Internal wave breaking and flow dynamics over obstacles

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Abstract:

Internal waves generated by stably stratified flow over obstacles can break as the waves steepen under low Froude number conditions. This situation was investigated in the laboratory via PIV measurements by towing Gaussian-shaped obstacles in linearly salt-stratified towing tanks. First, while it is shown that the width of the obstacles and the side-wall confinement help induce breaking, the Reynolds number of the flow is also shown to be important. Second, it is shown that while the physically correct no-slip condition on the obstacle itself is crucial since it allows a wave-induced boundary layer separation on the lee-side of the obstacles, resulting in a trapped lee-wave with rotors, changing the lower boundary condition after the obstacle only leads to secondary changes in the flow pattern. Finally, first estimates of the turbulence statistics in the vertical symmetry plane of the flow in the breaking zone at high Re (10\textsuperscript{4}) have been obtained.

Résumé:

Les ondes internes générées par un écoulement stratifié au-dessus d’un obstacle à faible nombre de Froude conduisent à un phénomène de déferlement. Cette situation a été étudiée par mesures de PIV en tractant des obstacles de formes gaussiennes dans des canaux linéairement stratifiés. Premièrement, il est montré que des paramètres que sont la largeur de l’obstacle et le confinement latéral interviennent dans le déclenchement du déferlement mais que le nombre de Reynolds joue aussi un rôle important. Deuxièmement, la condition aux limites de non-glissement sur l’obstacle est primordiale, car elle permet la séparation de la couche limite par les ondes sous forme de d’ondes piégeées avec rotors, mais la modification de cette condition d’adhérence en aval de l’obstacle, quant à elle, n’induit que des modifications mineures sur la topologie de l’écoulement. Enfin, de premières estimations des statistiques turbulentes dans le plan de symétrie vertical de l’écoulement, dans la zone de déferlement, ont été obtenues à hauts Re (10\textsuperscript{4}).

Key-words:

Internal-wave breaking, turbulence, PIV

1 Introduction

Stably-stratified flow over obstacles such as mountains can lead to wave breaking and clear-air turbulence under low Froude number conditions. Associated with wave breaking are severe downslope windstorms of which a well known example is the 1972 storm over Boulder, Colorado [Lilly and Zipser (1972)]. Although airplane measurements were able to locate zones of turbulence encountered in the flight path, computed horizontal spectra by Lilly (1972) from the available wind measurements did not reveal a -5/3 inertial range, likely due the difficulties in interpreting the measurements. More recent measurements campaigns such as PYREX in 1990 over the Pyrenees or MAP in 1999 over the Alps have attempted to measure mountain wave-breaking but were hampered by the difficulty in coordinating the airplane flights with the spatially and temporally hard to predict breaking phenomena.
Other orographic phenomena that can occur under stratified conditions are trapped lee-waves and rotors, which are loosely defined as vortices with horizontal vorticity in the lee of the orography. These are usually associated with inversions and non-uniform flow [e.g., Doyle and Durrant (2002)], and not linked to wave-breaking, which can occur under uniform flow and stratification conditions. These phenomena were observed, however, for uniform conditions when wave-breaking occurs [Eiff and Bonneton (2000)]. Most numerical simulations of wave-breaking also do not reveal trapped lee-waves or rotors [e.g., Afanasyev and Peltier (1998)], but the results of Gheusi et al. (2000) show that this is probably due the common use of a slip-condition on the lower boundary of the obstacle. Gheusi et al. (2000) showed that the use of a slip-condition on the obstacle’s surface reduces the downslope windspeed and helps induce a trapped lee-wave with embedded rotors. Furthermore, the structure the turbulent wave-breaking zone is strongly modified by the lower boundary-condition. The interaction, however, is not yet clearly understood.

Laboratory measurements can provide more controlled measurement opportunities, although at lower Reynolds numbers than the atmospheric case. Our earlier studies [Eiff and Bonneton (2000)] were performed near \( Re \sim 10^2 \), but our more recent studies have started to investigate the flow at \( Re \sim 10^4 \) [Eiff et al. (2005)], with preliminary statistical results in the horizontal plane. The results suggest a -5/3 inertial range and large-scale structures - toroidal vortices - similar to those found at low Reynolds numbers.

In this paper we present results of an ongoing investigation on the turbulent statistics in the wave-breaking region at high Reynolds numbers, including the first experimental results in the vertical plane of the flow. We also look at the effect of the lower boundary condition behind the obstacle on the flow dynamics and discuss new results on the generation of wave breaking.

