# 3D computer simulation of Rayleigh-Benard instability in a square box

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## Résumé :

*La stabilité des rouleaux bidimensionels est étudié au moyen des calculations bi- et tridimensionels utilisant le nombre de Rayleigh* $1.7 \cdot 10^3 < Ra < 1.1 \cdot 10^4$, *nombre de vague de rouleau* $0.5 < k_0 < 7.0$, *nombre de Prandtl* $Pr = 0.71; 1$. *Les scénarios differents d'évolution de mode d'écoulement en partant de mouvement potentiellement instable jusqu'à mouvement stable sont discutés. En particulier, nous avons observés trois instabilités vraiment tridimensionels, le varicose oblique, l'instabilité oscillatoire et le si-nommé spiral defect chaos. Les resultats sont comparés aux donneés theoretiques de Busse et colleagues sur la stabilité de couche horizontale de liquide (Busse's balloon)[1],[2] et aux donneés experimentales.*

## Abstract :

*The stability of two-dimensional rolls is investigated by means of 2D and 3D calculations for Rayleigh number in the range* $1.7 \cdot 10^3 < Ra < 1.1 \cdot 10^4$, *roll's wave number* $0.5 < k_0 < 7.0$ *and Prandtl number* $Pr = 0.71; 1$. *Different scenarios for the flow pattern evolution from potentially unstable motion to a stable one are discussed. In particulary truly three dimensional forms of instabilities, such as skewed varicose, oscillatory, spiral defect chaos, have been observed. The results are compared with Busse and coworkers theoretical data on the stability of horizontal layer of fluid (so called Busse's balloon) [1],[2] and experimental data.*

**Mots clefs : Rayleigh-Benard instability, computer simulation**

## 1 Introduction

Rayleigh-Benard convection, i.e. convection in a horizontal layer heated from below, permanently attracts attention of the researchers as a meaty example of hydrodynamical system in which transition to various types of instabilities can be studied. When the temperature difference between lower and upper plate is large enough, buoyancy forces lead to the destabilization of the quiescent state and convection evolves. The first stable convection pattern takes the form of rolls, i.e., two-dimensional structures with roughly circular streamlines in alternating directions. The strength of buoyancy forces are determined by Rayleigh number $Ra$, the size of rolls are described by wave number $k$. For different values of Prandtl number stability area of roll structures in the plane $(Ra, k)$ has been obtained in [1]. Though RB system has been extensively studied both theoretically and experimentally, there is a lack of quantitative and reliable comparisons between theory and experiment. The theory usually deals with infinite layers, the short, linear stages of flow development, ideal flow patterns and etc. The main goal of the paper is to study numerically evolution and stability of roll structures and to produce in calculations overriding types of instabilities predicted by theoretical data.

## 2 Governing Equations and Numerical Procedure

Rayleigh-Benard problem is governed by 3D time-dependent fluid flow and heat transfer equations. Fluid motion is described by Navier-Stokes equations in the Oberbeck - Boussinesq approximation. In Cartesian coordinates $(x, y, z)$ the non dimensional equations take the form

$$\partial_t V + (V\nabla)V = -\nabla p + \Delta V + GrTe_z \quad (1)$$

$$\nabla V = 0 \quad (2)$$

$$\partial_t T + (V\nabla)T = \frac{1}{Pr}\Delta T \quad (3)$$

where $\partial_\xi \equiv \dfrac{\partial}{\partial\xi}$, $\nabla = (\partial_x, \partial_y, \partial_z)$, $\Delta = \nabla^2 = \partial^2_{xx} + \partial^2_{yy} + \partial^2_{zz}$, $(x, y, z) \in \mathcal{D}$, $\mathcal{D} = [0, L] \times [0, L] \times [0, 1]$, $V = (V_x, V_y, V_z)$ is the velocity vector, $t$ - time. The non-dimensional variables are introduced by scaling the

length with the liquid depth ($H$), time is scaled with $H^2/\nu$, temperature with $\delta T$.
On the domain boundary the velocity satisfies the no-slip and non-penetration condition : $V = 0$. Border conditions for temperature are : $T = 1$ at $z = 0$, $T = 0$ at $z = 1$ and $\partial_n T = 0$ on the remainder of the boundary, where $n$ is the normal to the boundary. Initially the fluid is at rest and $T = 1 - z$.
To solve the governing equations a sequential procedure is applied. The velocity is advanced using the temperature from the previous layer in the force term. The calculated velocity field is inserted into the heat transfer equations. The problem is approximated at staggered grid using control-volume approach. Approximation of convective terms ensure kinetic and heat energy conservation [3]. The scheme is implicit, has second order in space and first in time. Algorithm was successfully used for simulation of subcritical thermo-diffusion convection [4],

