Stability of a Rotating Cylindrical Shell Containing Axial Viscous Flow

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Abtract :
A viscous flow model was developed to investigate the stability of a rotating cylindrical shell containing axial flow because the inviscid model was shown to be inadequate due to singularities in the solution. It was found that the stability of the system is very sensitive to the treatment of the shell-fluid interface and that a small rate of rotation tends to stabilise the system.

Résumé :
Un modèle d'écoulement visqueux a été développé pour étudier la stabilité d'une coque cylindrique contenant un écoulement axial parce que le modèle de fluide parfait a été démontré comme étant inadéquat. Il a été montré que la stabilité du système est très sensible à la modélisation de l'interface coque-fluide et qu'un faible taux de rotation tend à stabiliser le système.

Key-words : fluid-structure interactions; shear flow ; swirling flow

1 Introduction

Ever since the seventies, the dynamics of thin cylindrical shells coupled with subsonic flows has been of interest to engineers. Païdoussis and Denise (1972) made the fortuitous discovery that shells containing low-speed flow do flutter and paved the way to a resumption of interest in this area of research in fluid-structure interactions (FSI). Since then, the research done on the dynamics of shells interacting with fluid has evolved in different branches. Païdoussis (2004) offers a full review of the field.

The present paper studies the stability of a rotating cylindrical shell containing a co-rotating axial viscous flow. Practical examples of such a system include swirling flow in dual spool aircraft jet-engines, rotating drums used in the process industry, some nuclear applications [Païdoussis (2004, section 7.6)], spin-stabilised rockets, and piping in a rotating space-station.

In Lai and Chow (1973), the stability of a rotating thin shell containing inviscid fluid flow was investigated. There, just as in the present study, both the fluid and the shell are rotating about the axis of the cylindrical shell at a constant rate. The linear Donnell-Mushtari shell theory is modified to account for solid body rotation and is coupled to an analytical inviscid fluid solution, similar in essence to that derived by Chow (1969) for swirling flow in a varicose tube. It was found that the critical flow rate in the shell decreases with increasing angular velocity. Using essentially the same method of solution, Chen and Bert (1977) studied the dynamics of a stationary (non-rotating) shell carrying a rotating flow. Once again, it was found that rotation severely decreases the stability of the system.

The validity of the results obtained by Lai and Chow and Chen and Bert was questioned by Cortelezzi et al. (2004), who found it impossible to reproduce the results of the former when rotation is present. Without rotation, the solution for pressure in the fluid is continuous, but in the presence of rotation, the solution is not bounded in particular regions of the parameter space. The presence of singularities in the flow solution makes it impossible to obtain the critical velocity of the system. It is emphasised that the problem lies within the flow solution and is possibly due to the absence of viscous effects.

The objective of the present study is to develop a model to predict the stability of a FSI system comprising a rotating cylindrical shell conveying a co-rotating fluid. To hopefully avoid the singularities in the flow solution with rotation, the flow model includes viscous effects.

2 Theoretical model

Consider a cylindrical shell of radius R, thickness $h \ll R$ and of infinite length. This shell contains an axial incompressible viscous flow. The whole system is in a frame of reference rotating about the axis of the cylinder at rate Ω as illustrated in Fig. 1. The deformation of the shell is quantified by the three orthogonal time-dependent components of deformation u, v and w (shown in Fig. 1). The stability of the system is assessed by studying whether the shell deformations grow or decay with time.

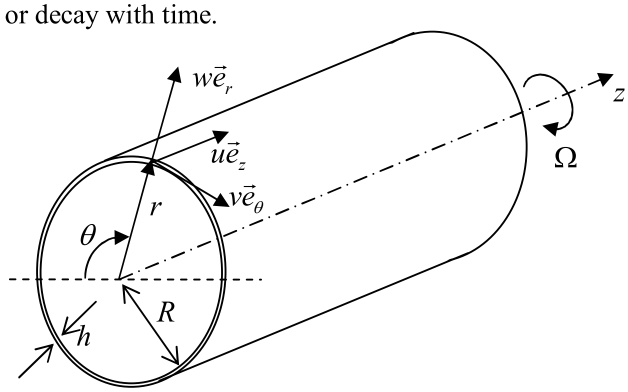

Fig. 1. Schematic of the system.

