Synoptic responses to mountain gravity waves breaking

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Abstract:

A heuristic model is used to study the synoptic response to mountain Gravity Waves (GWs) absorbed at directional critical levels. The model is a Semi-Geostrophic version of the Eady model for baroclinic instability adapted by Smith (1986) to study lee cyclogenesis. The GWs exert a force on the large-scale flow where they encounter directional critical levels (Shutts 2003). This force is taken into account in our model, and produces Potential Vorticity (PV) anomalies in the mid-troposphere.

For the case of an idealized mountain range in which the orographic variance is well separated between the small-scales (that produce the Gravity Waves) and large-scales (that produce Synoptic Eady Waves), the PV produced by the GWs force has a surface impact that is significant compared to the surface response due to the large scales of the mountain. For a cold front, the GWs force produces a trough over the mountain and a larger amplitude ridge immediately downstream. It opposes somehow to the large scale Eady wave response to the mountain, which is anticyclonic aloft and cyclonic downstream.

Key-words:

Lee Cyclogenesis, Gravity Waves, Potential Vorticity

1 Introduction

The large-scale flow response to the breaking of vertically propagating gravity waves (GWs) has been the subject of many studies during the last 40 years. The importance of mountain GWs for the atmospheric circulation is now well established. The parameterization of mountain GWs in General Circulation Models (GCMs) reduces the cold bias these models present near the tropopause in the Northern Hemisphere mid-latitudes, more recent parameterizations of Subgrid-Scale Orography (SSO) include low-level flow effects (Lott and Miller 1997) which also reduce biases in the low-level winds.

Another large scale process related to mountains is lee cyclogenesis, for which various dynamical mechanisms were proposed. One of them is the triggering of Eady Waves by synoptic scale mountain ridges (Smith 1986). Nevertheless, on this problem of lee-cyclogenesis, and in contrast with that of the planetary scale general circulation, very few studies address the role of the breaking gravity waves.

This paper presents a theoretical model of the large-scale effect of the GWs generated by an idealized front passing over a mountain range. This model accounts for the GWs through the large-scale momentum deposit they induce where they encounter critical levels, in the mid-troposphere. For this purpose, we adopt a Semi-Geostrophic (SG) version of Smith’s model (1986) of lee cyclogenesis in which we include a GWs force following Shutts (2003). The use of a balanced formalism here is supported by Lott (2003) which has shown by direct 2D simulations that after 12hrs typically, the balanced part dominates the inertio gravity waves part in the total response to GWs absorption at a critical level.
2 Model

A central assumption of our model is that the power spectrum of orography shows a clear separation between the large scales and the small scales. We adopt for this an idealized mountain range profile given by,

\[ h(x) = H_0 e^{-\frac{x^2 + y^2}{2L^2}} (1 + \cos(k_w x)) = \mathcal{H}(x) (1 + \cos(k_w x)) , \]  

where \( k_w = k_x e_x + k_y e_y \). In Eq. (1), \( L \) is the characteristic large scale, \( k_w \) is the dominant horizontal wavenumber associated with the small scales, \( 2H_0 \) is the maximum altitude of the mountain range and \( \mathcal{H}(x) \) is the large scale orography profile, that is the envelope of the ridges (Fig. 1a,b). If the GWs encounter directional critical levels (Fig. 1c), they are absorbed and can deposit the momentum they transport. For the large scale flow, this effect can be translated into a force per unit mass (see Martin and Lott 2007),

\[ \mathcal{F}(x, z) = \mathcal{F}(x, z) e_x + G(x, z) e_y , \]  

whose impact adds to that of the large scale orography profile \( \mathcal{H}(x) \).