# 3 Numerical results

In a series of papers [1],[2], Busse and coworkers determined the region in $(k, Ra)$ plane, so called "Busse Balloon", where steady rolls are predicted to be stable with respect to infinitesimal disturbances, outside the region, rolls are destabilized by different kinds of secondary instabilities. There are Eckhaus mode that is the only 2D type of instability, zig-zag, skewed varicose, knot, oscillatory instability, etc. [5],[6]. All the results relate to the infinite horizontal layer.

## 3.1 2D simulation

Firstly we have done a set of 2D calculations to estimate the effect of sidewalls on the rolls stability.
To initiate convective flow in the form of rolls with the wave number $k_{dis}$, the temperature field in the plane $z = 0.5$ is disturbed with perturbation in the form $0.02cos(k_{dis}x)$, $0.5 < k_{dis} < 7.0$. Horizontal domain size $L$ is chosen to be a multiple of $\pi/k_{dis}$ and around 15. The grid size in space is $128 \times 16$. The calculation has been done for Rayleigh number in the range $1.7 \cdot 10^3 < Ra < 1.1 \cdot 10^4$.
Numerical results show that the presence of sidewalls, no matter how distant, substantially restrict the possible wave-numbers which can occur in the bulk of the system. Near the convective threshold the band of available wave numbers is reduced to a range $|k| \sim (Ra - Ra_{cr})/Ra_{cr}$ instead of a size $|k| \sim [(Ra - Ra_{cr})/Ra_{cr}]^{1/2}$ predicted for Eckhaus instability in the infinite systems, ($Ra_{cr} = 1708$ is the critical Rayleigh number). This is compatible with theoretical data [7].
Two scenarios of the flow pattern evolution, entitled *quasi stable* and *roll diffusion*, have been registered in our 2D calculations.
Under *quasi stable* scenario, the initial disturbance evolves into well developed roll pattern with $k = k_{dis}$ all over the domain. Then, after a certain period of quasi stability, flow transition occurs : rolls adjoining the sidewalls are extruded from the region by the neighbouring ones and the wave number of the roll structure decreases. Such flow transitions can recur several times until stable roll pattern with $k_e \neq k_{dis}$ is set on.
Another scenario of flow evolution is *roll diffusion*. In this case inside the bulk the initial perturbation grows slowly, while near the side walls, appears the roll structures with a new wave number $\tilde{k} \neq k_{dis}$, which is near stable. The cells with new wave number quickly gain the kinetic energy. Then next pair of rolls with the same wave number $\tilde{k}$ appears near the first one and gradually the roll structure fills up the whole box.

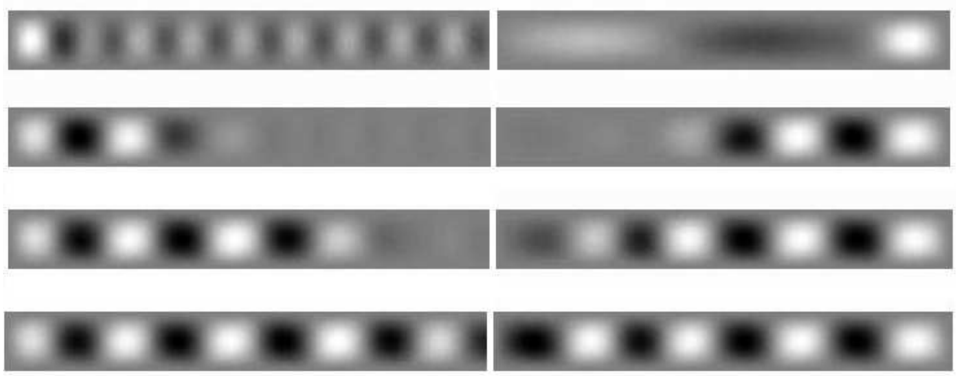

FIG. 1 – *roll diffusion*. Evolution of the stream function for $k_{dis} = 6.8$ $k_e = 3.8$ (left) and $k_{dis} = 1.2$ $k_e = 3.2$ (right), $Ra = 3Ra_{cr}$

2D calculations are refined by the series of time - dependent 3D simulations with similar initial data have been done. In this case we have observed the scenarios mentioned above as well as the essentially three dimensional flow pattern evolution. There are some results of the simulation.

