Finite element model of the ultrasonic propagation in cortical bone : application to the axial transmission device

G. Haïat¹, S. Naili², Q. Grimal³, M. Talmant³, C. Desceliers⁴ et C. Soize⁴

¹ CNRS, Laboratoire de Recherches Orthopédiques, UMR CNRS 7052 B2OA, Université Paris 7, 75010 Paris, France.
² Université Paris 12 – Val de Marne, Laboratoire de Mécanique Physique, 94010 Créteil, France.
³ Université Pierre et Marie Curie, Laboratoire d’Imagerie Paramétrique, UMR CNRS 7623, 75006 Paris, France.
⁴ Université Paris-Est, Laboratoire de Modélisation et Simulation Multi-Echelle, MSME FRE3160 CNRS, 77454 Marne la Vallée, France

Résumé :

La technique de transmission axiale est utilisée en clinique pour l’évaluation de l’os cortical. Cependant, la propagation d’onde ultrasonore dans ce milieu multi-échelle isotrope transverse reste mal comprise, notamment du fait de la nature hétérogène de l’os cortical. À l’échelle macroscopique, la distribution de la porosité induit un gradient de propriétés matérielles orienté dans la direction radiale. L’objectif de cette étude est d’évaluer l’effet d’un gradient spatial de porosité sur la réponse ultrasonore de cette structure isotrope transverse. Une méthode de simulation par éléments finis dans le domaine temporel est développée afin de simuler la propagation d’onde en régime impulsionnel dans un tri-couche constitué d’une couche de solide hétérogène isotrope transverse prise en sandwich entre deux couches de fluide et excitée par une source acoustique linéique située dans une des couches de fluide et engendrant des pulses ultrasonores large bande (1 MHz de fréquence centrale). Le modèle prend en compte le couplage de la propagation acoustique dans les deux fluides avec la réponse élastodynamique du solide. Un gradient constant de porosité dans la direction perpendiculaire à la couche est considéré dans le solide pour deux valeurs d’épaisseur de solide h correspondant à un os relativement épais (h=4 mm). Pour des fortes valeurs d’épaisseurs corticales (4 mm), les résultats sont en accord avec la propagation d’ondes latérales. Nos résultats permettent l’estimation d’une profondeur de pénétration équivalente de l’onde latérale dans le cas d’un solide transverse isotrope hétérogène (entre 0.46 et 0.63 mm pour un gradient de porosité).

Abstract :

The aim of this work is to evaluate the effect of a spatial gradient of porosity of cortical bone on its ultrasonic response obtained with an axial transmission device. Therefore, a two-dimensional finite element time-domain method is derived to model transient wave propagation in a three-layer medium composed of an inhomogeneous transverse isotropic solid layer sandwiched between two acoustic fluid layers and excited by an acoustic linear source located in one fluid layer, delivering broadband ultrasonic pulses. The model couples the acoustic propagation in both fluid media with the elasto-dynamic response of the solid layer. A constant spatial gradient of porosity is considered for a value of bone thicknesses corresponding to relatively thick bone width. For a thick bone (4 mm), the results are in agreement with the propagation of a lateral wave and allow the derivation of an equivalent contributing depth in the case of a transverse isotropic inhomogeneous solid layer. Our results allow the estimation of an equivalent penetration depth of the lateral wave in the case of an inhomogeneous transverse isotropic solid (between 0.46 and 0.63 mm for a gradient of porosity).

Mots clefs : Cortical bone, Quantitative Ultrasonic (QUS) parameters, axial transmission

