Numerical analysis for an interpretation of the pressuremeter test in cohesive soil

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Abstract:

We present the theory used for the interpretation of the pressuremeter test in cohesive soil and its extension to the conventional limit pressure, which is defined as the pressure at the borehole wall for a volume increase $\Delta V$ equal to the initial volume of the borehole. This conventional limit pressure can be directly measured with the pressuremeter whereas the determination of the theoretical limit pressure needs an extrapolation to an infinite expansion and cannot be directly measured.

The validation of this theory is made by the finite element method with the results of the Tresca standard model of Plaxis, which is compared with the theoretical expression. Conclusions are drawn on the validity of this new theory which allows the measurement and the control of shearing modulus and shear strength of the natural soil.

Résumé:

Nous présentons la théorie de l’interprétation de l’essai pressiométrique dans le sol cohérent et son extension à la pression limite, qui est définie comme la pression appliquée au forage pour laquelle le sonde de la sonde a doublé. La pression limite conventionnelle peut être mesurée directement au moyen du pressiomètre alors que la pression limite théorique nécessite une extrapolation à une expansion infinie et ne peut pas être directement mesurée. La validation de cette théorie est réalisée à l’aide de la méthode des éléments finis par le modèle de Tresca standard de Plaxis. Des conclusions en sont tirées par rapport à la capacité de la théorie à mesurer et à contrôler le module de cisaillement et la cohésion du sol naturel.

Key-words:

Pressuremeter, shear strength, limit pressure

1 Introduction

The pressuremeter is a well-known apparatus (Ménard 1956). It is widely used nowadays for foundation engineering (Amar et al. 1991, Clarke B.G. 1996) with mostly empirical rules. The commonly used methods for the interpretation of the pressuremeter measurement can be found in some states of the art (Ladanyi 1995, Clarke 1997).

The elasto-plasticity is the general frame of this study because it allows to cover the total range from small reversible displacements to large irreversible displacements. The present approach may be considered as following the elasto-plastic method (Gibson and Anderson 1961) extended to the determination of the conventional limit pressure, which is influenced by the equilibrium in the vertical plane. The pressuremeter test is considered as an in situ shearing test, so that it measures soil deformability, shear resistance of the soil, and can be made in any soil, without sampling.
2. Behaviour of cohesive soil around the pressuremeter

2.1 Hypothesis

We assume an test with an elastic behaviour at low level of stress. Numerical results with constitutive model (Cambou and Bahar 1993) show that the test should be assumed as undrained with a permeability lower than $10^{-10}$ m/s. We assume a standard plasticity for a high level of shearing and positive stress in compression. The Tresca relation gives the failure of the soil between the maximum compression stress $\sigma_1$ and the minimum compression stress $\sigma_3$, with the associated flow rule and the scalar $\xi$

\begin{equation}
F(\sigma) = (\sigma_1 - \sigma_3) - 2c_u ; \quad d\varepsilon^p = \xi . \frac{dF(\sigma)}{d\sigma}
\end{equation}

Three different areas of soil are considered from the borehole wall to the infinite radius (Fig. 1). Plasticity appears between the radial stress $\sigma_r$ and the circumferential stress $\sigma_\theta$ in the first zone. This first plastic area extends between radius $r_a$ (borehole wall) and $r_b$ (external radius of the first plastic area). For a cohesive soil the plasticity may appear in the vertical plane (Wood and Wroth 1977) between the vertical stress $\sigma_z$ and the circumferential stress $\sigma_\theta$ in an area between radii $r_b$ and $r_c$ (external radius of both plastic areas). An elastic area extends beyond $r_c$.

2.3 Equilibrium condition

In the horizontal plane and in the vertical plane the equilibrium of an element of soil is given by:

\begin{equation}
\sigma_r - \sigma_\theta + r . \frac{d\sigma_r}{dr} = 0 ;
\end{equation}

\begin{equation}
\frac{d\sigma_z}{dz} = \gamma
\end{equation}

2.4 Pressuremeter relation with two plastic area

Monnet and Chemaa (1995) have shown that the continuity for the stress between the three different areas allows finding the $C_1$ constant, and the relation between the pressure applied by the pressuremeter probe and the displacement at the borehole wall, eq.[4].

\begin{equation}
\frac{m_0}{r_n} - \frac{C_1}{2} = \frac{1}{c_u} p - \frac{\gamma z}{c_u} + \ln \left( \frac{\gamma z}{2G(1 - K_0)} \right) - \frac{C_1}{2}
\end{equation}

with \[5 \quad C_1 = -\frac{c_u}{G}\]

The value of coefficient $C_1$ is usually equal to the hundredth of the radial strain. It can be neglected. Eq. [4] shows a linear relation between the logarithm of the radial strain at the borehole wall and the pressure applied by the pressuremeter as previously found (Gibson and Anderson 1961). Such a relation allows the unique and accurate determination of the shear strength $c_u$ by the slope of the straight line between the variables.