The equations of motion for the coupled fluid-structure system can be found by summing the structural and the fluid forces acting on the shell. The structural forces are obtained via the linear Donnell-Mushtari thin shell theory modified to account for the rotating frame of reference (Lai and Chow 1973). The shell deformation imposes a displacement of the fluid at the shell-fluid interface which produces a flow perturbation. This flow perturbation, obtained with the linearized Navier-Stokes equations, generates a force on the shell-fluid interface.

A solution to the motion of the shell is sought in the form of a travelling-wave:

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} \equiv \begin{Bmatrix} \bar{u} \\ \bar{v} \\ \bar{w} \end{Bmatrix} \exp\left(-ikz - in\theta + i\omega t\right) = \begin{Bmatrix} \bar{u} \\ \bar{v} \\ \bar{w} \end{Bmatrix} \exp\left(i\alpha\right), \tag{1}$$

where ω, k and n are the frequency, axial wavenumber and circumferential wavenumber, and $\bar{u}$, $\bar{v}$ and $\bar{w}$ are the initial amplitudes much smaller than R. A dispersion relation (an equation relating the admissible frequencies to the wavenumbers) is obtained after the travelling-wave solution is substituted in the equations of motion of the system.

We define the fluid vector field components as V_z, V_θ, V_r. For a no-slip boundary condition, the velocity of the fluid at the deformed position of the wall is simply the Lagrangian derivative of the displacement of the wall:

$$V_r = \frac{Dw}{Dt}, \qquad V_\theta = \frac{Dv}{Dt}, \qquad V_z = \frac{Du}{Dt}, \qquad \text{at } r = R + w. \tag{2}$$

The velocity of the fluid is decomposed into two distinct parts, namely the steady mean flow velocity and the unsteady perturbation velocity:

$$\vec{V}\left(r,\theta,z,t\right)=\vec{V}_0\left(r\right)+\vec{v}\left(r\right)e^{i\alpha}=\begin{Bmatrix}0\\0\\U\left(r\right)\end{Bmatrix}+\begin{Bmatrix}v_r\left(r\right)\\v_\theta\left(r\right)\\v_z\left(r\right)\end{Bmatrix}e^{i\alpha}, \tag{3}$$

where the basic flow is defined as an axial flow with a power law profile. Substituting eq. (3) into eqs. (2), the perturbation boundary conditions can be approximated at the instantaneous position of the wall via a Taylor expansion about the mean position of the wall and keeping only first order terms:

$$v_r=\left[i\omega-ikU\right]\overline{w},\ v_\theta=\left[i\omega-ikU\right]\overline{v},\ v_z=\left[i\omega-ikU\right]\overline{u}-\frac{\partial U}{\partial r}\overline{w},\quad \text{at } r=R. \tag{4}$$

One can notice the added complexity when comparing the boundary conditions of eqs. (4) with those for the inviscid theory of Lai and Chow (1973); for free-slip and impermeability conditions, only the transverse displacement w affects the flow, as shear cannot be transmitted by inviscid flow. With the inclusion of viscosity in the model, all three shell perturbation components influence the flow. Hence, we are dealing with a fluid flow subjected not to one but three perturbations. We develop a "triple-perturbation" method, in which three perturbation flow fields are superposed, each proportional to one of the three components of the shell deformation.

These three superposed flow solutions are obtained by numerically integrating the Navier-Stokes equations for the three perturbation components using a finite difference scheme. To maximise simplicity and efficiency, the numerical scheme makes use of flux quantities [see Verzicco and Orlandi (1996)] and a staggered grid [see Harlow and Welch (1965)] to avoid iterating, get rid of the singularity at the centre of the flow and maximise the accuracy for the number of grid points.

