Fluid Bearing Failure Probability Evaluation

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Résumé :

Cet article présente une démarche permettant d'évaluer la probabilité de défaillance d'un palier fluide, un composant mécanique primordial utilisé dans la conception de machines tournantes, de systèmes mécatroniques et de bancs de mesure de haute précision. Le comportement statique et dynamique d'un palier fluide dépend de plusieurs paramètres tels que la charge à supporter, les dimensions du palier, la pression d'alimentation, la qualité de fabrication des surfaces du palier et les propriétés du lubrifiant. Dans cet article, on présente l'approche adoptée pour l'approximation de la fonction de performance d'un palier en utilisant un Plan d'Expériences numériques pour estimer par FORM (First Order Reliability Method) et la Simulation de Monte Carlo, la probabilité de défaillance d'un palier fluide.

Abstract :

This paper presents a methodology for the failure probability evaluation of a fluid bearing which plays a significant role in stability of machines rotors, mechatronic systems and high precision metrology systems. The static and dynamic behavior of a fluid bearing depends on several parameters such as external load, the dimensions of the bearing, the supply pressure, the manufacturing capability and fluid properties. In this paper, we present how the failure probability of a fluid bearing is evaluated using Monte Carlo Simulation and FORM (First Order Reliability Method) after the approximation of the performance function by a numerical Design Of Experiments (DOE).

Key Words : Fluid Bearing, Finite Element Method, Design Of Experiments (DOE), Stochastic Response Surface Method (SRSM), Monte Carlo Simulation (MCS), First Order Reliability Method (FORM)

1 Introduction

Fluid bearings are critical components for machines rotors, mechatronic and metrology systems. The design approach of a fluid bearing is usually based on deterministic static characteristics. However, it is subjected to load and pressure fluctuations or to fluid film gap perturbations induced by defects of the slideways surfaces geometry and of the type of supply inlet (groove, orifice, pocket, etc.). These factors induce excitations in the bearing dynamic response; which may eventually lead to bearing instability. The prediction of the reliability of a fluid bearing under operating conditions is then necessary for applications requiring high accuracy movements or positioning within a micrometer to nanometer repeatability.

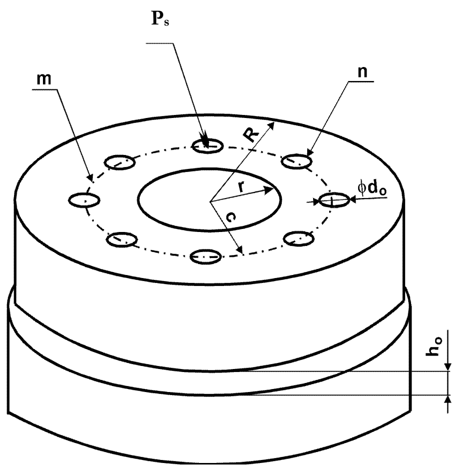
The literature is sparse regarding research that investigates the reliability estimation of such mechanical systems specially those involving fluid flow. Gorla [1] conduct a probabilistic study of fluid interaction for a combustor liner which takes into account several uncertainties in the aerodynamic, structural, material and thermal factors.

In reliability analysis, the performance function (which is commonly defined as stress-strength limit performance function in structural reliability analysis, Lemaire [2]) should be defined precisely for a fluid bearing. Different approaches can be used to define the limit performance function for a fluid bearing. For example, reliability is studied based on a maximum load capacity of the bearing that results in the smallest film gap possible. Alternatively, reliability is studied based on a given load capacity for a fixed film gap higher than that of the corresponding maximum load capacity. The first case is more of interest because of

the instability of the bearing may occur around the maximum load capacity [3] which immediately leads to the failure of the system supported by the bearing.
In this article, we propose to evaluate the failure probability of a fluid bearing using a Design Of Experiments (DOE) followed by a Monte Carlo Simulation and FORM. The performance function of the fluid bearing chosen for the example is evaluated thanks to the calculation of the pressure distribution obtained by Finite Element Method (FEM).

2 Fluid Bearing Performance Function

The performance of a thrust bearing depends on the choice of the different parameters presented in figure 1. Several approximate analytical approaches of the behaviour of fluid bearings have been presented in literature. Frêne [4] and Bassani [5] develop analytical models to analyze the influence of geometric parameters on the stability of aerostatic bearings.



