**Discharge control by a side weir in a triangular main channel**

Contrôle des débits par un déversoir d'orage latéral sur un canal triangulaire

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**RESUME**

Les déversoirs d'orage latéraux ont été utilisés de manière extensive pour contrôler le niveau d'eau pour l'irrigation et les systèmes de canal de drainage, afin de dévier les excès d'eau dans les canaux des ouvrages de protection contre les crues. Ils ont également été utilisés comme déversoirs d'orage pour les réseaux d'assainissement urbain. Une solution analytique complète pour les équations qui régulent les débits dans les déversoirs d'orages latéraux n'est pas possible. Jusqu'à peu, des méthodes approximatives fondées sur des expérimentations réalisées à partir d'un éventail limité des nombreuses variables impliquées ont été utilisées. Dans de nombreux cas, l’usage de ce type de méthode a engendré des erreurs dans les déversements calculés. Le débit d’un déversoir d’orage latéral sur un canal rectangulaire a fait l’objet de nombreuses études, mais il n’y a pas d’études sur les déversoirs d’orages latéraux sur un canal triangulaire dans la littérature.

Cette étude fait état des études numériques de débit dans les déversoirs d’orages latéraux sur des canaux triangulaires. Les modèles numériques sont obtenus à partir de principes énergétiques sur la base d’une hypothèse d’énergie constante et solutionnés par la méthode de différence finie. Les résultats sont présents sous forme graphique. L’étude recouvre les régimes de débits sub-critiques et super-critiques. Les expressions dérivées sont comparées avec les résultats expérimentaux pour le déversoir d’orage latéral et pour les profils de surface d’eau pour ses régimes.

**ABSTRACT**

Side weirs have been used extensively for water-level control in irrigation and drainage canal systems, as a means of diverting excess water into relief channels for flood protection works, and as storm overflows from urban sewage systems. A complete analytical solution of the equations governing the flow in the side weir channels is not possible. Until quite recently, approximate methods based on experiments conducted over a limited range of the many variables involved have been used. In many cases, the use of such methods caused very substantial errors in the calculated spill discharge. The flow over a side weir in a rectangular channel has been the subject of many investigations but there are no more studies about side weirs in a triangular channel reported in the literature.

In this study, numerical investigations of flow over side weirs in triangular channels are reported. Numerical models are obtained from energy principles on the basis of a constant energy assumption and solved by a finite difference method. The results are presented in graphical form. The study covers both sub- and supercritical flow regimes. Derived expressions are compared with experimental results for the side weir discharge and water surface profiles for these regimes.

**KEYWORDS**

Side weir; discharge; discharge capacity; water surface; triangular channel.
INTRODUCTION

Side weirs have been used extensively for water-level control in irrigation and drainage canal systems, as a means of diverting excess water into relief channels for flood protection works, and as storm overflows from urban sewage systems. A complete analytical solution of the equations governing the flow in side weir channels is not possible. Until quite recently, approximate methods have been used based on experiments conducted over a limited range of the many variables involved. In many cases, the use of such methods cause very substantial errors in the calculated spill discharge. The flow over a side weir in a rectangular channel has been the subject of many investigations (Allen, 1957; Coleman and Smith, 1923; Collinge, 1957; Engels, 1920; Forchheimer, 1930; Frazer, 1954; Kumar, and Pathak, 1987; Ranga, et al 1979; Tyler, et al 1929). Probably the first theoretical approach to the hydraulics of a side weir in a rectangular channel was reported by De Marchi (1934). Theoretical and experimental studies for a side weir in a circular channel were reported in the literature include Uyumaz, (1982); Uyumaz, and Muslu (1985, 1987); Hager (1987); Uyumaz, and Smith (1981). Practically all the experimental work and theoretical analysis are confined to the flow over side weirs in rectangular and circular channels. There are no studies about side weirs in a triangular channel reported in the literature.