2.5 Pressuremeter relation with one plastic area
The continuity of stress between the two different areas gives a null constant \( C_1 \). The general equilibrium condition between stress and strain is:

\[
\begin{aligned}
L_\nu \left[ \frac{H_\nu}{E_\nu} \right] - \frac{1}{c_u} \cdot p \cdot \left[ \frac{K_0, \gamma' z}{c_u} \right] - 1 + L \nu \left[ \frac{C_u}{2G} \right]
\end{aligned}
\]

The proportionality between the axial strain at the borehole wall and the pressure applied by the pressuremeter is also obtained. The difference between the two cases is linked to the value of the radial stress for the radius of the external area of plasticity \( r_c \).

### 2.6 Conventional limit pressure with two plastic areas

In the two cases, we obtain the conventional limit pressure \( p_{LM} \) with the assumption of a volume of the probe which is double the initial one and a radial equal to \( \sqrt{2} - 1 \). The main interest of this conventional pressure is that it can be directly measured with the pressuremeter test, which is not the case of the theoretical limit pressure found by an extrapolation for an infinite expansion of the cavity. This particular value of the radial strain is put in eq. [4] and we obtain the conventional limit pressure:

\[
\begin{aligned}
p_{LM} &= \gamma' z + c_u L \nu \left( \frac{2G \left( \sqrt{2} - 1 \right) + c_u}{\left( 1 - K_0 \right) \gamma' z + c_u} \right)
\end{aligned}
\]

This relation is quite different from the Ménard experimental correlations, proposed by the European Regional Technical Committee (Amar et al. 1991):

\[
\begin{aligned}
p_{LM} &= 5.5c_u + K_0, \gamma' z & \text{if } p_{LM} - K_0, \gamma' z < 300 \text{kPa}
\end{aligned}
\]

\[
\begin{aligned}
p_{LM} &= 10\left( c_u - 2S \right) + K_0, \gamma' z & \text{if } p_{LM} - K_0, \gamma' z > 300 \text{kPa}
\end{aligned}
\]

The Ménard relation was a result of the experience on many pressuremeter tests at mean depth. Theoretical considerations show that the shearing takes place between the radial stress \( \sigma_r \) and the circumferential stress \( \sigma_\theta \), which lie in the horizontal plane. For a cohesive soil, the plasticity condition shows that the level of shearing is independent of mean stress. For the pressuremeter test the mean stress is proportional to the vertical stress and the level of shearing must be independent to \( \sigma_z \). The eq. [7] and eq. [8] show that the net conventional limit pressure is not linked to a particular value of the vertical stress.

### 2.7 Conventional limit pressure with one plastic area

The particular value of the radial strain is put in eq. [6] to infer the conventional limit pressure. It appears that the net conventional limit pressure is independent of the vertical stress:

\[
\begin{aligned}
p_{LM} &= K_0, \gamma' z + c_u \left[ 1 + L \nu \left( \frac{2G \left( \sqrt{2} - 1 \right)}{c_u} \right) \right]
\end{aligned}
\]

### 3. Numerical validation of the elasto-plastic theory

The theoretical expressions of the conventional limit pressure in the cohesive soil depend on the vertical stress, the coefficient of pressure at rest, the shearing modulus and the shear strength. We use the finite element program Plaxis with the Tresca model to compute the value of the conventional limit pressure, which is compared to the results of the theory. The model used is elasto-plastic with a constant shearing modulus and five parameters (Young modulus and Poisson ratio, undrained shear strength, no friction angle, no dilatancy angle). The method used for the validation is a variation of only one parameter when the other ones stay constant.
Table 1. Values of the mechanical parameters used in the numerical analysis

<table>
<thead>
<tr>
<th>Parameter</th>
<th>G (MPa)</th>
<th>$K_0$</th>
<th>$\sigma_z$ (kPa)</th>
<th>$c_u$ (kPa)</th>
<th>$E$ (MPa)</th>
<th>$\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_z$ 1 zone</td>
<td>13.3</td>
<td>0.667</td>
<td>100-300</td>
<td>100</td>
<td>40</td>
<td>0.499</td>
</tr>
<tr>
<td>$\sigma_z$ 2 zones</td>
<td>13.3</td>
<td>0.667</td>
<td>300-600</td>
<td>100</td>
<td>40</td>
<td>0.499</td>
</tr>
<tr>
<td>G 1 zone</td>
<td>3 - 67</td>
<td>0.667</td>
<td>250</td>
<td>100</td>
<td>10 - 200</td>
<td>0.499</td>
</tr>
<tr>
<td>G 2 zones</td>
<td>3 - 67</td>
<td>0.4</td>
<td>250</td>
<td>100</td>
<td>10 - 200</td>
<td>0.499</td>
</tr>
<tr>
<td>$c_u$ 1 zone</td>
<td>13.3</td>
<td>0.667</td>
<td>250</td>
<td>100 - 700</td>
<td>40</td>
<td>0.499</td>
</tr>
<tr>
<td>$c_u$ 2 zones</td>
<td>13.3</td>
<td>0.667</td>
<td>600</td>
<td>80 - 200</td>
<td>40</td>
<td>0.499</td>
</tr>
<tr>
<td>$K_0$ 1 zone</td>
<td>13.3</td>
<td>0.65 - 1.0</td>
<td>250</td>
<td>100</td>
<td>40</td>
<td>0.499</td>
</tr>
<tr>
<td>$K_0$ 2 zones</td>
<td>13.3</td>
<td>0.3 – 0.55</td>
<td>250</td>
<td>100</td>
<td>40</td>
<td>0.499</td>
</tr>
</tbody>
</table>

