Electromagnetic Stirring Effect on Thermal Conductivity of a Levitated Sample

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Résumé :
La calorimétrie modulée est une technique de mesure indirecte de la chaleur spécifique et de conductivité thermique. Combinée à la lévitation électromagnétique, elle permet la mesure sur des alliages métalliques fondus de très haute réactivité ou bien en état de surfusion. Ce travail présente l’utilisation d’une nouvelle procédure, issue du génie des procédés, d’utilisation de cet instrument. Pour prouver la validité de notre approche, nous utilisons à la place des enregistrements de température expérimentaux des résultats de simulations 2D-axisymétriques. Ces signaux sont traités suivant la procédure choisie. Ce traitement permet de retrouver les données attachées au cas de diffusion (échantillon solide) et de quantifier l’effet de la convection sur les mesures. La convection conduit à surestimer la conductivité thermique, mais n’a pas d’impact négatif sur la mesure de la capacité calorifique.

Abstract :
Modulation calorimetry is an indirect method to measure specific heat capacity and thermal conductivity. When using electromagnetic levitation, this technique allows the measurement for highly reactive molten and undercooled alloys. This work presents the application of a new procedure, coming from Process Engineering, to use this tool. To prove the validity of our approach, 2D-axisymmetric simulations are used instead of experimental temperature records. These signals are processed by the chosen procedure. This allows both to recover the data attached to diffusion (solid sample) and to quantify the convective effects on the quality of the measures. Convection inside sample leads to an overestimation of the thermal conductivity. No negative impact is observed on specific heat capacity measurement.

Mots clefs : Lévitation électromagnétique, calorimétrie modulée, convection

1 Introduction
The modulated calorimetry using inductive levitation consists in deducing thermophysical properties of a sample located in an alternating magnetic field from its thermal behavior. Induced currents generate a total Joule power $P$ which amplitude is modulated. Polar surface temperature is recorded. This signal is then analyzed using an analytical model to calculate specific heat capacity and thermal conductivity.

In this article, we present the experimental device and briefly sum up the modulation calorimetry technique as well as the new procedure that we propose. Instead of using recorded data, temperatures of of levitated sample are obtained from unsteady numerical simulations [1]. The calculated temperatures are processed for two cases solid and liquid. A spectrum of the transfer function, linking power modulation to polar temperature fluctuations is found as a function of the modulation frequency. Eigen-frequencies of this function are representative from heat transfers both between the sample and the surrounding atmosphere and inside the sample. The analysis of obtained results allows to quantify the negative electromagnetic stirring
effects on thermophysical properties measures and to propose a future assembling for the levitator.

2 Modulation calorimetry measurement

2.1 Principle

Figure 1 is a schematic representation of the principle of the measure.

A spherical metallic sample of radius $R$ and electrical conductivity $\sigma_e$ is located in an inductor powered by an alternating current of $I_o$ peak intensity and $\omega_1/2\pi = 350$ kHz frequency. This inductor generates a magnetic field which induces, inside the sample, electrical currents localized in a surface layer called the electromagnetic skin depth $\delta$ such that:

$$\delta = \frac{2}{\sqrt{\mu_0 \omega_1 \sigma_e}}$$

where $\mu_0 = 4\pi \times 10^{-7} \text{H/m}$ is the vacuum permeability. Those currents compose with the magnetic field generating a force field able to center the sample and a Joule heating power $P$ inside the sample. Moreover, when the sample is liquid, the body force causes an electromagnetic stirring. Resulting velocity in the sample is of the order of magnitude of the Alfvén velocity $U_A = I_o C \sqrt{\mu_0 / \rho}$, where $\rho$ is the density and $C = 170.1$ is a geometrical factor corresponding to the inductor geometry described in figure 3. Reynolds number in such a system is defined as $R_A = U_A R / \nu$ where $\nu$ is the kinematic viscosity.

Current modulated calorimetry consists in perturbing the thermal equilibrium of the sample by modulating the total Joule power $P$ around its mean value $\bar{P}$, i.e. $P = \bar{P} + \tilde{P}$, where $\tilde{P}$ is the fluctuating part of the Joule power assumed to be harmonic, i.e. $\tilde{P} = \alpha \bar{P} \sin(\omega_2 t)$, $\alpha$ and $\omega_2$ (small compared to $\omega_1$) being the relative perturbation amplitude and (0.01 to 100Hz) and the modulation angular frequency respectively. For clarity sake, all time dependant variables $X(t)$, the following convention is taken:

$$X(t) = \bar{X} + \tilde{X}$$

where $\bar{X}$, $\tilde{X}$ are the time average and the fluctuating parts respectively.

$$\bar{X} = \frac{1}{t_X} \int_{0}^{t_X} X(t) dt \quad \text{and} \quad \tilde{X} = X(t) - \bar{X}$$

where $t_X$ is the duration of the experiment. For an established harmonic regime, the resulting polar temperatures $T_p(t)$ is written in the same way i.e. :

$$T_p(t) = \bar{T}_p + T_{p,o} \sin(\omega_2 t + \phi_p)$$

In experiments, $T_p(t)$ is measured at the sample surface by using a pyrometer. The sinusoidal modulation of
the power, of amplitude, generates an oscillation of the polar temperature of $T_{p,o}$ amplitude around its mean value $\bar{T}_p$. The principle of the analysis consists in i) measuring the ratio $T_{p,o}/\alpha P$ for a set of angular frequency $\omega_2$, ii) choosing an optimal frequency and iii) using the following analytical model which describes of heat transfers in the sample to calculate the thermo-physical properties of the sample: $C_p$ and $\kappa_{th}$.

