The effect of buoyancy in controlling vortex breakdown in swirling jets

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Abstract

In this study we explore the effect of buoyancy in controlling vortex breakdown (VB) in swirling jets. The experimental apparatus consists of a vertical swirling water jet which discharges into a large tank, in which the temperature difference between the jet and its surrounding is controlled. Three non-dimensional parameters govern the flow: the jet exit Reynolds number, the swirl ratio and the Richardson number (buoyancy). Vector maps of the vertical mid-plane and horizontal cross-sections are obtained by PIV measurements. It is demonstrated that VB can be effectively suppressed (enhanced) by prescribing a negative (positive) temperature difference between the jet core and its surrounding fluid. Moreover, the experimental critical swirl ratio for the appearance of VB agrees with a value predicted by simple criterion. Finally, the transition of VB from a closed bubble to an open cone configuration is mapped in terms of the Reynolds and Richardson numbers.

Key-words: vortex breakdown, swirling jet, buoyancy

1 Introduction

Vortex breakdown (VB) is a remarkable phenomenon which may occur in swirling flows. It is characterized by a sudden deceleration of the flow near the axis and the formation of a stagnation point accompanied by the divergence of the stream surface. Downstream of the stagnation point a region of reverse flow forms and the wake of the expanded vortical structure may undergo large-scale velocity fluctuations. As such VB can have either detrimental effects, for example over delta wings at high angle of attack, or beneficial ones as a flame stabilizer utilizing the fast spreading and rapid mixing of the incoming flow with its surrounding. Thus there is a strong need for better understanding of the phenomenon in an attempt either to prevent the breakdown or to promote it. In isothermal flows VB occurs when the ratio between the swirling and axial velocities exceeds a certain value of order unity. The inclusion of buoyancy effects in swirling jets can result in the increase or decrease of this ratio, i.e., providing possible VB control.

Thermal effects on axisymmetric VB have been reported in various flow configurations, e.g. in a spherical gap (Arkadyev et al. (1993)) and in a container with a rotating bottom disk (Herrada & Shtern (2003)), where it was found that negative (positive) axial gradient of temperature can enforce (suppress) VB. In this article we extend the simple VB criterion, derived and experimentally verified by Billant et al. (1998) for an isothermal swirling jet, to include thermal effects due to lateral temperature gradients. Our experimental results confirm the generalized criterion and show how it can be used to suppress/enhance VB.

2 A simple vortex breakdown criterion including buoyancy effects

In this section the simple necessary criterion for the onset of vortex breakdown, derived by Billant et al. (1998, hereinafter referred to as BCH), is extended to include buoyancy effects. Accordingly, a free vortex undergoing conical breakdown in a jet of infinite extent
(figure 1) is considered. The cylindrical coordinate system is used $\{x, r, \theta\}$, where $x$ and $r$ are the axial and radial distances and $\theta$ is the azimuthal angle. The corresponding velocity components are $(V_x, V_r, V_\theta)$. The flow is assumed to be steady and laminar. Viscous, diffusive and entrainment effects are supposed to be negligible. Applying the Bernoulli equation between two cross-stream planes, $X_1$ (far upstream of the stagnation point) and $X_2$ (the plane containing the stagnation point), along the centerline, and away from it where the ambient fluid is at rest, respectively yields

$$P_{j_1} + \rho_j g H + 0.5 \rho_j V_\theta^2 (X_1,0) = P_{j_2}, \tag{2.1}$$

$$P_{amb_2} = P_{amb_1} + \rho_{amb} g H, \tag{2.2}$$

where $P$ is the pressure, $\rho$ is the fluid density, $g$ is the acceleration due to gravity and $H = X_2 - X_1$ is the vertical distance between the two planes. The subscripts $j$ and $amb$ correspond respectively, to the jet and ambient fluid properties, whereas the subscripts 1 and 2 indicate the associated cross-stream planes. At the $X_1$-plane the radial pressure gradient is balanced by the centrifugal force

$$P_{amb_1} = P_{j_1} + \int_0^\infty \rho_j \frac{V_\theta^2(r,X_1)}{r} dr. \tag{2.3}$$

where $\beta$ is the coefficient of thermal expansion. As the cone is open to the surrounding fluid and the flow is almost at rest, it is assumed that $P_{j_2} = P_{amb_2}$. Finally, substituting (2.2)-(2.4) into (2.1) leads to the following necessary condition for a vortex breakdown ($VB_{crit}$):

$$VB_{crit} = \frac{1}{V_x^2 (X_1,0)} \int_0^\infty \frac{V_\theta^2}{r} dr + \frac{g H \beta \Delta T}{V_x^2 (X_1,0)} = S_{int} + Ri \geq \frac{1}{2}, \tag{2.5}$$

where $S_{int}$ and $Ri$ are the swirl integral and the Richardson number (buoyancy effect), respectively. The inequality sign is applied to a bubble vortex breakdown configuration in which the stagnant zone is separated from the ambient quiescent fluid. In §4, this criterion is verified by experimental results.
3 Experimental setup

A schematic view (drawn to scale) of the experimental apparatus, consisting of a vertical swirling jet discharging into a large transparent cylindrical tank, is shown in figure 2. After exiting the tank, the water circulate through a temperature control unit and return to the rotating chamber. Water circulation through the system is maintained by using two piston-type metering pumps driven by a single control-motor.

