On the baroclinic vortex instability: the scattering problem

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Abtract :

The stability of an isolated vortex in a two-layer stably stratified fluid on a rotating plane (f-plane approximation) is studied. In the case of axially symmetric vortex structure consistent of a set of vortex patches, the problem of linear instability with respect to small disturbances of the patches' boundaries may be solved analytically. Dispersion relationships allow studying the stability properties of the vortex, depending on the respective thickness of the layers, radii of the vortex patches, the medium stratification and the value of potential vorticity.
Contour Dynamics Method (CDM) allows investigation of the non-linear stage in the evolution of unstable vortex structures, and, in particular, the time law of scattering of newly formed two-layer vortices of smaller size. When unstable vortices with zero total vorticity (heton) decays, the scattering is shown to follow the ballistic law. The obtained results can be used in the problems of deep convection for describing the process of spreading of heat and density anomalies.
The use of CDM gives also possibility to examine the stability and peculiarities of the further evolution on non-axially symmetric vortex structures – the hetons with "tilted axis" and two-layer quasi-elliptical vortices.

Key-words : heton, CDM, instability

1 Introduction

We investigate the stability of two-layer vortices with piecewise constant distribution of potential vorticity in the layers, and in particular, the non-linear stage of evolution of unstable vortex patches.

If the summary potential vorticity of the two-layer vortex system is equal to zero, we deal with the so called hetons (Hogg and Stommel 1985). In the conditions of a stable stratification and a hydrostatic equilibrium such vortices possess abnormal heat content. After the unstable heton decay there form two-layer vortices of smaller scale which scatter from the core of the initial vortex, and in that way they are the carriers of heat anomalies. Legg and Marshall (1993) proposed a heton analogy for explaining the mechanism of deep convection formation in the ocean that has become widely known (Marshall and Schott 1999). In the present work a special attention is paid to the numerical investigation of the thermal anomaly front motion, which is associated with evolution of the external boundary of the vortex structure.

2 On linear stability analysis of axially symmetric heton

Let us examine an undisturbed axisymmetric heton, which consists of two-layer cylindrical structure of unit radius circular patches located strictly one under another. Such state is a stationary solution of the equations of potential vortex conservation

$$\frac{d_j \Pi_j}{dt} = 0 ,$$

where

$$\Pi_j = \nabla^2 p_j + F_j \left(p_{3-j} - p_j \right), \quad j = 1, 2 .$$

Here $\frac{d_j(\ )}{dt} \equiv \overset{\bullet}{(\ )} + u_j \frac{\partial(\ )}{\partial x} + v_j \frac{\partial(\ )}{\partial y}$ and $\nabla^2(\ ) \equiv \frac{\partial^2(\ )}{\partial x^2} + \frac{\partial^2(\ )}{\partial y^2}$ - are two-dimentional operators: a total derivative with respect to time and the Laplacian one; a dot above a variable means a partial derivative with respect to time ; u_j, v_j - are the components of the velocity vector of liquid particles in a j layer of thickness h_j along the axis of the orthogonal coordinate system x and y correspondingly; $F_j = 4\rho^* \Omega^2 D^2 h_j / g(h_1 + h_2)\Delta\rho$, ρ^* - is the mean value of the density, $\Delta\rho = \rho_2 - \rho_1$; D - is a characteristic linear scale, g - the gravitational acceleration, p_1 and p_2 - are anomalies (with respect to the equilibrium hydrostatic state) of the hydrodynamic pressure in the layers, connected with the velocity components by geostrophic relationships

$$u_j = -\frac{\partial p_j}{\partial y}, \quad v_j = \frac{\partial p_j}{\partial x} .$$

The question about the stability of the corresponding axisymmetric solution is lawful. Let the liquid lines, coinciding with external boundaries of the vortices, be described with parametric relations

$$r = f_j(\theta, t; \alpha), \quad \alpha = f_j(\theta, 0; \alpha), \quad j = 1, 2,$$

where the parameter α characterizes a radial Lagrangian coordinate of the points belonging to the contours, and θ- is a polar angle. Let's write the function f_j in the form

$$f_j(\theta,\ t;\ 1) = 1 + \varepsilon_j(1) \exp[im(\theta - \delta_m t)], \quad \left|\varepsilon_j\right| << 1, \quad j = 1, 2, \quad m \geq 1 ,$$

here m - is the number of the azimuthal mode. It is obvious, that for the modes, unstable in the linear approximation, the condition $Im\delta_m > 0$ should be fulfilled. The algorithm of the stability examination is given in (Kozlov *et al* 1986). In essence, the problem reduces to the analysis of the system of linear algebraic equations with respect to ε_j . The stability analysis shows that at $h_1 = h_2 = 1/2$ (the discussed below parameters correspond to this particular case) the modes with $m \geq 2$ and $\gamma = \sqrt{F_j / h_{3-j}} > 1.7$ may be unstable.

