Strong amplification of water waves at “non-reflecting” beaches

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In the case when the water depth changes smoothly and the slope of the beach is small, WKB approach can be applied. Since the wave reflection on such beaches is weak, water wave amplification can be significant. Here we demonstrate a similar character of the significant wave amplification even for the case, when the depth is an arbitrary (not smoothed) function of the offshore coordinate. Several examples of a special bottom profile, when the wave equation has a rigorous solution in the form of a traveling wave are found. The reflection and transmission of the wave approaching the non-reflecting beach are studied.

We solve the linear shallow-water wave equation for the water displacement

\[ \frac{\partial^2 \eta}{\partial t^2} - \frac{\partial}{\partial x} \left( c^2(x) \frac{\partial \eta}{\partial x} \right) = 0, \quad c(x) = \sqrt{gh(x)}, \tag{1} \]

where \( \eta(x,t) \) is a vertical water displacement, \( h(x) \) is a water depth, and \( g \) is gravity acceleration. An exact analytical traveling wave solution of this equation exists in the case of \( h(x) \sim x^{4/3} \) \cite{1}

\[ \eta(x,t) = A_\pm(x)f[t \mp \tau(x)], \quad \tau(x) = \int dx/c(x), \quad A_\pm \sim h^{-1/4}(x), \tag{2} \]

where \( \pm \) corresponds to the wave propagating onshore/offshore, and the function \( f \) satisfies to

\[ \int_{-\infty}^{+\infty} f(t)dt = 0. \tag{3} \]

The velocity field in the traveling wave is described by

\[ u(x,t) = U(x) \left[ f(\zeta) + \frac{\sqrt{gh}}{4h} \frac{dh}{dx} \Phi(\zeta) \right], \quad U(x) = A(x) \sqrt{\frac{g}{h}}, \quad \Phi(\zeta) = \int f(\zeta) d\zeta. \tag{4} \]

It is important to mention that this exact solution coincides with an asymptotic solution of the wave equation for slowly varying depth, but now it is valid for any values of the bottom slope. The wave (2)-(4) propagates above uneven bottom without reflection. That is why we call such bottom profile \( h(x) \sim x^{4/3} \) the “non-reflecting” beach. Similar solution can be found for the wave equation written for the water particle velocity

\[ \frac{\partial^2 u}{\partial t^2} - c^2(x) \frac{\partial^2 u}{\partial x^2} = 0. \tag{5} \]

In this case the wave of velocity has a sign-variable shape \cite{2}. The rigorous solutions for traveling waves are also found for waves in inhomogeneous water channels (for example, an inclined channel of a parabolic cross-section) in both, linear and nonlinear cases. The role of the traveling waves in the water wave dynamics is discussed.

References