1 Introduction

Osteoporosis is a systemic disease of the skeleton characterized by a decrease of bone mass and micro-architectural deterioration of bone tissue. The investigation of cortical bone is important since it accounts for about 80% of the skeleton, supports most of the load of the body and is mainly involved in osteoporotic fractures of many kinds. The so-called axial transmission (AT) technique has been shown particularly suitable for cortical bone evaluation. The earliest event or wavelet (usually called First Arriving Signal, FAS) of the multicomponent signal recorded at the receivers has been the most often investigated and its velocity measured in the time domain was shown to be able to discriminate healthy subjects from osteoporotic patients. Both experimental [1] and simulation [2] studies have indicated that the FAS velocity was related to bone properties. When the wavelength is comparable or smaller than the cortical thickness, the type of wave contributing to the FAS corresponds to a lateral wave, whereas when the wavelength is large compared to the cortical thickness divided by four, the received signal corresponding to the FAS comes from the first symmetric Lamb wave mode.
Bone is heterogeneous at several scales. Its elastic behavior has been described as transverse isotropic [4]. At the macroscopic scale, porosity in the radial direction is heterogeneous [5, 6]: the mean porosity in the endosteal region (inner part of the bone) is significantly higher than in the periosteal region (outer part of the bone). Modeling the FAS in AT experiment is a time-domain elasto-acoustic problem. Time-domain analytical methods have been used in the past to solve the elasto-acoustic wave system in simple AT models [7, 8]. However, such methods can not account for an arbitrary heterogeneity of materials. On the other hand, numerical methods such as finite elements or finite differences allow accounting for continuous variations of material elastic properties, which is relevant to model the heterogeneity of bone. In this paper, a finite element library is used. The aim of this paper is to assess the effect of the heterogeneous nature of cortical bone on its ultrasonic response obtained with an AT device. Bone is modeled as an anisotropic material (transverse isotropic) and a gradient of material properties in the radial direction is considered.

2 Methods

2.1 Axial transmission configuration

Bone is modeled as a two-dimensional multilayer medium composed of one elastic transverse isotropic solid layer (corresponding to cortical bone) sandwiched between two acoustic fluid layers. The upper medium corresponds to soft tissues and the lower medium corresponds to bone marrow. Both soft tissue and bone marrow are modeled by the same fluid with an acoustic wave velocity of $1500 \text{ m.s}^{-1}$ and a mass density of $1 \text{ g.cm}^{-3}$. The cortical thickness is denoted $h$, as shown in Fig. 1. The geometrical arrangement attempts to model that of an actual probe developed by the ‘Laboratoire d’Imagerie Paramétrique’ (France) [9].

![Fig. 1 – Schematic representation of the simulation domain. The emitter and receivers are indicated by indents.](image)

2.2 Finite element model

The acoustic wave propagation equation is solved in both fluid media where the formulation is written in terms of pressure. Conversely, the formulation in the solid layer for the wave propagation without dissipation is given in terms of displacement. At both interfaces between the fluid layers and the solid layer, the boundary conditions in terms of displacement and normal stresses are taken into account. The model therefore fully describes the fluid-structure interaction between the three sub-domains, accounting for all reflection, refraction, and mode conversion effects. The system of wave propagation equations is solved using a stiffness tensor 2-D transverse isotropic for the solid layer:

$$
\mathbf{C} = \begin{pmatrix}
C_{11}(z) & C_{13} & 0 \\
C_{13} & C_{33} & 0 \\
0 & 0 & C_{55}
\end{pmatrix},
$$

where only the stiffness coefficient $C_{11}$, written using the Voigt notation, only depend on $z$. We only consider a variation of $C_{11}$ because only the case of a thick bone width (4 mm) will be considered in what follows. The
interested reader can refer to [10] in which other thicknesses are discussed. The plane of isotropy corresponds to the \((y, z)\) plane in Fig. 1. Note that in this present work, the transverse isotropy imposes the relation \(C_{15} = C_{12}\). Subscripts 1, 2 and 3 are associated with \(x, y\) and \(z\)-axes respectively. We used realistic values to define the spatial gradient of \(C_{11}\) by combining measurement performed in [4] with the thermodynamical conditions of stability. The boundary value problem associated with the 2-D model is solved using the finite element library of the COMSOL Multiphysics software [11]. For each computation, around 186,000 triangular elements are used, resulting in about 393,000 degrees of freedom. Simulations were performed with a solid layer thickness \(h = 4\) mm.