Methods of analyzing spatially varied flow in a channel with a side weir have recently been developed to give accurate computations for certain cases, which include subcritical flow along the weir, and supercritical flow both in the upstream channel and along the weir (De Marchi,1934; El-Khashab, 1975; El-Khashab and Smith, 1976).

In side weir channels, the flow is not affected by curvature and the rate of depth varies with distance along the main channel. In the case of subcritical flow, the water surface profile at the main channel axis rises from the upstream to downstream, whereas the opposite occurs in supercritical flow (El-Khashab, 1975; El-Khashab and Smith, 1976).

However, when the upstream and along the weir flows are supercritical, the depth remains below the critical depth just before the start of the weir. Also, the water surface is appreciably curved to the extent that it affects the distribution of pressure in the flow (Smith, 1973).

In the present study, general expressions for the surface profile along the side weirs in triangular channel are derived on the basis of a constant specific energy assumption.

FLOW EQUATION FOR SIDE WEIRS

The specific energy of the flow in the triangular main channel is assumed to remain constant. Referring to Fig.1, the specific energy at any point is

\[ H = h + \frac{v^2}{2g} \]  

in which “H” is a specific energy; “h” is the depth of water; “v” is the mean velocity of flow in the main channel; and “g” is the gravitation constant.
If “a” is the cross sectional area of flow and “Q” is a discharge at any point in the main channel, then

\[ H = h + \frac{Q^2}{2ga^2} \]  

(2)

and

\[ H = h + \frac{Q^2}{gh^2 \tan^2 \alpha} \]  

(3)

in which “\( \alpha \)” is the bottom angle of the triangular cross-section. Eq.3 can be rewritten as:

\[ Q = \frac{\tan \alpha}{\sqrt{2}} h^2 \sqrt{g(H - h)} \]  

(4)

General assumptions are that the flow in the side weir channel is approximately two-dimensional, and pressure in the channel is approximately hydrostatic despite some curvature and irregularity on the water surface. Although the flow over the side weir makes a considerable angle with the direction of the weir, a conventional weir equation for discharge per unit length is assumed as,

\[ -\frac{dQ}{ds} = q = m \sqrt{2g(h - p)(h - p)} \]  

(5)

where “m” is the side weir discharge coefficient; “p” is the weir height; and “s” is the distance measured along the channel. Provided that the proportion of the flow over the side weir is not excessive, these assumptions are approximately valid (El-Khashab, 1975; El-Khashab and Smith, 1976; Smith, 1973, 1974).

The change of discharge per unit length of a triangular channel due to the weir can be obtained by taking the derivative of Eq.4 with respect to “s” and substituting it into Eq.5. The results are given below:
\begin{align*}
\frac{dQ}{ds} &= \frac{\tan \alpha \sqrt{g}}{\sqrt{2}} \left(2h\sqrt{H-h} - \frac{h^2}{2\sqrt{H-h}}\right) \frac{dh}{ds} = -m\sqrt{2g(h-p)(h-p)} \\
(6)
\end{align*}

and

\begin{align*}
\frac{\tan \alpha}{2m} \left(2h\sqrt{H-h} - \frac{h^2}{2\sqrt{H-h}}\right) \frac{dh}{ds} &= -\sqrt{h-p(h-p)} \\
(7)
\end{align*}

Eq. 7 can be changed into dimensionless from through the following parameters that are obtained as a result of dividing the variables by the specific energy:

\begin{align*}
\frac{h}{H} &= z; \quad \frac{s}{H} = x \\
(8)
\end{align*}

These dimensionless parameters are then substituted into Eq. 7 to obtain a differential equation for the water surface profile along the side weir. The result is given below:

\begin{align*}
\frac{dz}{dx} &= -\frac{\sqrt{z - \frac{p}{H}(z - \frac{p}{H})}}{\frac{\tan \alpha}{2m}(2z\sqrt{1-z} - \frac{z^2}{2\sqrt{1-z}})} \\
(9)
\end{align*}