## 3.2 3D simulation

To study evolution and stability of roll structures, computer simulation has been done for $1.7 \cdot 10^3 < Ra < 1.1 \cdot 10^4$, $Pr = 0.71; 1$. Spatial grid of size $128 \times 128 \times 16$ has been used, time step has been varied from $10^{-4}$ to $10^{-2}$. In the calculations the stability threshold is about $Ra \approx 1750$, which is close to theoretical critical value $Ra_{cr} = 1708$. At $Ra = 1750$ the flow has roll pattern with wave number $k \approx k_{cr} = 3.117$, where $k_{cr}$ - the wave number that corresponds to $Ra_{cr}$. The flow structure keeps the same with mesh refinement in space and time. It means that the grid size we use provide a good compromise between the computation time and accuracy.

For $Ra > 1750$ amplitude of the initial perturbation grows uniformly throughout the space, when $k_0$ is close to $k_{cr}$. When $k_0$ strongly differs from $k_{cr}$, convective structures with wave number $\tilde{k}$ form near the vertical boundaries and begin to spread into the bulk. Resulting flow has a roll structure and again can be stable or not depending on $Ra$ and wave number values. At this stage, 3-dimensional results are similar to two-dimensional data. Further flow transition can be two or three dimensional.

As an example let's analyse the flow structure evolution for $Pr = 1$, $Ra = 3500$, $k_{dis} = 5.8$ (fig.2).

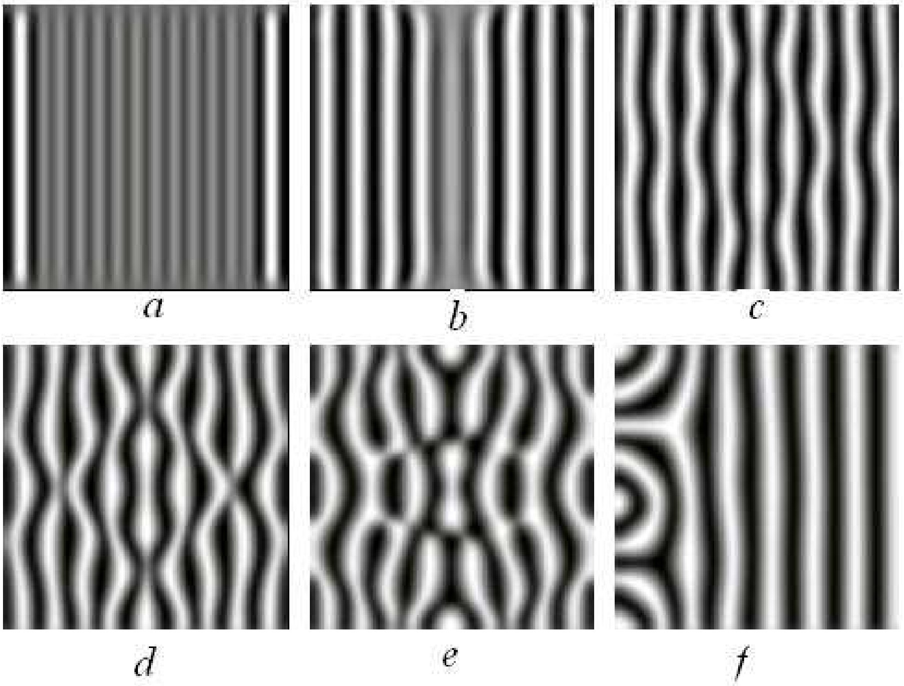

FIG. 2 – Flow evolution : a–t=0.2,b–t=1, *roll diffusion* ; c–t=5,d–t=8, e– t=9,f–t=62, skewed varicose instability. $Ra = 2Ra_{cr}$, $Pr = 1$, $k_{begin} = 5.8$ $L_x = 16.25$. Temperature field in the plane $z = 0.5$ plane. Light areas are hot, dark are cold.