In the inviscid model, there are two terms in the fluid boundary conditions which transmit the structure's slope (or position) $-ikw$ and its velocity $i\omega w$ to the flow solution. This generates centrifugal (added stiffness), Coriolis and added mass forces among others acting on the structure. But for the linear boundary conditions of viscous flow, since the mean flow velocity is zero at the wall, the position-dependent term disappears from eqs.(4), as $U\left(r\right)\big|_{r=R}=0$. Because all effects relative to the slope of the wall are removed, the dynamics of the system is greatly impoverished.

We resolve this problem in a similar manner as was done by Dowell (1971) for the shear flow over a flat plate: instead of applying the boundary conditions at the undeformed shell position $r=R$, they are applied at an assumed deformed position $r=R-\delta$. The fluid forces acting on the wall are also taken at $r=R-\delta$ instead of $r=R$; cf. El Chebair et al. (1990).

It is convenient to describe the dynamics in terms of dimensionless quantities by defining

$$\gamma=R\sqrt{\rho_s\frac{1-\nu^2}{E}},\ \bar{U}_R=\frac{\bar{U}\gamma}{R},\ \delta'=\delta/R,\ \Omega'=\Omega\gamma,\ \omega'=\omega\gamma,\ k'=kR, \tag{5}$$

respectively the dimensional timescale, the mean reduced velocity, the deformation scale parameter, the dimensionless rotation rate, frequency and axial wavenumber.

3 Purely axial flow

It has been shown (Païdoussis 2004) that a FSI model of a cylindrical shell conveying fluid using linear inviscid theory can predict acceptably well the critical $\bar{U}_R$. Therefore, we expect that in the absence of rotation the added realism of taking viscous effects into account should not change the predicted critical flow velocities too much vis-à-vis the inviscid theory results of Lai and Chow (1973).

The simulations are done for a rubber tube conveying water with parameter values equal to those defined in Lai and Chow. In Fig. 2, the frequency evolutions obtained with the inviscid and the viscous theories are compared. The parameter δ' is increased from 0, to 0.1 and 1 in Fig. 2 (a-c). For $\delta'=0$, the system is overly unstable. The reason for this is believed to be related to the application of the fluid boundary conditions at the mean position of the shell wall instead of its instantaneous deformed position as discussed in Section 2. In Fig. 2 (b) and (c), as δ' is increased, the frequency curves become more similar to the inviscid ones and so does the onset of instability. This last point is emphasised in Fig. 3, where the critical flow velocity predicted with the viscous model approaches the inviscid prediction for increasing values of δ'. If δ' is varied between 0.005 and 0.01 the critical flow velocity does not change appreciably. This is reassuring in the sense that even if the value of the deformation scale parameter is roughly estimated, the critical flow velocity prediction is robust.