The evolution of the numerical conventional limit pressure is compared to the value found by the theoretical expression. The values of the mechanical characteristics are shown in Table 1. The mesh is composed of 9199 nodes with 1013 triangular elements of 15 nodes. The mesh is refined close to the borehole to have a correct numerical evaluation of the radial stress in the plastic area. The left limit is the borehole wall placed at 3cm from the axis to simulate a 6 cm diameter borehole and no horizontal displacement are allowed above the pressuremeter probe, but vertical displacements are allowed. The right limit is placed at a radius of 5m from the axis with an horizontal at rest pressure and displacements allowed in both directions. The lower limit is the horizontal plane, which intersect the probe at its mid length with vertical displacements not allowed. The upper limit is an horizontal plane at 2m from the mid-length of the probe. The L/D ratio is 7.5 adapted to the dimension of the apparatus, which is commonly used.

Fig. 2: Influence of the vertical stress $\sigma_z$ on the conventional limit pressure with one plastic zone

Fig. 3: Influence of the $K_0$ coefficient on the conventional limit pressure with a test with one plastic zone

3.1 Influence of the vertical stress

The theory takes into account the vertical stress as the intermediate stress between the radial and the circumferential stresses. It shows that shearing takes place mainly in the horizontal plane. For cohesive soil, the net conventional limit pressure (difference $p_{cm} - p_0$) is independent of the vertical stress as shown by eqs. [7, 9] where the vertical stress is an additive...
factor into the theoretical conventional limit pressure so that the increase of the vertical stress gives an equivalent increase of the conventional limit pressure. The FEM (Fig. 2) shows the same variation of the conventional limit pressure with an underestimation in the range of 8%. The Ménard eq. [8] assumes that the net conventional limit pressure does not depend on the vertical stress, with an overestimation in the range of 7%.

Fig. 4: Influence of the Shear Modulus on the conventional limit pressure with a test with one plastic zone

Fig. 5: Influence of the undrained strength on the conventional limit pressure on a test with one plastic zone

3.2 Influence of the coefficient of pressure at rest $K_0$

The coefficient of pressure at rest $K_0$ should increase the horizontal pressure at rest and consequently should increase the conventional limit pressure. This evolution is found by the theory for one plastic areas (Fig. 3) with an underestimation in the range of 6%. It can be seen that the relation proposed by Ménard gives more or less a larger difference with the Plaxis results with a difference in the range of 10%.

3.3 Influence of the shearing modulus

For the conventional limit pressure value, if the soil is stiffer the deformation of the soil should be smaller, and should reach twice the initial volume for a high value of the pressure. On the reverse side, for a soft soil and for a define value of the pressure, the deformation of the soil should be larger, and should reach twice the initial volume for a low value of the pressure. This evolution is described by the theory, and we can see (Fig. 4) that the shearing modulus have an increasing influence on the conventional limit pressure. Furthermore, the theory can predict with a precision of 14% the conventional limit pressure obtained by the Plaxis program. But if we consider the correlative relation of Ménard eq.[8] we find that there is no influence of the shearing modulus on the conventional limit pressure. The new theory improve the interpretation of the pressuremeter test by the use of the Young modulus as a parameter of the limit pressure.

3.4 Influence of the shear strength

The shear strength acts as a resistance factor for the deformation of the soil, and when the shear strength increases the conventional limit pressure increases also. This is found by the theory of the expansion of the pressuremeter probe with a conventional limit pressure, which is function of the shear strength. This variation is also obtained by the finite element analysis.
made by Plaxis. It can be seen (Fig.5) that the evolution of the limit pressure is in the same range as the numerical results with a mean difference of 70kPa on the limit pressure, which validates the theory for the variation of the shear strength. If we consider now the correlative relation of Menard, we find an increasing difference with the numerical results of Plaxis and a large underestimation of the shear strength, with a mean difference of 360kPa.

4. Conclusion

We present the numerical validation of a theory which takes into account the three dimensional state of stress around the pressuremeter, the plasticity which occurs between the radial stress and the circumferential stress, and the plasticity which occurs between the vertical stress and the circumferential stress.

The theory shows that the linearity between the radial stress and the logarithm of radial strain at the borehole wall allows the measurement of the shear strength. This value can be controlled by comparison between the theoretical and experimental pressuremeter curves and by comparison between theoretical and experimental conventional limit pressures.

It shows that four mechanical parameters have an influence on the conventional limit pressure ($\sigma_z$, $G$, $c_u$, $K_0$). The numerical calculation of the pressuremeter test by Plaxis software has been made with a variation of one of these variables while the other ones remained unchanged. The theory shows the same variation of each variable as the numerical results and a close agreement with Plaxis. This allows the validation of the theory in the range of variation for the four variables identified.

References


