Dynamics of vesicles under shear flow


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Abstract :

Giant vesicles are deformable lipid membranes enclosing a fluid, with diameters of several microns. They are useful models for the study of biological flows. Their behaviour under shear flow reveals a complex dynamics involving several regimes: stationary tank-treading motion, periodic flipping or tumbling motion, and periodic vacillating-breathing motion. Our experimental study reveals that the deformability of vesicles strongly influences the dynamics. One noticeable feature is a slowing down of the tumbling motion with respect to the shear rate when the later increases. A semi-phenomenological model including the deformability of objects helps understand the connection between the slowing down of the tumbling motion and a coupling between rotation and deformation. A new vacillating-breathing motion is also predicted and a complex phase diagram can be established.

Résumé :

Les vésicules géantes sont des membranes lipidiques déformables renfermant un liquide, d’un diamètre de quelques microns, utiles pour la modélisation d’écoulements biologiques. Leur comportement en Écoulement de cisaillement est une dynamique complexe qui se manifeste par l’existence de plusieurs régimes : mouvement stationnaire de chenille de char, mouvement périodique de bascule, et mouvement d’oscillation-respiration. Notre étude expérimentale révèle une grande sensibilité de la dynamique à la déformabilité de l’objet. Cela se traduit par une décroissance notable du rapport entre fréquence de tumbling et taux de cisaillement lorsque celui-ci augmente. Une modélisation semi-émélogique prend en compte la déformabilité des objets permet de démontrer le lien entre le ralentissement relatif du tumbling et un couplage entre rotation et déformation. Ce couplage permet par ailleurs de prévoir l’existence d’un nouveau mode dynamique d’oscillation-respiration et d’Otablir un diagramme de phase complexe de la dynamique.

Key-words :

vesicle ; fluid-structure interaction ; complex fluids

1 Introduction

Vesicles are closed fluid membranes suspended in an aqueous solution. They may be viewed as a simple model to represent the viscoelastic properties of a real cell and their behaviour under shear flow has been the subject of several theoretical (Keller & Skalak (1982); Seifert (1999); Noguchi & Gompper (2004); Misbah (2006)) and experimental (Haas (1997); Abkarian & Viallat (2005); Kantlsr & Steinberg (2005); Mader et al. (2006); Kantlsr & Steinberg (2006)) studies. Vesicles under flow have so far revealed several nonequilibrium features that are shared by red blood cells. Capsules, which bear some resemblance with vesicles have also received a lot of attention (Barthes-Biesel & Sgaier (1985); Ramanujan & Pozrikidis (1998)).

The mechanical characteristics of vesicles are simple. Their internal fluid is usually a viscous liquid, and their membrane is a bidimensional fluid that is almost incompressible (high stretch modulus) and quite flexible, its equilibrium shape being governed by Helfrich’s curvature energy. Two types of motion are predicted and observed under shear (fig. 1): for
slightly deflated vesicles, when the viscosity ratio between the interior and the exterior is low, $(\nu = \eta_{\text{in}}/\eta_{\text{out}}, \text{with } \eta_{\text{in}} \text{ and } \eta_{\text{out}} \text{ the internal and the external viscosity respectively})$ vesicles reach a stationary orientation while its membrane undergoes a tank-treading motion. Above a critical value of the viscosity ratio, $\nu > \nu_c$, which depends on the reduced volume $\tau$ (the ratio between the volume of the vesicle and the volume of a sphere having the same surface area) a transition to a periodic tumbling motion occurs.

In this paper, we present the main results of our experimental study of tumbling dynamics and show the influence of deformability on the observed motion. Semi-phenomenological models are also presented, which recover the main qualitative features of the experimental dynamics. A new dynamical mode called vacillating-breathing, also revealed by recent theoretical models, and possibly seen in other experiments, is predicted by this model.

2 Experimental study of tumbling dynamics

![Image of vesicles](image)

Figure 1: Top: Tank treading motion of a vesicle under shear flow. Bottom: tumbling motion of a viscous vesicle filled with dextran (Viscosity ratio $\nu = 10.1$, reduced volume $\tau = 0.878$, $R_0 = 7.5\mu m$, time interval between pictures $\Delta t = 0.4 s$, shear rate $\dot{\gamma} = 1.6 s^{-1}$).