![Figure 1: Schematic representation of the idealized mountain range and of the background flow used to derive the model equations in the cold front case: a) vertical section with horizontal axis along \( k_w \), b) view from top, c) spectrum of the small-scale orography \( h^\prime = \mathcal{H}(x) \cos(k_w x) \).](image)

To study the response to \( \mathcal{F} \) and \( \mathcal{H} \), we adopt the Boussinesq approximation and consider an idealized front such that the background wind \( U \) and the potential temperature \( \Theta_b \) have uniform shears:

\[ U(z) = U(z) e_x + V_0 e_y = (U_0 + \Lambda z) e_x + V_0 e_y , \]  

\[ \Theta_b(y, z, t) = \Theta_r + \Theta_0 z + \Theta_y y + \Theta_{ad}(t) . \]  

In Eqs. (3)-(4), \( V_0 \) is the surface wind, \( \Lambda \) is the vertical wind shear, \( \Theta_r \) is a constant reference temperature, \( \Theta_0 z \) is the vertical stratification, \( \Theta_y \) is the cross front potential temperature gradient. In this framework, the constant Brunt Väisälä frequency \( N \), and \( \Theta_y \) can be written

\[ N^2 = \frac{g \theta_0 z}{\Theta_r} , \quad \Theta_y = -\frac{\Lambda f \Theta_r}{g} . \]  

In Eqs. (5), the thermal wind balance relates \( \Theta_y \) to \( \Lambda \), \( f \) is a constant Coriolis parameter, and \( g \) is the gravity constant.

If the scale \( L \) is such that the large scale Rossby number \( \frac{V_0}{fL} \) is near or below 1, the linear response to \( \mathcal{F} \) and \( G \) can be evaluated using the disturbance potential vorticity \( (q) \) Semi-Geostrophic budget (see Hoskins 1975 for the Semi-Geostrophic approximation):

\[ (\partial_t + U \nabla) \rho_r q + \nabla \cdot J_N = 0 , \]  

where
\[
\rho_r q(x, z, t) = \theta_{0z} \left( (1 - R i^{-1}) \partial_x v_y - \partial_y u_g \right) + \Lambda \partial_y \theta + \Theta_y \partial_z u_g + f \partial_z \theta, \quad \text{and} \quad \text{(7)}
\]

\[
J_N = -\theta_{0z} \left( 1 - R i^{-1} \right) G e_x + \theta_{0z} F e_y - \Theta_y F e_z. \quad \text{(8)}
\]

In Eqs. (6) and (8), \(J_N\) is the non-advective PV flux, \(R_i = \frac{N^2}{\Lambda}\) is the background flow Richardson number, \(u_g = -\partial_y \phi / f\) and \(v_g = \partial_x \phi / f\) are the geostrophic components of the wind perturbation, \(\phi = p / \rho_r\) is the geopotential perturbation, \(p\) is the perturbation pressure and \(\rho_r\) is a constant reference density. Note also that the disturbance potential Temperature inversion of the Elliptic Equation Eq. (7).

This numerical integration only request vertical integral s, it does not request a temporal numerical integration.

We will note detail here the methodology used to invert the two Eqs. (6)–(9) (see Martin and Lott 2007 for details). Briefly, we pass in the Fourier space for the two horizontal directions. In this space, there is an analytical form for \(\hat{q}\) (the Fourier transform of \(q\)):

\[
\hat{\phi}_p(k, z, t) = e^{-\lambda z} \int_0^z e^{2\lambda z'} \int_{z'}^\infty -\frac{g \rho_r}{f \theta_r} \hat{q} e^{-\lambda^* z^*} dz^* dz', \quad \text{where} \quad \lambda = \lambda_r - i \lambda_i \quad \text{and} \quad \lambda^* = \lambda_r + i \lambda_i. \quad \text{(11)}
\]

This numerical integration only request vertical integrals, it does not request a temporal numerical integration.