R - outer radius (m)
r - inner radius (m)
c - radius of row of orifices (m)
d_o - diameter of the orifice (m)
n - number of orifices in the row
m - number of rows of orifices
h_o - film gap (m)
P_s - absolute supply pressure (Pa)
C_d - coefficient of discharge
Feeding type - inherent orifices/ groove /pocket

FIG.1 - Bearing Details with Orifices

Stiffler [3] provides a theoretical analysis of a thrust bearing with inherently compensated orifices by using a small perturbation of the Reynolds equation and concludes that an unstable range occurs when the stiffness is maximum. Bonis and Charki [6] analyzes the stability of thrust fluid bearings fed with inherent orifices using FEM. Simulations are performed in order to describe the influence of geometrical parameters and static equilibrium conditions. The numerical results show that an optimum stable position may be reached. Experimental investigations show that an unstable state occurs around the maximum axial applied load (F_a shown in figure 2) to the bearing with different types of feeding, by exciting the bearing system with a variable force about its static equilibrium position.

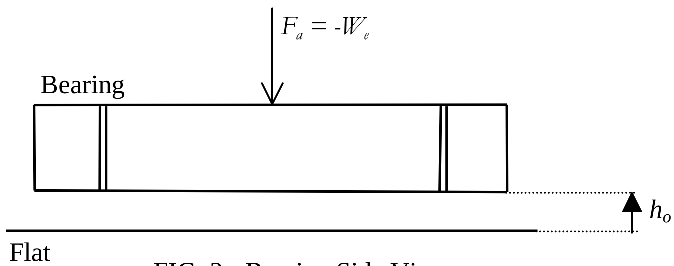


FIG. 2 - Bearing Side View

The failure of a fluid bearing may generally occur because of the instability phenomenon due to the quality of lubrication, the geometry of machined surfaces, the rotor misalignment, the choice of the characteristics of the bearing such as the type of feeding, the supply pressure and Reynolds number and others. Generally, the failure occurs when the friction coefficient of the film fluid becomes very high. Boffey [7] concludes that a critical aspect (a pneumatic hammer phenomenon) may occur if the bearing is not correctly designed. In this case, the failure of a fluid bearing generally causes the destruction of systems or machines rotors parts.

We determine the failure probability of a fluid bearing using the common definition utilized for mechanical structures; namely stress-strength relationship. The equivalent of the strength for a fluid bearing is the maximum load capacity which can be supported by the film fluid taking into account the optimum of stiffness obtained. In this condition, the fluid film gap is very small (lower than 10 µm for air thrust bearings) and a solid contact is possible if an unstable state appears.

Thus, the performance function is defined as:

$$g(X) = W_e - W_{e-\max} \tag{1}$$

where W_{e-max} is the maximum load capacity of the bearing, W_e is the operating load capacity of the bearing. The values of W_{e-max} and W_e depend on the parameters presented in figure 1. The Reynolds equation [4,6] gives us the pressure distribution in the fluid film of the bearing. Assuming the statical case and a thrust aerostatic bearing as shown in figure 1, the Reynolds equation is defined for a compressible and isothermal flow as follows:

$$\nabla . \left(\frac{h_o^3 p}{12\mu} \nabla p \right) = 0 \tag{2}$$

The pressure distribution is calculated using FEM in taking into account the mass conservation in the flow [4,6]. The solution is determined over a surface on which the boundary conditions are given by the pressure along the external boundary of the bearing (see figures 3 and 4) and the mass flow rate. The inlet mass flow rate in orifices q_o and the mass flow rate q_f in the fluid film are expressed as follows:

$$q_o = nC_d \pi d_o h_o P_s \sqrt{\frac{2\gamma}{(\gamma-1)\Re T}\left[\left(\frac{P_o}{P_s}\right)^{\frac{2}{\gamma}} - \left(\frac{P_o}{P_s}\right)^{\frac{\gamma+1}{\gamma}}\right]} \quad \text{if} \quad \frac{P_o}{P_s} \geq \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma-1}{\gamma}} \tag{3}$$

$$q_o = nC_d \pi d_o h_o P_s \sqrt{\frac{\gamma}{\Re T}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}} \quad \text{if} \quad \frac{P_o}{P_s} \leq \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma-1}{\gamma}} \tag{4}$$

$$q_f = -\frac{h_o^3 p}{12\mu \Re T} \nabla p . \overline{u} \tag{5}$$

where $\overline{u}$ is the normal unit vector in the fluid film, P_0 is the outlet pressure of orifices calculated using the mass conservation in the flow [4,6], P_a is the atmospheric pressure. γ, $\Re$, μ and T are respectively the isothropic exponent, the gas constant, the dynamic viscosity and the temperature at supply conditions. See the definition of other parameters in figure 1.