Giant vesicles were prepared from dioleoyl-phosphatidylcholine (DOPC) using the electroformation method of Angelova et al. (1992). Vesicles were swollen in aqueous solution with different concentrations of glucose and a polymer (referred to as interior solution), to adjust the inner osmolarity and viscosity. Two types of polymer where used: dextran or sodium carboxymethylcellulose (CMC) to adjust the viscosity ratio between the internal and external fluids $\nu = \eta_{\text{in}}/\eta_{\text{out}}$ between 8.7 and 19.9. Vesicles were diluted in an external sucrose solution in order to deflate them slightly by osmose and change their reduced volume $\tau$.

Vesicles were then injected in a flow chamber with dimensions $L = 45 \ mm$ along the flow axis $x$, $b = 10 \ mm$ along the $y$ axis and $a = 1 \ mm$ along the $z$ axis, and a constant flow rate was imposed with a KDS syringe pump. A nearly 2D Poiseuille flow takes place inside the chamber with a parabolic velocity profile across the thickness $a$, which is approximately invariant in the $x$ and $y$ directions. The flow chamber is mounted on a phase contrast microscope stage for observation.

For each observed vesicle, we recorded the orientation angle of the longest axis of inertia, and the axis lengths during the motion. An exemple of dynamics is shown of fig. 2.

The reference model for vesicle dynamics under shear was established by Keller & Skalak (1982) for ellipsoidal objects with fixed shape and a fluid membrane. In this model (denoted KS), the vesicle’s inclination is governed by:

$$\frac{d\psi}{dt} = A + B \cos(2\psi) \quad (1)$$

where $A = \gamma / 2$ is the rate of rotation imposed by the external shear flow and $B$ is the amplitude of tumbling velocity fluctuations, due to the elongational part of the flow, and depends on vesicle properties.
Figure 2: Left: Orientation of a vesicle during tumbling. Right: fluctuations of the long axis (black) and short axis (white).

Experimental measurements of \( \psi \) vs. time were fitted using equation 1 to provide values of \( A \) and \( B \) for each vesicle. A striking result is that the dynamics significantly deviates from the KS model when the capillary number \( Ca = \eta_o uR^3\dot{\gamma}/\kappa_c \), comparing external viscous forces and the vesicle’s bending energy which tends to bring it back to equilibrium shape, in increased. Figure 3 shows that both \( A \) and \( B \) undergo a sharp decline above a critical value of the capillary number, leading to a global slowing down of the tumbling motion. This observation hints at a nontrivial coupling between deformation and rotation.

Figure 3: Experimental tumbling parameters \( A \) and \( B \) vs. capillary number for 22 different vesicles.

3 Models

In order to understand the interplay between deformation and rotation in vesicle dynamics, we propose phenomenological models that capture the main physical ingredients and qualitative features observed in experiments. One reason is that more exact calculations are either purely numerical (based for instance on boundary integral methods), or restricted to small deformation theory in the case of analytical methods, which involves heavy algebra to go beyond the smallest order of expansion. Therefore we believe simple, but reasonably realistic semi-
phenomenological models can bring a lot of physical insight on the phenomena.

To set this model up, one needs i) an evolution equation for the vesicle’s orientation angle that reasonably describes tumbling dynamics for a fixed shape, and ii) an evolution equation for the shape.

3.1 A rectangular vesicle

In order to get a simple and fully analytical expression for the orientation angle, we propose a simple model of 2D rectangular vesicle, which captures the essence of tumbling dynamics far from the transition from tumbling to tank-treading Mader et al. (2007). The orientation $\psi$ follows the same equation as in KS, with the following expressions for tumbling parameters:

$$A = -\frac{1}{2}, \quad B = \frac{(\bar{\ell} - 1)(\bar{\ell}^2 + 4\bar{\ell} + 1)}{2 \left[(\bar{\ell} + 1)^3 - \frac{12\lambda \bar{\ell}^3}{(\nu(\bar{\ell} - 1)^2 + \lambda(\bar{\ell} + 1))} \right]}$$

(2)

where $\bar{\ell}$ is the aspect ratio of the vesicle (length/width), $\bar{\lambda}$ is a friction coefficient and $\nu$ the viscosity ratio.