### 2.3 Determination of a realistic range of variation of elastic bone properties

In the case of bone, all homogenized material properties are expected to exhibit coupled spatial variations because they are all related to porosity, which increases from the periosteal to the endosteal part [5]. When porosity increases, the values of the homogenized elastic constants and of mass density are expected to decrease, having opposite and competing effects on the wave velocity. Here, spatial variations of types 1 and 2 are considered for the porosity (noted \(\bar{P}\)), with the minimum and maximum values of porosity \(P_m\) and \(P_M\) equal to 3 and 15\%, respectively. In the case of spatial variations of types 1 and 2, the porosity \(P\) writes respectively:

\[
P(z) = P_M + \delta_P \times z
\]

\[
P(z) = \frac{(P_m + P_M)}{2} + \delta_P \times (z + \frac{h}{2}),
\]

where \(\delta_P\) corresponds to the gradient of porosity. Following a simple rule of mixture (\(i.e\). the total mass density is given by a linear combination according to each of the volume fraction), a variation of porosity induces an affine variation of mass density given by:

\[
\rho(z) = \rho_m + \delta_\rho \times (P - P_m),
\]

where \(\delta_\rho\) corresponds to the gradient of porosity and \(\rho_m\) is the minimum value of the mass density. Here, we choose \(\rho_m\) in order to obtain a variation of mass density from 1.753 to 1.66 g.cm\(^{-3}\) when \(P\) varies from 3 and 15\%, which leads to \(\delta_\rho = 7.7 \times 10^{-3}\) g.cm\(^{-3}\). These values correspond to a mass density equal to 1.722 g.cm\(^{-3}\) when \(P = 7\%\).

The variation of the elastic coefficient \(C_{11}\) with porosity is taken from the literature. Affine dependence with porosity was derived from [12] where a variation of porosity between 3 and 15\% corresponds approximately to a change of \(C_{11}\) of 7.8:

\[
C_{11}(z) = C_{11}^m + \delta_C \times (P - P_m),
\]

where \(\delta_C\) corresponds to the gradient of \(C_{11}\). The variation of \(C_{11}\) is centered on its reference value. Therefore, \(C_{11}^m\) is equal to 19.7 GPa ; the quantity \(\delta_C\) is equal to 0.65 GPa. Note that taking into account a slight non linear variation of \(C_{11}\) as a function of porosity should not significantly modify our results. We did not consider any variation of \(C_{12}, C_{33}\) and \(C_{55}\) which were taken equal to its reference value given in Tab. 1 [4].

<table>
<thead>
<tr>
<th>(C_{12})</th>
<th>(C_{33})</th>
<th>(C_{55})</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.7 GPa</td>
<td>15.1 GPa</td>
<td>4.7 GPa</td>
</tr>
</tbody>
</table>

Tab. 1 – Mean of the three elastic constants affecting the ultrasonic propagation in the framework of the 2-D model of Fig. 1. These values are taken from [4].

### 3 Results and discussion

When the thickness \(h\) is large compared to wavelength, the FAS velocity tends towards the bulk longitudinal wave velocity inside the material constituting the solid layer:

\[
v_b = \sqrt{\frac{C_{11}}{\rho}}.
\]

Figure 2 shows the results obtained for \(h = 4\) mm with a gradient of porosity. The black and gray solid lines show the FAS velocity obtained numerically for a gradient of porosity of types 1 and 2, respectively. The dashed lines of Fig. 2 are obtained by considering the longitudinal wave velocity \(v_b\) in the direction perpendicular to the normal of the surface at \(z = 0\). This velocity \(v_b\) is shown to slightly overestimate the simulation results, which is consistent with results obtained in [2]. The dependence of the FAS velocity to the gradient of porosity is slightly different to that predicted by the velocity \(v_b\). These last results can be exploited and lead to the
FIG. 2 – Thick solid layer is $h = 4$ mm. Variation of the FAS velocity as a function of a gradient of porosity. The black lines correspond to the variation of type 1 (constant value of porosity at $z = 0$) and the grey lines correspond to the variation of type 2 (constant value of porosity at $z = 0.5h$). The continuous lines indicate the results obtained from the finite element model. The dashed lines correspond to the longitudinal bulk wave velocity in the material at the upper interface.

determination of an equivalent penetration of the lateral wave simulated in this study, which correspond to the average depth investigated by the AT device in the case of a thick bone width. The value of this equivalent penetration depth is equal to 0.46 and 0.63 mm in the case of a gradient of types 1 and 2 respectively. This slight difference can be explained by the first order approximation used in the method of determination of the equivalent penetration depth.

References