Figures 3 and 4 show respectively the fluid surface meshing and the pressure distribution obtained by the FEM calculation.

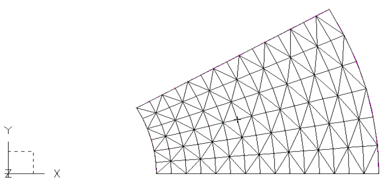


FIG. 3 - Fluid Film Meshing

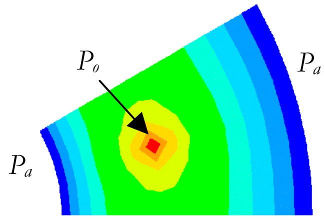


FIG. 4 - Pressure Distribution

Computationally, the gap and pressure step are chosen to be 0.05 µm and 0.0001 bar respectively in order to reach a very small ratio error (to maintain the conservation of mass flow rate through orifices and in the fluid film gap). This aspect needs to be maintained especially for a very small gap approaching zero.

In order to approximate the implicit performance function *g(X)*, the load capacity is deduced by the integration (over the fluid film surface $\partial\Omega$) of the pressure distribution (obtained by the Reynolds equation) as follows:

$$W_e = \int_{\partial\Omega} pdS \tag{6}$$

From the calculation of W_e versus the film fluid gap h_o , $W_{e-\max}$ is taken equal to a fixed fluid film critical gap $h_{o\text{-}min}$.
Thus, the results enable us to study the influence of several design and working parameters on the characteristics of fluid film gap such as discharge coefficient, supply pressure, diameter and number of orifices and position of orifice rows and external load.

3 Failure Probability and DOE

The structural reliability problem seeks the estimation of the probability that a structure exceeds a critical state defined by a state function indexed by a vector of so-called basic variables X, which obeys a joint density function $f_X(X)$. Hence, the problem is written as follows:

$$P_f = \int_{g(X)<0} f_X(X)dX \tag{7}$$

where P_f is the failure probability and *g(X)* is the performance function or the state function that separates the failure and safe domains. The performance function *g(X)* is expressed such that a negative sign for the function (i.e. $g(X) < 0$) indicates a failure of the component whereas a positive sign for the function (i.e. $g(X) > 0$) indicates a survival of the component.

Direct numerical integration of the equation (7) is rarely used to calculate P_f of a complex system due to the computational inefficiency and difficulty of defining $f_x(X)$ and *g(X)* explicitly.
The Stochastic Response Surface Method allows us to approximate function *g(X)* if we have an implicit form. In the Stochastic Response Surface Method (SRSM), *g(X)* is generally approximated by the following quadratic polynomial:

$$g(X) = a + \sum_{i=1}^{n} b_i X_i + \sum_{i=1}^{n} c_i X_i^2 \tag{8}$$

where a, b_i and c_i are the unknown coefficients. The values of these coefficients can be determined using a set of sample points from the true limit state function, $g(X) = 0$. Among various sampling methods, a common approach consists in evaluating *g(X)* at $2n+1$ combinations of μ and $\mu_i \pm h_f.\sigma_i$, where μ and σ are the mean and standard deviation of X_i , and h_f is an arbitrary factor.

In order to capture the non-linearity of the true limit state more precisely, mixed terms can be included into the following quadratic polynomial *g(X)*:

$$g(X) = a + \sum_{i=1}^{n} b_i X_i + \sum_{i=1}^{n} c_i X_i^2 + \sum_{i=1}^{n-1}\sum_{j=2}^{n} d_{ij} X_i X_j \tag{9}$$

The polynomial response function is determined using the Central Composite Design. In this case, $2n+2^n+1$ points are used. The detailed procedure for the calculation of polynomial coefficients can be found in [9]. The polynomial response function found may be inaccurate because the sample points which are located around the mean values may be far from the limit state, $g(X) = 0$ [8].

A new approximated response function is necessary around the design point X_D. Bucher and Bourgund [10] proposes an algorithm to locate a new centre of sample points X_m, closer to the true limit state function than the mean value by a linear interpolation given by:

$$X_m = \mu - g(\mu)\frac{\mu - X_D}{g(\mu) - g(X_D)} \quad (10)$$

where X_D is the design point defined as the closest point to μ lying on the approximated limit state. A new set of sample points, which are combinations of X_{mi} and $X_{mi} \pm h_f.\sigma_i$, is used to construct a second approximated limit state function and the reliability is estimated with this new approximation. Note that the choice of h_f contributes to the accuracy of the approximation. In the example proposed in this article, we assumed that $h_f = 1$.