The equation governing shape is:

$$\frac{d\bar{\ell}}{dt} = -\frac{\bar{\ell} - \bar{\ell}_{eq}}{\chi} + \zeta \bar{\ell} \sin(2\psi)$$

(3)

where $\chi$ is a capillary number comparing the shear rate and the relaxation time of the vesicle shape, and $\zeta$ a friction coefficient. The first term on the right side of eq.3 is a relaxation term and the last term is the extension rate of the vesicle due to the elongational part of the flow.

After solving eqs. 1 together with 2 and 3 for a given set of parameters and by varying the shear rate (or capillary number), one can determine the effective constant $A$ and $B$ parameters, denoted $A_{eff}$ and $B_{eff}$, as plotted in fig. 4. The generic qualitative features observed in the experiment are then recovered, namely a slowing down of the motion and a decrease of $A_{eff}$ and $B_{eff}$ as $\chi$ increases.

![Figure 4: A and B vs. $\chi$ for the rectangular vesicle model](image-url)
3.2 A phenomenological modification of Keller and Skalak’s model

To study dynamics close to the tumbling-tank treading transition, and account for possible 3D effects (including saturation of deformation), we chose to complement the KS model with a phenomenological equation for deformation based on a simplified balance between external stresses due to the elongational flow, membrane curvature forces and viscous friction inside the vesicle. The resulting equation is:

\[
\frac{\nu \, dr_2}{r_2 \, dt} = -\frac{1}{\chi} \frac{\partial U(r_2, r_3, \tau)}{\partial r_2} - \sin 2\psi
\]  

(4)

where \(r_2\) is the ratio of short axis over long axis in the plane of shear (as defined in KS) and \(U(r_2, \tau)\) is the curvature energy of a vesicle of reduced volume \(\tau\) and aspect ratios \(r_2\) and \(r_3\). This energy is computed from Helfrich’s model and the aspect ratio in the third direction \(r_3\) is determined by the constant reduced volume condition.

A rich variety of dynamical modes can be computed from this regime. For a vesicle whose properties (reduced volume and viscosity ratio) place it in the tumbling regime and not too far from the transition curve according to KS (\(B = 0.49\) for the equilibrium shape), one gets tumbling at low \(\chi\), a slowed-down tumbling at intermediate \(\chi\), then vacillating-breathing (VB), where the angle oscillates with a finite amplitude while shape undergoes strong oscillations. This regime has been predicted by small deformation theory Misbah (2006), and one experimental example of a similar regime has been reported (called "trembling" in Kantlser & Steinberg (2006). Finally at higher capillary number, the amplitude of the VB mode drops to zero and the inclination relaxes to a tank treading regime with a negative stationary angle (see fig. 5). This is quite remarkable since in the classic tank treading regime, the negative solution to \(0 = A + B \cos \psi\) in the KS model is unstable.

For vesicles with viscosity ratios below the KS critical ratio, one gets tank treading at all capillary numbers, while for viscosity ratios higher than a second threshold, tumbling is always observed.

![Figure 5: Different regimes of \(\psi\) vs. time computed for a given vesicle (\(\tau = 0.9\) and \(\nu = 5.6\)) when increasing the capillary number \(\chi\) (from left to right: tumbling at \(\chi = 0.1\), slower tumbling at \(\chi = 5\), vacillating-breathing at \(\chi = 6\), relaxation to tank-treading with negative stable angle at \(\chi = 10\)).](image)

4 Conclusions

Our experimental study of the tumbling dynamics of vesicles under shear flow revealed that deformability would not be neglected at high shear rates and was responsible for a relative slowing down of the tumbling motion. Simple models coupling a classic equation for tumbling dynamics and a phenomenological equation for deformation can recover the main features of
experimental, numerical and analytical results, which underlines the generic character of the observed dynamics. A new dynamical mode, vacillating-breathing, is predicted by this and other models, near the transition curve from tumbling to tank-treading, but has not been clearly characterized experimentally yet, although one author shows some evidence about it. A detailed experimental study of this mode is clearly needed since the details of the different modes and transitions between them should strongly influence the rheology of vesicle suspensions or other suspensions of deformable objects such as blood.

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