2 Experimental procedure

The experiments consist in towing a Gaussian-shaped quasi two-dimensional obstacle in an upright configuration on the bottom of a hydraulic tank at constant velocity \( U_0 \). The tank is filled with linearly stratified salt water, characterized by a uniform Brunt-Väisälä frequency \( N \sim \text{rad.s}^{-1} \) (see Eiff and Bonneton (2000)). All experiments are carried out at \( F_H = U_0/NH = 0.6 \) where wave breaking occurs with \( H \) the height of the ridge. The particle imaging velocimetry (PIV) technique developed by Fincham and Spedding (1997) is used to measure two simultaneous velocity fields. The images were acquired with two CCD cameras (Pulnix TM9701, 768 x 486 resolution, 30Hz frame rate) and two continuous laser sheets (at 488nm and 514nm wavelengths from a Spectra Physics Beamlock 2080, 25W Argon-Ion laser), all fixed to the towing carriage. The no-slip condition behind the obstacle is realized with a thin base-plate attached to the trailing edge of the obstacle. Two tanks, whose dimensions \( H_t \times W_t \times L_t \) are \( 0.7 \times 0.8 \times 7 \text{ m}^3 \) and \( 1 \times 3 \times 22 \text{ m}^3 \), enable \( Re = U_0H/\nu \) of order \( 10^2 \) and \( 10^4 \) to be reached, respectively.

3 Confinement and aspect ratio effects

Any stratified flow studied in the laboratory must accept the presence of walls, which are not usually present in the natural environment. Their confinement effect can be overcome by reducing the blockage ratio, but this comes at the expense of reducing the size of the obstacle, i.e., the Reynolds number. Another option is to increase the size of the tank, especially its width, while continuing to use relatively small obstacles. This was done by Eiff et al. (2005) for Reynolds numbers of about 600, to investigate the effect of sidewall confinement on the occurrence of wave breaking. Also included was the effect of the obstacle’s ridge-to-height aspect ratio. The
resulting regime diagram as a function of the aspect ratio of the obstacle, \( \beta = W/H \), and the confinement due to the side walls, \( \gamma = W/W_t \), is shown in Figure 1 (open and filled circles). Here \( W \) is the width of the ridge, \( H \) is its height and \( W_t \) is the width of the tank. The results were obtained in the large tank \( (W_t=3m) \) with \( H = 35 \text{mm} \).

Several observations can be made. First, for asymmetric obstacles \( (\beta = 0) \), wave breaking does not occur. Second, for low confinement ratios \( \gamma \) less than about 0.2, the transition from no wave-breaking to breaking appears to be independent of the obstacles’ aspect ratio \( \beta \), occurring near \( \beta \approx 9 \). On the other hand, for obstacle aspect ratios \( \beta \) less than about 5, the no-breaking to breaking transition tends to be independent of \( \beta \), occurring near \( \gamma \approx 0.25 \). The latter implies that for small \( \beta \), over which the wavefield more easily diverges, breaking is artificially induced by increasing the confinement. The former implies that under negligible confinement effects, there is a minimum required ridge-width for breaking.

One might ask how these results depend on the \( Re \), especially higher ones. Since it is not feasible to investigate higher \( Re \) with low confinement ratios, we have investigated only lower \( Re \). The results are included in Figure 1 (triangles). Although there are not enough measurements to conclude on the shape of the transition curve, it can be seen that the breaking to no-breaking transition is shifted to higher \( \gamma \) and \( \beta \) values. The transition is thus dependent on the \( Re \), approaching \( (\beta, \gamma) \) values closer to zero as the \( Re \) increases. Whether wave-breaking can occur in these regimes for the higher atmospheric Reynolds numbers is still an open question to our knowledge. For a better understanding, the instability mechanism leading to wave-breaking needs to be investigated.

4 Influence of lower boundary-condition

As mentioned in the introduction, use of a physically correct no-slip boundary condition on the obstacle was shown to induce a trapped lee-wave with embedded rotors, thus significantly affecting the wave field [Gheusi et al. (2000)]. Here, we investigate the influence of the boundary condition after the obstacle. This was done by attaching a thin plate to the base of the obstacle, extending 10\( H \) behind the crest. Figures 2(a) and (b) show the resulting \( u/U_0 \) velocity isocontours with and without slip, respectively, after overturning has occurred and the flow has become stationary, for \( Nt \approx 100 \). In both cases, the same low and negatively valued region aloft, the overturning region, can be identified. In contrast to the boundary condition on the obstacle itself, the boundary condition beyond the obstacle does not influence the flow field significantly, except for a weaker secondary rotor. The negative velocities of the first rotor both attain about 10% of the freestream velocity. Computation of the horizontal vorticity (not shown) reveals a strong vorticity sheet enveloping the first wave crest, of significantly higher amplitude than the
ambient wave vorticity, supporting the idea that the boundary layer separates in phase with the first wave crest of the trapped lee-wave. The boundary-layer separation and lee-wave generation are thus interlocked, in agreement with the small influence on separation of the post-obstacle boundary condition. Such an interaction was also observed by Doyle and Durran (2002), but in a non wave-breaking situation with non-uniform flow and stratification.

5 Turbulence statistics

Eiff et al. (2005) presented the first turbulence statistics resulting from laboratory measurements at $Re \sim 10^4$, limited to the horizontal plane. The results in the vertical plane, especially the central symmetry plane, are more difficult to obtain due the large distance separating the glass sidewalls from the imaged plane (1.5m in the large tank). Optical distortions due to variations of the index of refraction across the turbulent breaking zone, which almost spans the width of the ridge of the obstacle, renders the interpretation of the images largely impossible. To circumvent this problem, we have taken images of the vertical symmetry plane through the free surface of the tank, which is essentially undisturbed during the towing. This greatly reduced the optical distance to the imaged plane and allowed index of refraction effects due to mixing to be minimized. However, due to the inclination (about 40°) the images are distorted. The PIV-computed displacement vector fields (the origin and arrival points of the vectors) were therefore transformed back into the undeformed $(x,z)$-coordinates by applying the transformation matrix obtained from an in situ-calibration grid (using 60 points) with the Levenberg-Marquardt non-square matrix-inversion technique.