The wave number of initial disturbance is $k_{dis} = 5.8$ that is close to neutral curve and has low amplitude growth rate. Rolls parallel to $OY$ with $\tilde{k} = 3.5$ appears near the walls at $t = 0.2t_\nu$. The flow velocity is still small and this stage of flow evolution can be considered as a linear one. The amplitude growth rate of the new wave number is higher than the initial one. With time rolls with the new wave number penetrate into the bulk (fig.2b). 2D calculations show that rolls with the wave number $k = 3.5$ are stable, but according to Busse's balloon this value is unstable with respect to skewed varicose instability [2]. This type of instability leads to rolls curvature and their periodical compression. In our calculations at $t = 5t_\nu$ straight rolls start to bend, then parts of the neighbour rolls glue to each other (the patchy process [2]) and complicated flow pattern is formed (fig. 2c-d). Newly formed flow pattern tends to form rolls with the wave number $k = 2.5$ that lies inside the Busse's balloon (fig. 2f).

Theoretical and experimental data shows that wave number of the freely evolving rolls decreases with $Ra$ [8]. Numerical calculations show similar dynamics and results are close to [8].

For $Pr = 0.71$, $Ra = 2500$, $k = 3.5$ and $Ra = 3500$, $k = 3$, developed roll structures also unstable and skewed varicose instability arises. For $Ra$ in the range $4000 < Ra < 7000$ evolution of skewed varicose instability leads to spiral defects chaos (fig.3).

Another example of instability is oscillatory. We have registered it for $Ra = 9000$, $Pr = 0.71$, $k = 1.5$(fig.4) Period of oscillations is about 0.17-0.19 and wave length is approximately 2.0-2.2, that is close to experimental data.

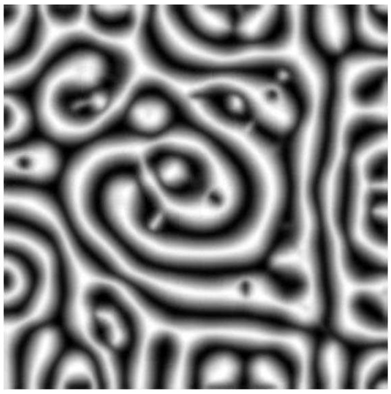

FIG. 3 – Spiral defect chaos. $Ra = 4Ra_{cr}$, $Pr = 0.71$,$k_{dis} = 1.8$

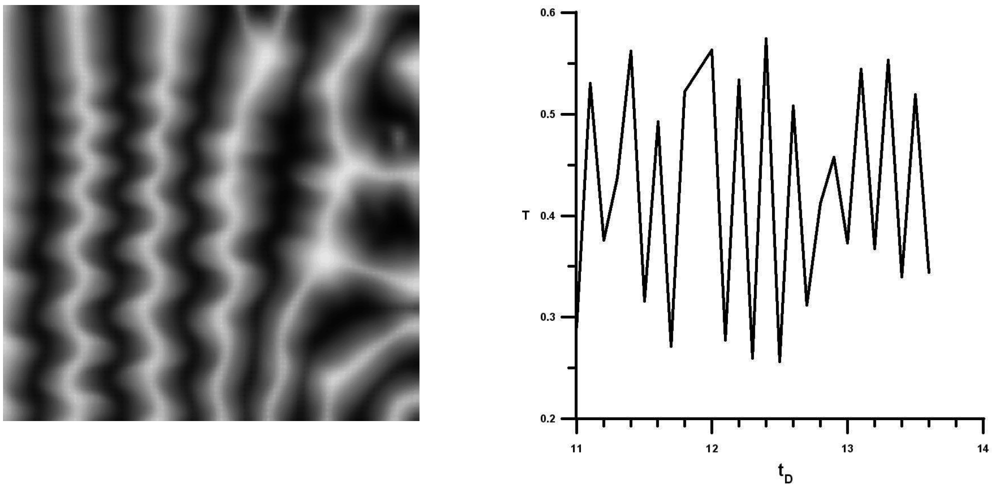


FIG. 4 – Oscillatory convection $Ra = 5.4Ra_{cr}$, $Pr = 0.71$,$k_{dis} = 1.8$

## 4 Conclusions

Convective stability of two-dimensional rolls in a closed box is investigated by means of 2D and 3D computer simulation. Different types of instability, well known from the theory of nonlinear stability, have been observed in calculations. Results are in good agreement with Busse and coworkers theoretical data on the stability of horizontal layer of fluid (so called Busse's balloon) [2] and experimental data.

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