Mechanical instabilities of swelling gels as a model for soft-tissue growth

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A gel is a liquid trapped within a diluted mesh of cross-linked polymer chains. The systematic study of gels, started in the 1940s, takes undoubtedly its roots within the field of physico-chemistry. Indeed, as a result of the complex interactions between the polymer chains and the solvent molecules, gels exhibits a volume phase transition during which they may absorb several thousands times their own weight in solvent in response to a tiny change in environmental conditions. Consequently, gels have initially been a subject of choice for the development of the theory of phase transitions, along with polymer chains and solutions. Thanks to fascinating studies from the 1970s and the following two decades, the interest for gels has flourished and they progressively became an important object in the mechanics of polymeric materials and volumetric growth. As it is well known, a growth process opens the door to the possibility of self-organization and the emergence of spatio-temporal patterns. Indeed, because the association of a solvent with a polymer network exhibits a rather complex mechanical behaviour, sharing properties of both liquids and solids, unique features were observed during the swelling process of gels thanks, among others, to the pioneering work of Toyoichi Tanaka.

Despite the considerable amount of work performed until the 1990s, some problems, such as the formation of the creasing instability, one of the hallmark of gel swelling, had not been fully understood. Recently however, highly controlled experiments have brought new results, shining light on pattern formation in swelling gels and allowing for detailed comparisons with theoretical and numerical models. Hydrogels are of particular interest because they have the advantage of being accurately monitorable. However it is yet unknown how closely swelling hydrogels can mimic growing biological bodies. How the microstructural differences between a network of polymers and a soft tissue affect the mechanical stability? Does the microscopic mechanisms underlying the volume variation affect the process of morphogenesis? Our goal is to clarify those aspects and precise the use of hydrogels as substitutes for growing soft tissues. For this purpose, specific models describing the variation of mass in each of those systems must be chosen.

A swollen hydrogel is a system at mechanical and thermodynamical equilibrium, a situation that contrasts markedly with that of a biological body. In addition, the swelling from a dry state does not involve any production of its elastic constituents, but rather a migration of solvent molecules inside a highly deformable elastic network, again in opposition with biological growth. In order to describe the swelling process, a poroelastic model has recently been proposed for the swelling of gels immersed in a solvent reservoir. The model introduces two phases, the solvent molecules and a highly compressible polymer network, in thermodynamic equilibrium with a reservoir of solvent particles. At equilibrium the chemical potential inside the gel equates that of the solvent.
On the contrary, volumetric growth is more simple and tractable analytically. Volumetric growth in soft tissues induces elastic stresses in at least three well identified cases: inhomogeneous and/or anisotropic growth, and incompatible boundary conditions. Then patterns result from the minimization of the elastic energy. They can be understood with the tools of bifurcation theory. Moreover, since soft-tissues are very often considered as incompressible, the use of volume-preserving transformations allows the elimination of the tensorial character of the equations of nonlinear elasticity, leading the possibility of deriving exact results.

To illustrate these two different formalisms, I will consider two typical experiments. First the experiment of Tanaka, where a thin layer of gel swells but remains attached to a solid substrate. The second concerns the swelling of a ring enclosing an elastic disc which cannot swell. Both experiments exhibit a bucking pattern for enough swelling, then creases appear. Both theory (constant volumetric growth and poroelastic model) can be tested on these two cases. It turns out that swelling at constant chemical potential induces elastic instabilities for lower elastic strains but physically, there is no major differences. Both models require a regularization at threshold as predicted by Biot. Biot's interpretation of this rather peculiar feature is that the wavelength of the instability is selected by effects occurring on scales much smaller than the thickness of the layer. Indeed, it has been found recently that several small-scale effects (such as the presence of a diffusive layer or surface tension) can regularize the selection of the wavelength which then scales as the thickness of the growing layer corrected by a logarithmic factor involving the small length-scale.

Finally one can wonder what will be the best model for growth of biological tissues. The behaviour of biological objects are undoubtedly the ultimate example of complex soft matter. Interestingly, many growing biological objects possess a gel-like structure and recent advances in developmental biology require the development of sophisticated models able to describe the concurrent mass production and reorganization processes that enable the reproducible generation of shapes in living objects. Although growing and swelling are obviously distinct processes, the fundamental understanding of mass reorganization in swelling gels is providing important tools to describe the more complex processes involved during biological growth. Gel swelling is spatially inhomogeneous, presents spontaneously “stress inhibition”, both features being inherent to biological growth. On the other hand it cannot mimic anisotropic growth. Our work aims at clarifying the distinctions between biological growth and swelling, but should also provide important results for the growth (biological or not) of stratified media.