An additional difficulty in obtaining turbulence statistics of this type of flow is the finite length of the towing tank, which limits the towing time and generates perturbations due to reflecting columnar modes. This raises the question if a stationary flow can be achieved. It was found that the flow was quasi-stationary for $80 < Nt < 200$, yielding 120 realizations for measurement intervals of $N^{-1} s$. Since this is a rather low number to achieve reasonable statistical convergence, an effort was made to repeat the measurements. Up to three runs under same conditions qualified to be considered repeatable, under the same salt-stratification filling. Furthermore, since the flow is only quasi-stationary, displaying low-frequency variations above the estimated integral scale of $7N^{-1} s$, the time-averages were performed over a shorter span ($20N^{-1} s$) to filter out these low-frequency variations.

Figures 3(a)-(d) show some preliminary results of the estimates of the turbulence intensities and Reynolds shear-stresses in the vertical plane [Figures 3 (a) and (b)] and in the horizontal plane [Figures 3 (c) and (d)]. Also included in Figures 3(a) and (b) are streamlines to help identify, from a topological point of view, the wave-breaking region. The region immediately behind the diverging streamlines is where the wave has overturned. The results in both planes were measured simultaneously.
Figure 3: Statistics in the vertical symmetry plane at $y = 0$: (a) $\frac{1}{2} \sqrt{\frac{u'^2 + w'^2}{U_0}}$, (b) $\frac{-u'}{\sqrt{u'^2 + w'^2}}$, and in the horizontal plane at $z = 2.5H$: (c) $\frac{1}{2} \sqrt{\frac{u'^2 + v'^2}{U_0}}$, (d) $\frac{-u'}{\sqrt{u'^2 + v'^2}}$. $Re \sim 10^4$.

It can be observed that the turbulence intensity in the vertical plane [Figure 3(a)] is relatively strong in the overturning zone, with maximum values close to 20%, close to those observed in the horizontal plane by Eiff et al. (2005). The intensities continue to be high downstream, up to the end of the measurement zone, and are concentrated in the lower shear-layer where Kelvin-Helmholtz instabilities have been proposed to occur [e.g., Smith (1991)]. This shear layer is the top end of the downslope wind. Here it has separated from the obstacle and envelops the trapped lee wave. The Reynolds shear-stress in the vertical plane [Figure 3(b)] reveal strong values along the shear layer as well as in the overturning region.

In the corresponding horizontal plane at $z/H = 2.5$ [Figure 3(c)], which slices through the center of the wave-breaking region, the maximum intensities match those in the vertical plane. The flow is clearly potential upstream but is seen to be turbulent beyond the overturning region, extending at least as far as the measurement zone. Although the intensities decrease, the Reynolds shear-stresses in the horizontal plane [Figure 3(d)], remain strong beyond the original overturning region as well. This supports the conjecture by Eiff et al. (2005) that turbulent vortices are "shed" from the wave-breaking region. The data still needs to be analyzed to confirm this directly.

6 Conclusion

While it is has been clear that the no-slip boundary condition on the obstacles is crucial in reproducing the correct dynamics of the flow under wave breaking conditions, it is shown that downstream of the obstacle the boundary condition is only of secondary importance.

The occurrence of wave breaking is seen to depend strongly on the aspect ratio of the obstacle and the confinement due to the side walls of the towing, the latter being a necessary artifact of hydraulic simulations. At low aspect and confinement ratios wave breaking is not observed. For negligible confinement ratios $\gamma$, an obstacle width-to-height aspect ratio $\beta$ of about 9 suffices to induce wave breaking. For lower $\beta$, increasing $\gamma$ to values greater than about 0.25, induces wave breaking. It is shown, however, that this transition depends on the Reynolds number, being about 600 for the numbers discussed. Decreasing the Reynolds number extends the transition to higher ($\beta, \gamma$) ratios, which implies that the transition at natural Reynolds numbers might occur for lower $\beta$ values.
At higher Reynolds numbers ($Re \sim 10^4$), an attempt was made to measure the turbulence statistics of wave breaking in horizontal and vertical planes intersecting the wave-breaking region. Although the mean flow in the overturning region is very calm, the intensities and shear stresses are high, with maxima reaching 20% and 50%, respectively. The lower shear layer is also marked by intense turbulent activity. It is also shown that the turbulent zone extends downstream beyond the overturning region, in agreement with the conjecture by Eiff et al. (2005) that turbulent structures are shed downstream. This is different from the simulations by Afanasyev and Peltier (1998) who used a slip boundary condition and whose results revealed long longitudinal structures and a downstream propagation of the turbulence. These structural dynamics need to be investigated more closely.

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References