On the other hand by considering the following functions

\begin{align*}
f_1(z) &= (2z\sqrt{1-z} - \frac{z^2}{2\sqrt{1-z}}) \\
(10)
\end{align*}

and

\begin{align*}
f_2(z) &= \sqrt{z - \frac{p}{H}(z - \frac{p}{H})} \\
(11)
\end{align*}

One can obtain from the rearrangement of Eq. 9, that

\begin{align*}
\frac{dx}{dz} &= \left[\frac{\tan \alpha}{2m} \left| \begin{array}{c}
f_1(z) \\
f_2(z)
\end{array} \right. \right] \\
(12)
\end{align*}

Making use of the aforementioned the dimensionless variables it is possible to see that

\begin{align*}
\frac{s}{H} &= x \rightarrow \frac{ds}{H} = dx \\
(13)
\end{align*}

Hence, its substitution into Eq. 12 leads to

\begin{align*}
\frac{ds}{dz} &= \left[\frac{\tan \alpha H}{2m} \left| \begin{array}{c}
f_1(z) \\
f_2(z)
\end{array} \right. \right] \\
(14)
\end{align*}
Furthermore, integration both sides yields

\[ \int_{s_1}^{s_2} ds = -\frac{\tan \alpha H}{2m} \int_{z_1}^{z_2} f_1(z) \, dz \quad \text{(15)} \]

Which can be defined as

\[ L = \frac{\tan \alpha H}{2m} \left[ \phi(z) \right]_{z_1}^{z_2} \quad \text{(16)} \]

or

\[ L = \frac{\tan \alpha H}{2m} \left[ \psi(z) \right]_{z_1}^{z_2} \quad \text{(17)} \]

in which "L" is the length of the side weir, and "\phi(z)" and "\psi(z)" are integral areas. In the derivation of the foregoing equations it is assumed that the width of the water surface in the triangular channel is small compared with the length of the side weir. For a side weir conveying flood waters from a natural channel, this condition may not be satisfied and it may be necessary to consider the flow in the channel as three-dimensional, requiring a different analysis, or model study. Eq.17 gives the length of the side weir for a triangular channel when the water depth varies from "h_1" to "h_2" along the side weir at the main channel. Eq.14 is solved by using the finite difference method and the results are shown graphically in Fig.2 for practical purposes of uses.

**DISCHARGE COEFFICIENT FOR SIDE WEIR**

In Eq.17, the parameter "m" presents the discharge coefficient of the side weir. In this approach the discharge coefficient for a side weir in a rectangular channel is modified for the discharge coefficient for a side weir within a triangular channel, where the equivalent rectangular channel width is calculated as the ratio of the wet cross-sectional area to water depth. A detailed procedure for the determination of the discharge coefficient for sub- and supercritical flow regimes for a rectangular channel is presented by Subramanya and Awasthy (1972). The discharge coefficient for subcritical regimes for Froude number \( F_1 < 1 \) was expressed as:

\[ m = \frac{2}{3} \left( 0.611 + \frac{3F_1^2}{F_1^2 + 2} \right) \quad \text{(18)} \]

and for supercritical regimes where \( F_1 > 2 \) as:

\[ m = \frac{2}{3} \left( 0.36 - 0.008F_1 \right) \quad \text{(19)} \]

Herein, "F_1" is the Froude number at the beginning of the side weir on the axis of the main channel.

Derived expressions for side weir water surface profiles, and discharge for subcritical and supercritical regimes have been compared with experimental results. The analysis indicated errors of less than 12% as shown in Figs.3-6 (Uyumaz, 1982; 1989 and 1991).
However, these equations will need further extensive checking from laboratory and field data to verify their applicability for design purposes.
CONCLUSIONS

The theoretical procedures have been presented for calculating flow profiles and discharges along a side weir in a triangular shaped channel. In the case of the subcritical regimes, the water profile at the main channel axis rises from upstream to downstream whereas the opposite occurs in the supercritical regime.

Nevertheless, computation techniques based on the constant energy solution give results that are usually considerably more correct than empirical curves or equations based on limited range experiments. However, in this study the specific energy in a triangular side weir channel remains constant.
REFERENCES