Thus, we determine if the polynomial response function found is the closest to the system response.

For the evaluation of the failure probability, FORM and MCS are used.
The FORM allows us to estimate the index reliability β and to calculate the failure probability $P_f = \Phi(-\beta)$, where Φ is the cumulative distribution function of the normal distribution. We compare the reliability index β at the current iteration with that of the previous iteration as follows:

$$\left|\beta_k - \beta_{k-1}\right| < \varepsilon_{tol} \quad (11)$$

where ε_{tol} is chosen between 10^{-6} and 10^{-3}.

MCS requires extensive computing times as it requires generating N sets of sample values of X to evaluate the limit state function $g(X)$ for each value. The failure probability is then estimated as the ratio of the number of events with $g(X) < 0$ to the total number of trials N. This approach requires N to be much larger than $1/P_f$ in order to provide a meaningful estimate of P_f.

The procedure for estimating the failure probability P_f of the bearing of the following example is summarized in the flow chart shown in Figure 5.

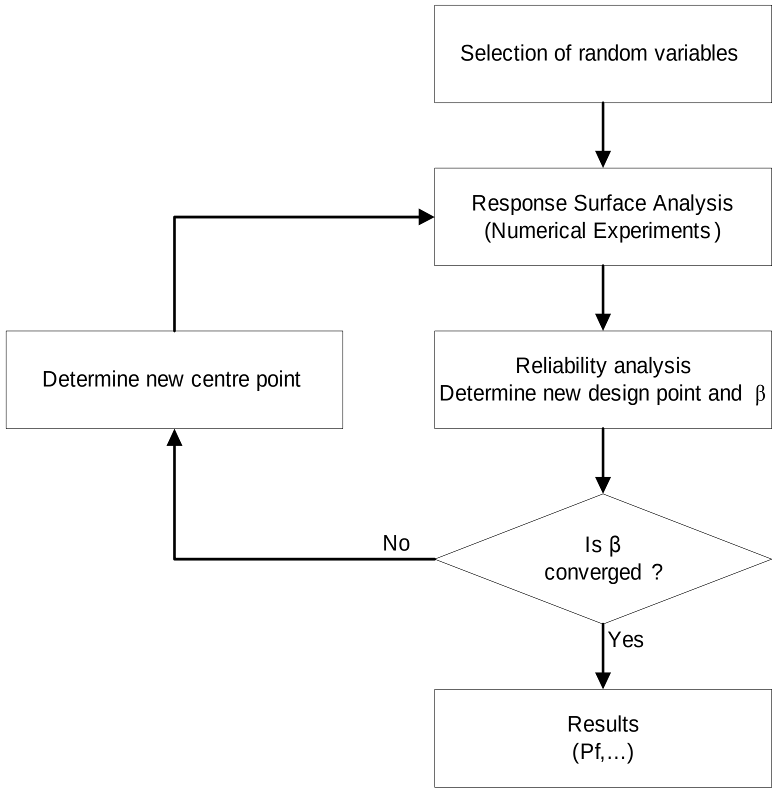


FIG. 5 - Procedure for estimating the failure probability

4 Example

In this section we demonstrate the use of the approach developed above in order to estimate the failure probability P_f of an aerostatic thrust bearing (see the configuration in figure 1). The parameter values of the bearing studied are presented in table 1. Four random variables (see table 2) are considered for the example.

Fluid bearing parameters	
Number of the orifices n	12
Coefficient of discharge C_d	0.7
Supply pressure P_s (bar)	5
Atmospheric pressure P_a (bar)	1
Isothropic exponent γ	1.4
Gas constant $\Re$ ($Jkg^{-1}K^{-1}$)	287
Dynamic viscosity μ (Pa.s)	18.38 .10^{-6}
Temperature at supply conditions (°K)	293

TABLE 1 - Nominal Values

Variables	μ	CV	Distribution
Outer radius R (mm)	75	10%	Normal
Inner radius r (mm)	30	10%	Normal
Radius of orifice row c (mm)	48	10%	Normal
Diameter of the feeding orifice d_o (mm)	0.15	10%	Lognormal

TABLE 2 - Random Variables Input

Figure 6 gives the fluid bearing load capacity versus the film gap calculated thanks to the Reynolds Equation (2). The failure probability is evaluated with the critical load capacity $W_{e\text{-}max}$ = 900 N for the film fluid critical gap $h_{o\text{-}min}$=10 µm. This critical load capacity $W_{e\text{-}max}$ and the operating load capacity W_e depend on all random variables considered.

