^4}{EI} (k^2 - 1) \left\{ 1 - \left[ \frac{\left( x_2 H_2 \right)^3}{24} - \frac{\left( \frac{H_2^3}{3} \right)^3}{6} + \frac{\left( \frac{H_2^3}{4} \right)^4}{4} \right] \right\} + \frac{WH_2^3 (k^2 - 1)}{6} \left( 3\xi_1^2 - 3\xi_1^3 + \xi_1^4 \right) \left( k^2 - 1 \right) y'(H_1)
\]

\[
\phi_{H_1} x_2
\]

\[
\phi_{H_1} = \frac{WH_2^3 (k^2 - 1)}{6} \left( 3\xi_1 - 3\xi_1^2 + \xi_1^3 \right) \left( k^2 - 1 \right) y'(H_1)
\]

\[
y'(H_1) = \frac{WH_2^3 (k^2 - 1)}{K^2} \left[ \frac{\xi_1 - \xi_1^2}{2} + \frac{\xi_1^2 - (1 - \xi_1)^2}{6} \frac{\tanh \xi_1 k_0 H}{(k_0 H)^2} \right] + \frac{WH_2^3 (1 - \xi_1)}{EI} \left[ \frac{1}{(k_0 H)^2} - \frac{1}{(K_0 H)^2} \right] \tanh \xi_1 k_0 H
\]

Where \( \phi_{H_1} \) represents the slope at the top of substructure 1 resulting from axial deformation of the columns where the slope of the frame, which at any level is equal to that of the wall, consists of a component due to racking just at that level, and a component due to axial deformations of the columns that accumulates from the base [Fig.4]
Fig. 4 Components of Drift: (a) Story Drift of Frame due to Racking; and (b) Story Drift of Frame due to Axial Deformation of Columns

The Optimum Level for Curtailment:
It is well known that in the beam element the maximum (or minimum) moment (mathematically, the local maxima or minima) corresponds to the zero point of the shear (mathematically, the zero point of the first derivative), so the level of curtailment that leads to delete the negative shear in the wall by making it equal to zero at the top of the wall, at the same time it makes the minimum moment (local minima) also in the top of the wall, but the moment in the top of the wall is equal to zero (according to the end conditions), therefore the moment over the entire height of the wall will be positive, in other words, the level of curtailment which leads to delete the negative shear is the same that leads to delete the negative momentum, as the result, the interruption of the shear wall at this level eliminates the reverse force applied by the wall on the frame, therefore the top deflection of the structure will be minimum (as it is illustrated by this research), this discussion lead to consider the optimum level of curtailment that which results in the minimum top deflection of the structure.

In order to determine the level of curtailment for which the top deflection of the curtailed wall-frame is a minimum (Nollet and Stafford Smith, 1993), the aforementioned expression for the top deflection y(H) of the curtailed structure, should be minimized. If the top deflection is defined as the function $F(\xi_1)$, this is a minimum when its derivative, $F'(\xi_1)$, is zero: $F'(\xi_1) = 0$; this equation is an implicit function of $\xi_1$; therefore an iterative process must be used to solve it. A program based on the Newton-Raphson algorithm was written to obtain the value of $\xi_{\text{opt}}$, the optimum level of curtailment, for which the top deflection of the curtailed wall-frame is a minimum for various values of the parameters $\alpha H$ and $k^2$. Curves of $\xi_{\text{opt}}$, with respect to the characteristic parameter $\alpha H$, for values of $k^2$ from 1.0 to 1.2 are shown in Fig. 5.

It is evident from Fig. 5, that for most of the range of values of $\alpha H$, the optimum level of curtailment generally lies between the points of inflection and zero wall shear in the corresponding full-height wall structure, regardless of the value of $k^2$. But there is some values which do not achieve this rule, for example: the values which concern the curve of $k^2 = 1.2$ and starting from the value of $\alpha H = 12$, in spite of existing negative moment and negative shear force in the wall, there is not necessary to curtail the wall; ($\xi_{\text{opt}} = 1$).
This observation have necessitated checking these curves, therefore it has been resolved the equation $F' (\xi) = 0$ and redrawn the curves of $\xi_{opt}$ as shown in [Fig.6]. Where the expression for the top deflection $y(H)$ of the curtailed structure has been written as follows:

$$y(H) = F (\xi) =$$

$$\frac{WH^4 (k^2 - 1)}{EI k^2} \left[ \frac{z^2_1}{4} - \frac{z^3_1}{6} + \frac{z^4_1}{24} - \frac{(1 - z_1^2)}{2} \frac{(\cosh \xi_1 k\alpha H - 1)}{(k\alpha H)^2 \cosh \xi_1 k\alpha H} \right] + \frac{WH^4}{EI} \frac{1}{k^2} \frac{(2 \xi_1 - z_1^2)^2}{2(k\alpha H)^2} - \frac{\tanh \xi_1 k\alpha H}{(k\alpha H)^3} +$$

$$\frac{(\cosh \xi_1 k\alpha H - 1)}{(k\alpha H)^4 \cosh \xi_1 k\alpha H} + \frac{WH^4}{EI} \left[ \frac{(1 - z_1^2)^2}{2(aH)^2} + (k^2 - 1) \frac{(1 - z_1^2)}{8} \right] + \frac{WH^4 (k^2 - 1)}{6} \left[ (1 - \xi_1) \left( 3 \xi_1 - 3 \xi_1^2 + \xi_1^3 \right) - \frac{WH^4 (k^2 - 1)}{K^2} (1 - \xi_1) \left[ \frac{z^2_1}{2} - \frac{z^3_1}{2} + \frac{z^4_1}{6} - \frac{(1 - z_1^2)^2}{2} \frac{\tanh \xi_1 k\alpha H}{(k\alpha H)^3} \right] \right] -$$

It is evident from the previous curves, Fig. 6, that:

- The optimum level of curtailment always lies between the point of inflection and zero wall shear in the corresponding full-height wall structure, regardless of the value of $k^2$. 

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Fig. 6. Location of Corrected Optimum Level for Curtailment

**Conclusion:**
In many cases, the shear walls in the upper part of wall-frame structures take negative role in resisting the lateral loads, the interruption of the shear walls at the optimum level for which the top deflection of the curtailed wall-frame is a minimum eliminates the reverse force which applied by the walls on the frames, this level of curtailment always lies between the point of inflexion and zero wall shear in the corresponding full-height wall structure.

**References:**