FIG. 2 Schematic view of experimental setup.

Three parameters govern the flow: the flow rate ($Q$), the angular momentum of the fluid and the temperature difference between the jet and the ambient fluid. These respectively correspond to the non-dimensional jet exit Reynolds number ($Re = 4Q/(\pi D \nu)$), where $D$ is the inner diameter of the nozzle at the exit plane and $\nu$ the kinematic viscosity of water at room temperature), the swirl ratio ($S_{int}$) and the Richardson number ($Ri$). The definitions of the latter two are given in (2.5).

The swirl velocity component is imparted onto the jet in the rotating chamber unit. The settling chamber (top part) is composed of a hollow inner perforated cylinder, having a conical end at its bottom, and a co-axial outer cylinder connected to a smooth converging nozzle shape at its bottom. Swirl is imparted to the flow while it is passing between these two cylinders. Only the outer cylinder (including the contraction nozzle) is rotated.

Water is supplied through three pipes symmetrically positioned on the top of the inner cylinder. During filling, the water flows through the network of holes to the gap separating the inner and outer cylinders. It then flows downwards through a perforated ring (attached to the inner cylinder) before entering the nozzle part of the chamber. The exit diameter is 19.6 mm whereas the outer diameter of the attached cylindrical test section is 226 mm and its height is 470 mm. The resulting ratio of diameters ($\approx 11.5$) is found to be sufficient to minimize the effect of confinement. Furthermore, to avoid any return flow, screens are placed at the bottom of the tank, upstream of the four exit pipes.

The temperature difference between the jet and the ambient fluid is the third independent parameter. To control the temperature of the jet, the supplied water is passed through a helical-coil heat exchanger immersed in a circulating water bath with a controlled temperature. A T-
type thermocouple, positioned at the entrance to one of the three water-supplying pipes (thermocouple 1), is used to regulate the jet temperature at the nozzle exit. Two additional thermocouples positioned at the inner top part of the test section (thermocouple 2) and at its bottom exit (thermocouple 3), are used to monitor the ambient temperature during the experiments. The difference between the readings of thermocouples 1 and 2 is identified as the temperature difference between the swirling jet and the ambient fluid.

Particle Image Velocimetry (PIV) system is utilized to visualize the flow and to measure the instantaneous and mean velocity fields. For measurements of the axial and radial velocities, the mid-plane is illuminated by a vertical light sheet which is generated by deflecting an argon ion laser beam on a rotating polygon mirror. The illuminated plane is imaged from the side on a double frame 768×480 CCD camera. A pair of two consecutive images, acquired with a controlled time delay is subsequently processed with a cross-correlation algorithm to produce vector maps of the instantaneous flow field. The azimuthal and radial velocities in a given cross-stream plane of the jet is measured by a similar procedure to that described above. In this case, however, the illuminated horizontal plane is viewed from the bottom of the tank using a 45° inclined mirror.

To obtain the critical conditions for the onset of VB, the rotation rate was increased (by a computer controlled motor) from zero in gradually decreasing small steps ($\approx 10\%$ of $\Omega_C$ - the critical rotation rate needed for initiating VB), while keeping the flow rate and $\Delta T$ constant. As $\Omega_C$ is approached, the rotation rate was increased by smaller steps $2\% - 5\%$ of $\Omega_C$ (depending on the Reynolds number). To avoid transient effects, a waiting period of $\approx 15 \text{ min}$ was allowed after each adjustment.

To experimentally verify the modified criterion for VB, the terms $S_{int}$ and $Ri$ of (2.5) had to be calculated. To calculate $S_{int}$, the azimuthal velocity profile as well as the axial velocity at the jet axis, in the plane $X_1$, sufficiently upstream of the stagnation point, had to be measured. To avoid the influence of the stagnation point on the upstream measured velocities, the procedure employed by BCH was adopted here. Accordingly, the measured velocities in $X_1$ ($4 \text{ mm}$ downstream of the jet exit plane) were obtained at a rotation rate of $\Omega_C = 0.97 \Omega_C$.