In this formalism, the boundary condition Eq. (9) becomes

\[
\lambda_r \left( \partial_t - i k. U(0) \right) \hat{\phi}_u - i k \Lambda \hat{\phi}_u = W \left\{ \left( \partial_t - i k. U(0) \right) \partial_z \hat{\phi}_p(0) + \Lambda \hat{F}(0) \right\} - \left\{ i k. U(0) N^2 (1 - R i^{-1}) \hat{H}(k) \right\}. \quad \text{(13)}
\]

In the right hand of Eq.(13) we have separated the forcings due to the GWs (\(W\)), and to the envelope \(\hat{H}(E)\). When this right hand side is null, this last Eq form a Semi-Geostrophic version of the classical Eady model for neutral edge synoptic waves. Finally, note that the temporal integration in Eq. (13) was solved numerically in the Fourier Space.

### 3 Results

We consider an idealized cold front moving toward the South in the Northern Hemisphere mid-latitudes, across an idealized mountain range. The flow and orography parameters are respectively:

\[
\theta_r = 300 \text{ K}, \rho_r = 1 \text{ kg m}^{-3}, f = 10^{-4} \text{ s}^{-1}, N = 10^{-2} \text{ s}^{-1}, \Lambda = 4.10^{-3} \text{ s}^{-1}, U_0 = 0 \text{ ms}^{-1}, V_0 = -20 \text{ ms}^{-1}. \quad \text{(14)}
\]
\[ H_0 = 800 \text{ m}, \ L = 200 \text{ km}, \ |k_w| = \frac{2\pi}{70000} \text{ m}^{-1}, \text{ with } k_w = l_w. \] (15)

The mountain range half-height width is \(2L \sqrt{\ln 2} \approx 330\text{ km}.\) It is typically constituted of 5 to 7 ridges, which are 70km wide and oriented South East - North West (Fig.1b). For this set of parameter, the vertical profile of the net force \(\mathbf{F}(z)\) (not shown) shows that each component reaches a maximum near \(z_w = 5\text{km},\) and is only substantial over a vertical depth of 1 or 2 km around \(z_w.\) In the rest of the paper we will refer to this area as the critical zone.

### 3.1 Potential Vorticity

The Figs. 2a,b,c show three horizontal sections of the PV anomaly due to \(\mathbf{F}\) in the critical zone at \(t=12\text{hrs}.\) At the three levels, the PV amplitude is between 0.4-0.8 PVU, and the PV patterns are predominantly oriented in the direction of the background wind \(U(z_w).\) This general orientation follows that once produced by \(\mathbf{F}\) aloft the mountain, the PV is advected by the background flow. To interpret the differences between those three levels, one visualizes schematically the non advective PV flux \(J_N\) in Fig. 3. The Fig. 3a shows that \(J_N\) is pointing upward to be parallel to the isentropes. In the \((y, z)\) plane, \(J_{nz}\) essentially takes the PV from below \(z_w\) to put it above (Fig. 3a), yielding the predominantly positive PV lobe above \(z_w\) (Fig. 2a) and the predominantly negative one below (Fig. 2c). Near the altitude \(z_w,\) the divergence of the vertical component of the non-advective PV flux is null,

\[ \partial_z J_{Nz} = \Theta_y \partial_z F = 0, \] (16)

therefore the PV is only due to the vector \(J_{Nxy} = J_{Nx} \mathbf{e}_x + J_{Ny} \mathbf{e}_y,\) which is nearly opposite to the wind, the force \(\mathbf{F}(z_w)\) being perpendicular to it (Fig. 3b). Hence, near \(z_w, J_{Nxy}\) tends to take PV from downstream to put it upstream, according to the direction of \(U(z_w).\) At small times, it results in a positive PV anomaly along the upstream flank of the mountain and a negative one downstream (Fig. 3b). Afterwards, the negative PV anomaly produced aloft the mountain is advected downstream at the velocity \(U(z_w),\) yielding the mid-tropospheric start-up anticyclone in Fig. (2b). Still near \(z_w\) but over the mountain, the long-term response is a steady-state cyclonic PV anomaly (Fig. 2b). There, the PV due to \(J_{Nxy}\) is exactly balanced by the advection term,

\[ (U, \nabla) \rho \partial_t q + \nabla \cdot J_{Nxy} = 0. \] (17)