### 2.2 Heat transfer model

Modulation calorimetry is an indirect technique to determine thermophysical properties, and as such heat capacity $C_p$ and thermal conductivity $\kappa_{th}$. The quality of the measurement is bound to the model used to describe the thermal behavior of the sample. Currently, the used analytical model used is related to a simplified description of the heat transfers as proposed by Fecht [2] and Wunderlich [3]. They describe the sample as two geometrical domains: an equatorial domain, of $g_e$ volume ratio and $s_e$ external surface ratio, is receiving the whole Joule power while a polar domain, of $(1-g_e)$ and $(1-s_e)$ volume and surface ratio, is heated by conduction with the equatorial domain. Those domains are assumed to be isothermal. External heat transfer is assumed radiative only and modeled by a global radiative heat transfer

$$h_{ext} = 4\pi\varepsilon\sigma\bar{T}_{surf}^3$$

where $A$ is the sample surface, $\varepsilon$ is the total hemispherical emissivity, $\sigma$ the Stefan-Boltzmann constant and $\bar{T}_{surf}$ the surface mean temperature of the sample. Heat transfer between the two domains are modeled by a global conductive heat transfer

$$h_{int} = 4\pi(R - \gamma \delta)\kappa_{th}$$

where $\gamma$ is a geometrical factor to be defined. The total Biot number of the system $Bi$ follows:

$$Bi = \frac{h_{ext}}{h_{int}}$$

Those assumptions lead to the following system of equations:

$$\begin{cases} 
C_p g_e \bar{T}_e = h_{int} \left[ \bar{T}_p - (1 + s_e Bi) \bar{T}_e \right] + \bar{P} \\
C_p (1 - g_e) \bar{T}_p = h_{int} \left[ \bar{T}_e - (1 + (1 - s_e) Bi) \bar{T}_p \right] 
\end{cases}$$

(6)

Because heating power fluctuation is considered harmonic of angular frequency $\omega_2$, analytical solution of equations (6) for the polar temperature fluctuation leads to:

$$\frac{T_{p,o}}{\alpha P} = \frac{h_{int} \frac{1}{C_p g_e (1 - g_e) [\lambda_{int}^2 + \omega_2^2] [\lambda_{ext}^2 + \omega_2^2]^{1/2}}}{\left[ \frac{\lambda_{ext}}{C_p} \right]}$$

(7)

where $\lambda_{ext}$ and $\lambda_{int}$ are typical angular frequencies related to external and internal heat transfer respectively. Assuming small Biot numbers, lead to the following approximated expressions for the angular frequencies:

$$\lambda_{ext} \equiv h_{ext}/C_p$$

(8)

$$\lambda_{int} \equiv g_e (1 - g_e) h_{int}/C_p$$

(9)

We propose to modify this procedure by using a different, i.e. non harmonic, modulation of the input power. We chose this modulation as a white-noise, i.e. all angular frequencies in the range larger than $\lambda_{ext}$ and $\lambda_{int}$. Using an identification procedure on the recorded temperatures, transfer functions linking the input power to the recorded temperatures are fund. The eigen-frequencies of the transfer functions are $\lambda_{ext}$ and $\lambda_{int}$. Therefore, even in the absence of any knowledge on the geometry, electrical characteristic of the levitator, the way it is used or the Biot number, we may get a reliable insight of the thermal behavior of the sample.

This is this advantage which is used in the following to study the impact of the electromagnetically driven
fluid flow on thermal behavior of a sample.
Indirect measurement of the properties consists in identifying the above described heat transfer model (system 6) to the experimental measurement. This will not be further described here.
The full procedure for signal analysis and thermophysical properties calculations is described in an article submitted to the Journal of Heat and Mass Transfer [4].

2.3 Numerical simulations

Some general assumptions are made in numerical simulations:
- Electrical, mechanical and thermal properties are assumed to be constant in the range of temperature of the sample.
- Alfvèn velocity is such that the magnetic Reynolds \( R_m = \mu_0 \sigma_{el} U_A \ll 1 \). Consequently, velocity can be neglected in the induction equation.

Those points make possible a separation between physical phenomena. Assumptions are summed up in figure 2. Consequently, the simulations of various physical phenomena are performed consecutively

Induction equations are solved by using a harmonic formalism of the electromagnetic field in axisymmetric 2D thanks to a module developed within EPM laboratory [5]. Equations are heat transfer equations in 2D axisymmetric. Reynolds number \( Re_A \) is low enough for the fluid flow to be laminar. Steady velocity field \( \vec{U} \) is obtained. Unsteady temperature field is solved for both solid and liquid cases. Simulation parameters are summed up in table 1.
The sample is a spherical sphere of constant electrical conductivity \( \sigma_{el} \) and density \( \rho \). Inductor geometry is described in figure 3.