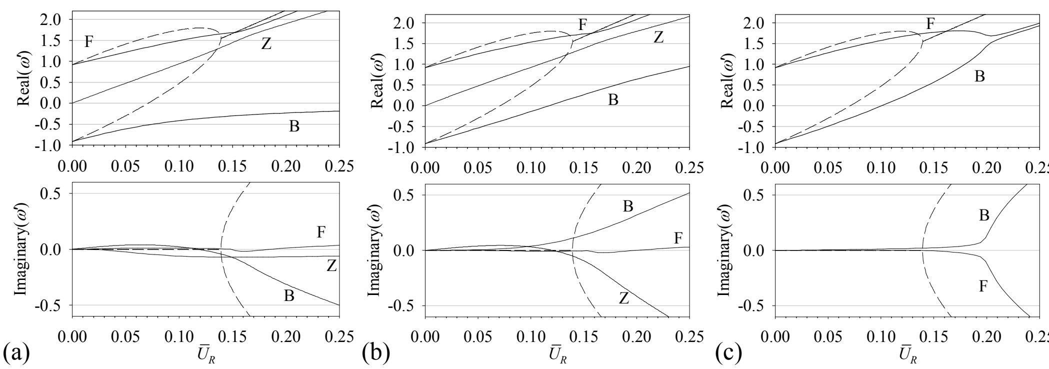

Fig. 2. Effect of the parameter δ' on the frequency evolution of a vibrating non-rotating tube with axial flow at Re=10^5 (——), for dimensionless wavenumbers *k'*=15 and *n*=8, for (a) δ'=0; (b) δ'=0.005; (c) δ'=0.01. The letters F and B indicate the forward and backward travelling waves and the letter Z indicates the travelling wave emanating from the origin. The inviscid results are plotted for comparison (--------).

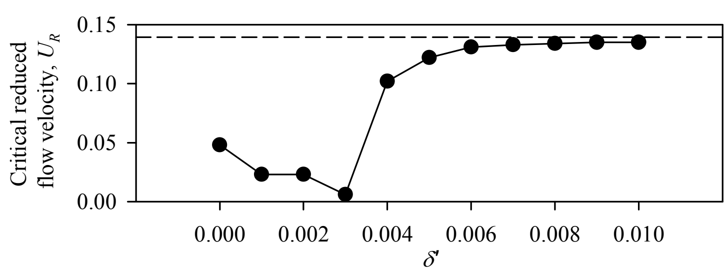

Fig. 3. Comparison of the prediction of the onset of instability of the viscous model (—●—) at Re=10^5 and the inviscid theory (--------) for the dimensionless wavenumbers *k'*=15 and *n*=8.

4 Effect of rotation

In these simulations, for a given frequency evolution plot, the Reynolds number is incremented with the reduced velocity so as to keep the Stokes number constant $\mathrm{St}=\mathrm{Re}/\bar{U}_R$.

The physical properties of a rubber tube containing water flow give $\mathrm{St} = 1\ 220\ 795$. Simulations are performed for this Stokes number and also for one 100 times smaller; this latter would be representative of a system with a fluid a hundred times more viscous, such as oil.

The effect of rotation on the stability of the system can be seen by comparing Fig. 4 (a) and (b) for a lightly viscous flow. With the addition of a rotation rate of $\Omega' = 0.05$, the onset of instability increases from $\bar{U}_R = 0.13$ to 0.15. The same effect can be seen for a more viscous flow, not shown here for brevity. The effect of the addition of a small rate of rotation is shown in Fig. 5. For the two modes studied and for high or low viscosity, the addition of a small rate of rotation stabilises the system. It would seem though that, if subjected to higher rates of rotation, the critical velocity of the system should decrease. Srinivasan and Lauterbach (1971) showed that, if spun fast enough, empty cylindrical shells become unstable. One would think that the same applies to fluid filled shells. If the shell-fluid system is spun fast enough, it will be unstable even without flow. So, before that critical rotation rate is reached, the critical flow rate should decrease.

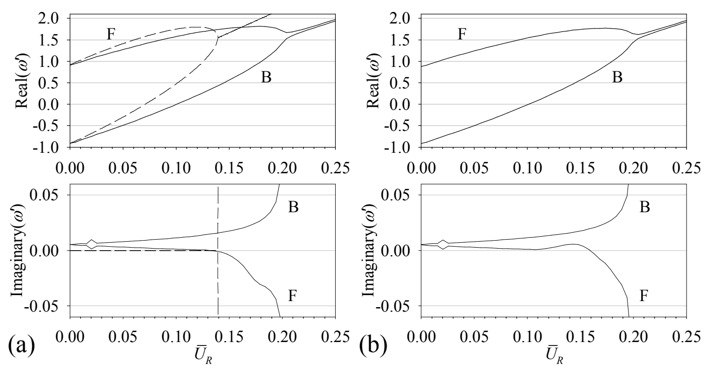

Fig. 4. Effect of rotation on the prediction of the frequency evolution for dimensionless wavenumbers k'=15, n=8, δ'=0.01 and St=1 220 795 (——), for (a) Ω'=0; (b) Ω'=0.05. The inviscid results for the case without rotation are plotted for comparison (--------). The letters F and B indicate the forward and backward travelling waves.