3 Numerical modeling of the nonlinear stage of the heton evolution

A convenient instrument for the numerical study of the evolution of unstable vortex structures is the Method of Contour Dynamics (CDM) which allows calculating the configurations of the boundaries of the vortex patches at any time moment. The two-layer version of CDM was for the first time given by Kozlov *et al* (1986); then it was used by Sokolovskiy and Verron (2000), Gryanik *et al* 2006 and in a series of other works.

The configurations of the vortex patches of the upper (solid line) and bottom (dashed line) layers are shown in the below figures for the indicated moments of the non-dimension time. A half of a rotational period of the undisturbed (circular) two-layer vortex is chosen to be a time unit.

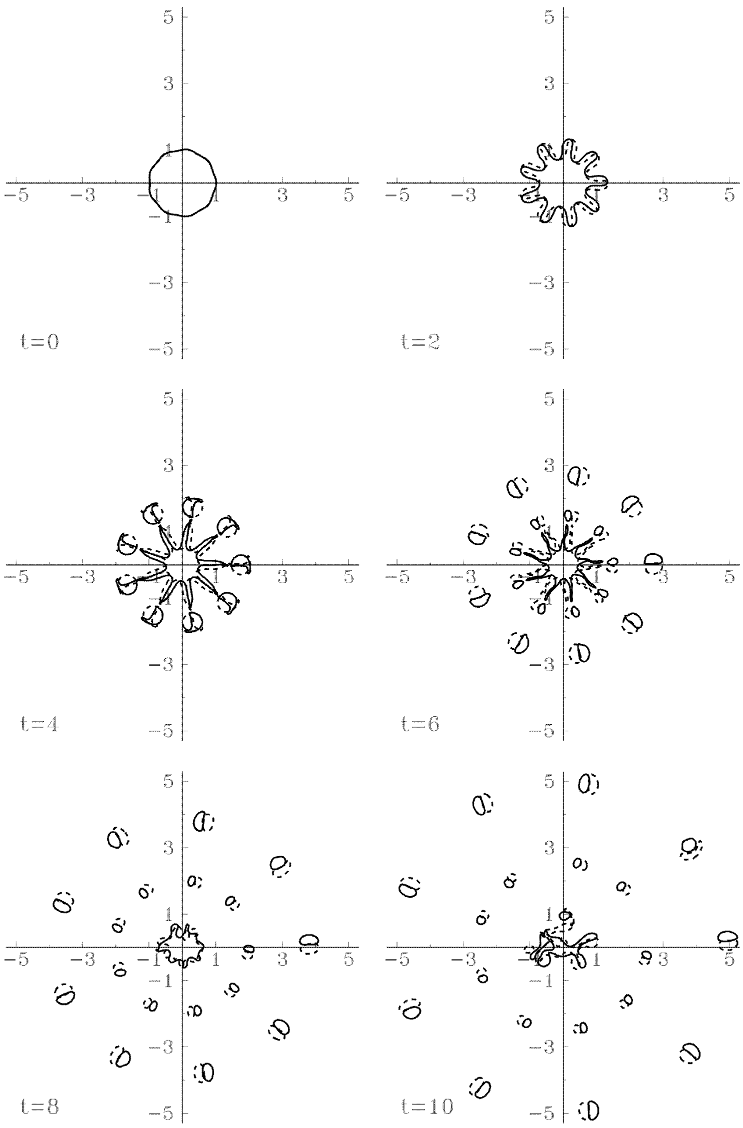

FIG. 1 – Time evolution of the circular unstable heton: cascade instability

Figure 1 shows the calculation result for the case $\gamma = 14$, when, according to the linear analysis of the stability, the mode with $m = 9$ occurs to be the maximum unstable. In the initial time moment the perturbations, corresponding to this mode, with amplitude $\varepsilon_j = 0.02,\ j = 1, 2$ were applied here.

This experiment gives an example of the process realization for the so called cascade instability:

- at the first stage in the upper and lower layers, there form 9-rayed figures, displaced one with respect to another;
- nine two-layer pairs with tilted axes break away from the extremities of these rays and scatter in the radial directions;
- at the periphery of the residuary core, there form 9 new rays;

- the extremity parts of the rays, shifted one with respect to other, irradiate a new series of 9 small-scale hetons, departing behind the first echelon;
- at the boundary of the central core, new (now irregular) vortex structures form, and they also move out from the center.

The motion law of the departing vortices is of obvious interest. The corresponding calculation results are shown in Figure 2 which demonstrates the behavior of radial coordinates of fore (first, second and third) and rear vortex fronts. The fore front of the vorticity we assume to be some fictitious material particle whose coordinates are calculated as an arithmetic average of $x-$ and $y-$coordinates of 20 contour markers that are at the maximum distance of the center of the initial vortex structure. For designating the rear front we took liquid particles which are at the minimum distance from the center.