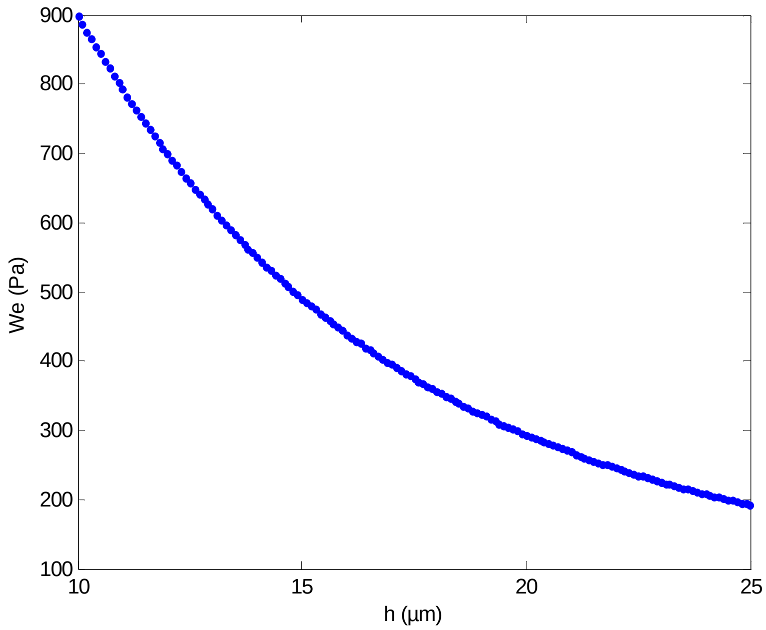


FIG. 6 - Load capacity versus fluid film gap

Table 3 and figure 7 show the results obtained with different methods (FORM and MCS). The sample size for MCS in simulations is taken to be 1.10^6. FORM offers the best alternative for the reliability assessment of fluid bearing problems. The latter method proves to be computationally more efficient than full scale Monte Carlo Simulation.

Method	P_f	*CPU*
MCS	0.00054	615,6
FORM	0.00075	0.053

TABLE 3 - Failure Probability P_f for h_o=18 µm

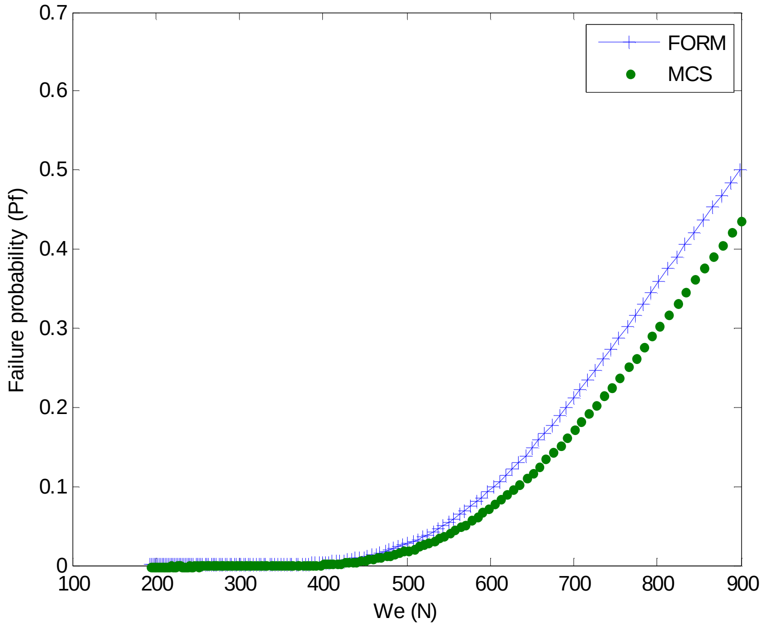


FIG. 7 - Failure Probability versus Load Capacity

Figure 7 presents the failure probability versus load capacity of the film fluid bearing with the configuration of table 1. The analysis of this figure gives us the critical margin of the load capacity not to be used in a system design.

5 Conclusion

This paper presents a method for the failure probability evaluation of a fluid bearing. The evaluation of failure probability is determined by FORM and MCS using SRSM for the approximation of the performance function. The method proposed can be utilized for studying the reliability of other types of fluid bearings (hydrostatic, hydrodynamic, aerodynamic) and for other geometries (cylindrical, spherical).

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