Results of this simulation are \( \chi \) and \( \vec{f} \), the Joule heating power density and the Laplace body force respectively. Steady fluid flow is solved for a spherical drop stirred by the body force \( \vec{f} \).

Velocity field \( \vec{U} \) is assumed to be constant and equal to \( \vec{U} \) despite the modulation. Boundary heat flux corresponds to a grey and diffuse surface in a black body enclosure at room temperature \( T_o = 300 K \).

\[
\begin{align*}
\frac{\partial T}{\partial t} + \frac{k_{th}}{\rho c_p} \Delta T + \frac{1}{\rho c_p} \nabla \cdot (\vec{U} T) &= \frac{\chi}{\rho c_p} \\
\frac{\partial T}{\partial r} &= \varepsilon \sigma (T^4 - T_o^4) \\
&\quad (r, \theta) \in [0; R] \times [0; \pi]
\end{align*}
\]

(10)
Internal heat source is such that \( P = \int \chi \, dV \). The chosen relative modulation amplitude \( \alpha \) is 5%.

Modulation function is fully described in [5].

Tab. 1. Simulations parameters

| Sample radius \( R, \text{ mm} \) | 4 |
| density \( \rho, \text{ kg.m}^{-3} \) | 3860 |
| heat capacity \( c, J.K^{-1}.kg^{-1} \) | 663 |
| thermal conductivity \( \kappa, W.m^{-1}.K^{-1} \) | 23.1 |
| electrical conductivity \( \sigma_{el}, \Omega^{-1}.m^{-1} \) | 5.26 \times 10^5 |
| total hemispherical emissivity \( \varepsilon \) | 0.4 |
| inductor current \( I_0, A.turn \) | 390 |
| current angular frequency \( \omega_1, rad.s^{-1} \) | \( 2\pi \times 350 \times 10^3 \) |
| Alfvèn velocity \( m.s^{-1} \) | 9.3 \times 10^6 |
| Biot (solid) | 0.06 |
| Reynolds | 37 |

3 Results

Calculated ratio \( T_{p,o}/\alpha P \) is reported in figure 4. It is a spectrum in which the changes of the slope allows us to calculate the two eigen-frequencies \( \lambda_{ext} \) and \( \lambda_{int} \). They are the inverse of the time of heat transfer between the sample and the surrounding atmosphere, and of the time of transfer inside the sample.

Solid and dashed lines are the thermal spectrum of a solid and a liquid samples respectively. Characteristic slopes are reported on both curves.

The two domains model is known to be a very good model of a solid sample thermal behavior [1]. As such, the simulated solid sample thermal behavior demonstrates a trend similar to the analytical solution given in equation (6). Three ranges of angular frequencies are observed, each one separated by natural frequencies \( \lambda_{ext} \) and \( \lambda_{int} \). In low frequency modulation range, the spectrum is flat meaning that the temperature field does not depend on \( \omega_2 \). It is almost stationary and we may affirm that in this range, the radiation is controlling the heat transfers. In high modulation frequency range, the heat transfer inside the sample controls the temperature field. When the sample is solid, it is a conductive regime. For intermediate values, both internal and external heat transfers control the temperature field. This is the range in which the modulation frequency of the input power has to be chosen, when modulation is harmonic, i.e. depending on one frequency only as in [3].

Fig. 3: Sample and inductor geometries

Fig. 4: Polar transfer functions calculated by procedure proposed in [4]
In the case of the liquid sample, we clearly see a shift toward the higher frequencies. This shift is significant of the impact of the convection on heat transfers. Measured internal frequencies \( \lambda_{int} \) are 9.0 and 25.7 rad.s\(^{-1}\) for solid and liquid respectively. Therefore using this value in system (6) and equation 4 to calculate \( \kappa_{th} \) will lead to an overestimation of this value.

These conclusion may appear as obvious, but, what is interesting in the present work is twofold. First, the used procedure recovers all expected phenomenological trends. Second, this trends are quantified which is not possible using the current procedure.

### 4 Conclusion

In this article we presented some numerical experiments leading us to affirm that the original method of measurement derived from traditional modulated calorimetry is valid. It differs from existing methods on the following 2 points:

- use of a temporal modulation of the Joule power dissipated into the sample in the a white noise form
- use of a signal analysis coming from Process Engineering to determine the eigen-frequencies attached to internal and external heat transfers.

All expected phenomenological trends are recovered. For example, the overestimation of thermal conductivity is calculated.

The method can be improved in two following directions:

- join a continuous magnetic field which intensity is high enough to the levitator, in order to damp fluid flow motions
- use an inverse method to calculate thermophysical properties from heat transfer function calculated from recorded time-depending temperatures.

Note that the above proposal is based on calculations only and that its practical implementation has not been achieved yet.

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### References