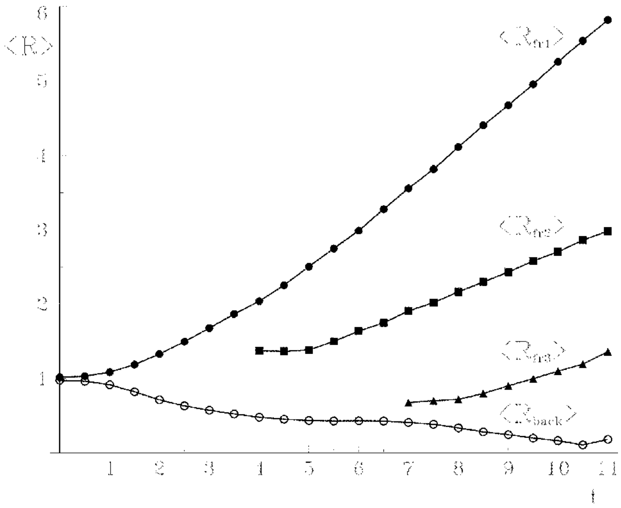

FIG. 2 – Time evolution of potential vorticity fronts

We see that the motion laws of the fore fronts reach asymptotically the ballistic (linear in time) law. Therefore we have reason to believe that the front of the heat anomaly also has to propagate with a constant velocity.

The example given above corresponds to the case when the initial boundary of the heton has the shape close to the circular one. In the case of a quasi-elliptical two-layer vortex, the linear analysis of the stability is much more complicated, but the calculations we carried out showed that the regimes of the cascade instability are also possible for the hetons of elliptical shape. Figure 3 gives the example of the decay of the initially compact vortex structure.

4 Conclusions

In the models of the general ocean circulation there is assumed that the heat transport has a diffusion nature with the law of the propagation of the temperature anomaly $\langle R \rangle \sim t^{1/2}$. At the same time it is known that the thermic component of the existing models is the least satisfactory. The heton theory, based on the results of the numerous numerical experiments (and, in particular, given here) on the modeling the non-linear evolution stage of unstable two-layered vortices certify in the favor of the mesoscale heat transport which more effective, i.e. $\langle R \rangle \sim t$. It seems that this parameterization has to provide more realistic results.

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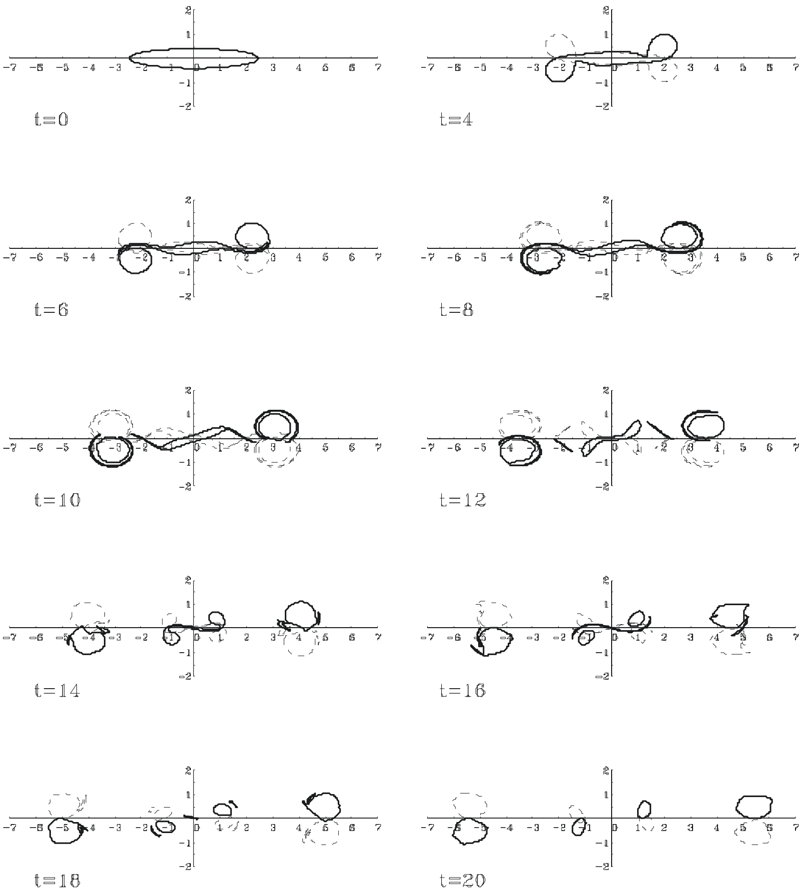

Fig. 3 – Time evolution of the elliptical unstable heton

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