To calculate $Ri$, the distance $H$, between the plane containing the stagnation point and $X_1$ had to be measured. For given Reynolds number and temperature difference, the rotation rate was increased gradually towards its critical value $\Omega_C$. $H$ is defined as the distance between $X_1$ and the cross-section plane where the stagnation point is first observed. This choice of definition was necessary because after its initiation, the stagnation point moved upstream affecting the velocity field, measured previously at $\Omega_C = 0.97 \Omega_C$.

### 4 Results and discussion

The theoretically obtained necessary condition for vortex breakdown (2.5) is compared with experimental results in figure 3 for a range of $-2.5 \leq \Delta T [\text{K}] \leq 5$ and for two Reynolds numbers of 246 and 500. In the plane of $S_{int}$ vs $Ri$ the criterion is a straight line. Good agreement between the modified theoretical model and the experimental data is obtained. It is also evident that the onset of vortex breakdown for all cases considered is slightly above the inviscid theoretical criterion, probably due to viscous effects. A further validation of the VB criterion is carried out for a specific case in which the jet is hotter than the ambient fluid and the end position of the stagnation point remains far downstream from the jet exit plane (figure 5 (d)). The criterion is checked at four streamwise positions corresponding to four different choices of $X_1$, indicated by the black dots. The average value of $\text{VB}_{crit}$ is 0.51 for all cases with a standard deviation of $\approx 0.05$. 


A quantitative measure of the vortex breakdown suppression and enhancement is presented in figure 4. This is done by measuring the reduction percentage of the critical dimensional swirl ($\Omega_C$), relative to its value at $\Delta T = 0$ (labeled as $\Omega_{C0}$). Colder jets (relative to the ambient fluid) would cause the suppression of VB whereas warmer jets would cause its enhancement. For all Reynolds numbers considered, the amount of suppression is increased monotonically as $\Delta T$ is decreased. (Here $\beta$ is used to normalize the temperature difference). For lower Reynolds numbers buoyancy becomes dominant and therefore the suppression is more effective. As the temperature difference between the jet and its surrounding is decreased higher rotation rates are needed to initiate VB.

The temperature difference between the swirling jet and ambient fluid affects the final form of VB, i.e. the shape of the recirculation zone. In the top part of figure 5 this effect is demonstrated for a fixed Reynolds number of $Re=246$.

FIG. 3 Experimental verification of the breakdown criterion.

FIG. 4 Delaying/enhancement of the onset of vortex breakdown by thermal effects in the range $150 < Re < 500$, $\Omega_{C0} = \Omega_C(\Delta T = 0 K)$.

FIG. 5. Effect of $Ri$ (a-d, $Re = 246$) and $Re$ (e-g, $\Delta T = 1.1K$) on the VB configuration at the vertical mid-plane (jet flows from top to bottom and shown by vector maps averaged over 100 sec). (a) $S_{int} = 0.64$, (b) $S_{int} = 0.48$, (c) $S_{int} = 0.28$, (d) $S_{int} = 0.13$, (e) $S_{int} = 0.47$, (f) $S_{int} = 0.36$, (g) $S_{int} = 0.15$. 
At \(Ri = -0.03\) (colder jet) the final configuration has a bubble shape (figure 5 (a)), i.e., the relative fast flow enclosing the stagnation zone (including the stagnation point and its downstream low-velocity recirculation region) first expands radially and then, beyond a certain downstream distance, contracts. As the jet temperature (and its associated Richardson number, \(Ri\)) are increased VB occurs at a lower swirl and the bubble size increases until it transforms into an open cone (figure 5 (b)). With a further increase of \(Ri\) a wide open cone is formed (figure 5 (c)) and then, for a larger temperature gradient (\(Ri = 0.42\)), a downstream movement of the stagnation point takes place (figure 5 (d)). Reducing \(Re\) while keeping \(\Delta T\) constant, has qualitatively the same effect as increasing the temperature of the jet.

The above results suggest the existence of two separate zones in the \(Re - Ri\) map where cones and bubbles may be observed. Therefore, for seven Reynolds numbers we have searched for the \(\Delta T\) associated with a boundary between these two configurations (figure 6).

![FIG. 6 The boundary between cone and bubble configurations.](image)

4 Conclusions

The modified simple criterion for VB, which includes buoyancy effects, is shown to predict the critical swirl number for the onset of VB as function of the Richardson number. Consequently, suppression (enhancement) of the vortex breakdown by buoyancy effects is demonstrated. The vortex breakdown configuration as function of the Reynolds and Richardson numbers is mapped. As the Reynolds number is increased the transition between cone and bubble configurations occurs at lower values of \(Ri\). For sufficiently high Reynolds numbers (\(Re > 300\)), the boundary between cones and bubbles seems to be independent of the Reynolds number. More details can be found in Mourtazin & Cohen (2007).

References