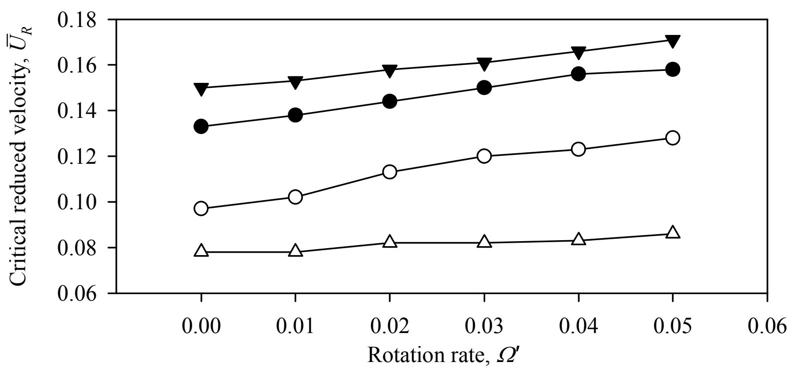

Fig. 5. Effect of rotation on the prediction of the onset of instability for δ'=0.01: —●— , St=1220795, n=8, k'=15; —▼— , St=12208, n=8, k'=15; —○— , St=1220795, n=0, k'=10; —△— , St=12208, n=0, k'=10.

The results for higher rates of rotation are not presented because they were deemed unsatisfactory. It seems that the phenomenon which gives rise to singularities in the inviscid theory leads to additional frequencies in the viscous model in the same region of the parameter space. In the viscous model, these are not singularities, but simply frequencies arising from the fluid. The problem is that the method employed (based on a Müller algorithm) for finding the roots of the dispersion relation is not appropriate for finding these additional frequencies. At high Ω', the program would jump from one solution to another and it is very hard to obtain smooth curves. It must be emphasized that this issue is not due to singularities or inconsistencies in the model; it appears to be solely due to the root-finding method employed.

Since for any non-zero rate of rotation the inviscid model has singularities, it is possible that, even in the results presented here for small rates of rotation, some additional frequencies exist, but due to the root-finding method, they were not found. On the other hand, engineering judgement would lead us to believe that a frequency arising from the addition of a small rate of rotation should not influence the dynamics of the system dramatically.

Conclusion

The stability of a rotating cylindrical shell containing a co-rotating axial viscous flow has been studied through the development of a theoretical FSI model. Learning from the flaws and breakdown of the inviscid model, it was believed that the added realism brought about by the introduction of viscosity in the theory would lead for a successful model.

Similarly to the problems encountered by Dowell (1971) and El Chebair et al. (1990), it was found that the usual technique for linear aeroelasticity studies, consisting of applying the fluid boundary conditions at the undeformed position of the wall instead of the instantaneous deformed position, greatly alters the stability characteristics of the system. Since for the viscous model the mean flow velocity is zero at the wall, the shell-position-dependent terms in the fluid boundary conditions disappear. To remedy this problem, the boundary conditions are applied at an assumed deformed position just off the shell mean position.

The dynamics of the system subjected to purely axial flow with no rotation was successfully studied. For acceptably small values of δ', the prediction of the onset of instability of the system tended towards that predicted by the inviscid model.

The results obtained for small Ω' showed that the system tends to be stabilised when subjected to a small rate of rotation. On the other hand, it seems reasonable to suppose that this trend should be reversed for higher rates of rotation; but it was impossible to show this, due to the limitations of the root-finding method employed. The important finding of this work is that the addition of viscosity in the theory allows successful modelling of the system subjected to rotation. The flow solution no longer breaks down as was the case with inviscid theory.

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