Diffusion-Driven Flow

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Abstract:

Diffusion-driven flow occurs whenever you have a density stratified fluid bounded by an inclined sidewall. The no flux boundary condition at the sidewall forces the density in a boundary layer adjacent to the wall to differ from that in the bulk, resulting in buoyancy driven flow. Here we summarize the results of a recent laboratory experiment that demonstrates diffusion-driven flow in action.

Key-words: Diffusion; Buoyancy.

1 Introduction

Convection in a stably stratified fluid, generated through the combined action of diffusion and gravity, was first identified by Phillips (1970) and Wunsch (1970). The phenomenon occurs at an inclined boundary across which there is a discontinuity in diffusivity, as shown in Figure 1. The necessity of flux continuity across the boundary gives rise to a thin buoyancy layer adjacent to the wall, in which otherwise horizontal isopycnals adjust to meet the wall at an angle. Fluid within this thin layer is buoyant with respect to the bulk, generating convection that develops to a steady state in which viscous and buoyancy forces balance. The net convective flux associated with this flow is up-slope. If the inclined wall bounds the fluid from above, the flow is reversed, generating a net flux down-slope. For salt-stratified fluids the characteristic flow velocity is of the order of centimetres per hour; such flows have applications in geophysical systems where the timescale is large (e.g. flow in a fissure), and may be applicable to transport in micro channels, where the length scale is small. For liquid metals, the effect is potentially strong enough to induce turbulence.

![Figure 1: Schematic of diffusion-driven flow.](image)

2 Experimental Arrangement

Our experiments were performed in a Plexiglas tank (40cm × 25cm × 2.5cm) with a sloping Plexiglas surface inclined at angle, ranging from 0.6° to 75.0°. The angle of inclination was measured from a digital image of the sloping wall and a plumb line to an accuracy of 0.1°. A linear stratification was achieved by filling the tank with salt water from below using a
double bucket system. The density profile was measured using a PME salinity probe calibrated using an Anton-Parr densiometer. The average buoyancy frequency for the experiments was $N=2.11 \pm 0.11 \text{rad/s}$. A small amount of Blue Dextran dye was carefully injected into a reservoir at the base of the inclined wall. Blue Dextran was chosen because its high molecular weight (40000) gives it a much lower diffusivity in water than salt, allowing it to follow the flow almost without diffusing. The density of the dye mixture was slightly greater (+5 kg/m$^3$) than that of the ambient fluid at the depth of injection, allowing the dye to settle in the reservoir.

Dye from the reservoir was drawn up by the diffusion-driven flow and transported along the wall. A sequence of three images portraying the advancing dye front for inclination $\alpha=3.6^\circ$ is shown in Figure 2. The pictures were taken using a digital camera with a resolution of 15 pixels/mm, and have been stretched by a factor of three in the vertical for improved visualization. The camera was inclined at a small angle to the horizontal, so that the buoyancy layer was not viewed directly from the side, but from slightly above. From this perspective it was easier to visualize the buoyancy layer because of its increased thickness. In agreement with the theoretical prediction of Phillips (1970) and Wunsch (1970) the characteristic thickness of the buoyancy layer was of the order of 1 mm and noticeably increased with decreasing angle.

![Figure 2: Three snapshots of the advancing dye front for $\alpha=3.6^\circ$. The reservoir of dye is located to the right of the images, and the black dye front can be seen advancing from right to left. The images were rotated by $-\alpha$ to facilitate processing. From top to bottom the three images correspond to times 10, 100 and 200 minutes after the start of the experiment. The units on the ruler are centimetres.](image)

Typically an image was recorded every 5 minutes. Each image was saved in RGB format, with the dye showing up most strongly in the blue field. To determine the position of the dye front as a function of time the first image was subtracted from all subsequent images. The position of the front was then defined to be the furthest pixel along the wall for which the blue value exceeded a minimum threshold (20 out of 255). This threshold was the smallest value that allowed for detection of the motion of the dye front above the background noise level. Choosing a higher threshold value did not noticeably affect the measured velocity.

3 Results

The results are presented in Figure 3, in which the measured speed is plotted as a function of the angle of inclination. Also presented are two theoretical curves, the maximum velocity $u_{\text{max}}$ and the mean velocity $u^*$ of the boundary layer flow. Since it was not possible to resolve the velocity profile within the buoyancy layer, the relevant velocity for comparison is $u^*$, which agrees remarkably well with the experimental velocity for angles larger than $5^\circ$. For angles smaller than $5^\circ$ the results in Figure 3 diverge from the theoretical solution. The inset of
Figure 3 highlights the angular dependence of the measured speed at small angles, showing that it tends to zero with the inclination of the wall. The angle of maximum velocity can therefore be determined to be $\alpha_{\text{max}}=2.8\pm1.0^\circ$. For comparison, Wunsch (1970) predicts that the theoretical solution breaks down in the range $0.03^\circ<\alpha<0.3^\circ$.

![Figure 3: Comparison of experimental results with the theoretical prediction. Each diamond corresponds to one experiment. The solid line is the theoretical mean velocity $u^*$ and the dashed line is the theoretical maximum velocity $u_{\text{max}}$. The errors bars denote the estimated experimental uncertainty. The inset shows the small angle data in more detail.](image)

4 Conclusions

In conclusion, we have presented the first experimental validation of the angular dependence of the Phillips-Wunsch diffusion-driven flow. Quantitative agreement between the measured velocity of a dye front and the mean theoretical velocity in the buoyancy layer is very good. The dependence of the velocity on the angle of the sloping wall is in accord with theoretical predictions for larger angles. For our experiment we deduce that the angle of maximum velocity for water stratified by salt is $\alpha_{\text{max}}=2.8\pm1.0^\circ$. In general, we expect that this value will depend on the Rayleigh number and the Prandtl number. Below $\alpha_{\text{max}}$ the velocity profile departs from the Phillips-Wunsch flow. Further experiments on these flows have been performed by the authors and are available if the following references (Peacock et al., 2004; Heitz et al. 2005; Peacock et al. 2007).

References