Figure 2: Horizontal sections of the PV anomaly in the critical zone, at \(t=12\text{hrs},\) and for different heights: a) \(z = 5.281\text{km},\) b) \(z = 4.969\text{km},\) c) \(z = 4.656\text{km}.\) CI=0.05 PVU, where 1 PVU=1.0*10^{-6}\text{K kg}^{-1} \text{ m}^2 \text{s}^{-1}, negative values dashed. The minimum and maximum values of \(q\) are indicated above each panel. The location and half-height width of the mountain range is indicated by the circle.
3.2 Surface Pressure

The Figs. 4a,b,c show the surface pressure perturbation induced by the GWs momentum deposit. At small time (Fig. 4a), it presents a small amplitude (approx. -0.1mb) trough over the mountain and a ridge downstream of it (towards \( U(z_w) \)), its amplitude is around 0.2mb. In a longer term (Figs. 4b,c), the trough over the mountain near disappears, while the downstream ridge extends along the direction of \( U(z_w) \). To the west of this extending ridge, a second trough is developing (Figs. 4b,c). At 36hrs, the maximum amplitude of the surface signal is 1.1mb.

It can be shown that this pattern is related to the surface Temperature anomaly, associated with the particular solution \( \partial_z \phi_p \). It is warm over the ridge, because the PV pattern aloft the
ridge is predominantly anticyclonic. This results in the surface pressure low seen over the mountain at \( t=12 \text{hr} \), but that vanishes with times (Fig. 4). Downstream of the ridge, and in the direction of the wind in the mid-troposphere (\( U(z_w) \)) the surface temperature presents a cold anomaly, because the PV aloft is predominantly cyclonic. Hence, there is a high surface pressure anomaly in the same direction and downstream of the ridge. Note that this high surface pressure is modulated to the South by a secondary low, witnessing that Surface Eady waves start to modulate the response.

To emphasises that this behaviour is cyclolitic, in the context of the development of down-stream lee cyclones, the time evolution of \( \phi_E \) (the signal due to the large-scale mountain) is shown in Figs. 4d,e, and f at=12hrs, 24hrs, and 36hrs respectively. A boundary Eady lee wave is developing and extending downwind according to \( U(0) = V_0 e_y \). The wave is bounded downstream by the start-up cyclone due to the warm potential temperature anomaly \( \theta_0 \mathcal{H} \) present over the mountain range when \( t < 0 \) and swept away by \( U(0) \) when \( t > 0 \). It is bounded over the mountain by an anticyclonic pattern due to vortex compression. The corresponding high keeps almost constant after 12hrs and reaches 1.2 mb at 36hrs. Immediately downstream of the mountain a trough is settling, whose amplitude reaches -1.3mb at 36hrs. It is the appearance of this trough that triggers lee-cyclogenesis according to Smith (1986). Hence our GWs response decreases significantly the amplitude of this trough as can be seen in the Figs. 4g, h, and f.

4 Conclusions

We have shown, with a comprehensive model, that the breaking of internal mountain waves can reduce the development of lee Cyclones. Because this model is very fast to run, we have made numerous tests. We found that the cyclolitic influence of the GWs is robust to changes in the mean flow and orography specification. It is also robust when we allow the presence of baroclinic instabilities, or when we use the Geostrophic approximation instead of the semi-geostrophic approximation. With the same degree of generality, we have also found that the GWs breaking tends to by cyclogenetic when we consider a warm front instead of a cold front.

Nevertheless, as our approach assumes that the large-scale flow response to the GWs is balanced, we need to pursue this kind of studies, by allowing the large scale flow to include non-geostrophic modes of baroclinic instabilities and Inertio Gravity Waves.

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