One-dimensional consolidation of poroelastic medium with rheological skeleton

Sawicki Eugeniusz
Wrocław University of Technology
Institute of Geotechnics and Hydrotechnics
Plac Grunwaldzki 9
50-377 Wroclaw
email: eugeniusz.sawicki@pwr.wroc.pl

Abstract:
In this paper the author presents the results of numerical investigations of poroelastic medium behaviour (consisting of a skeleton and the fluid flowing through the voids in this skeleton) called Biot’s body. It is assumed that the stress-strain state does not achieve the failure limits of the body.

The starting point for these considerations is the phenomenon of one-dimensional consolidation of such a medium described by the fundamental poroelastic Biot’s equation and completed by the Biot’s-Darcy flow equation. Additionally in this description the rheological properties of the skeleton of Biot body are modelled by linear visco-elastic Zener’s body (standard body).

Key-words:
consolidation, Biot’s body, poroelasticity

1 Introduction

This article presents an attempt to answer the following question: how do the viscosity properties (effects) of the soil matrix influence the stress-strain process of the two-phase poroelastic medium (water saturated soil), called the Biot’s body?

In order to find the solution, numerical investigations were conducted on one-dimensional consolidation of the Biot’s body with Zener’s rheological skeleton, which allows for viscosity of the solid phase. The dynamic viscosity as well as the volumetric viscosity was taken into consideration. The works of Kisiel et al. (1982) and Strzelecki et al. (2007) have shown that in the case of cohesive soils, the assumption of viscosity properties of the skeleton is not unfounded. It is assumed, that during the consolidation process, the motion of the solid particles takes place on the contact between: solid - solid, or in the majority of cases, on the film water bound to the surface of the solid. Hence a combination of two independent processes is obtained: i) consolidation connected with density increasing of the porous medium only due to the outflow of free water (according to Kisiel and Lysik (1966) this process corresponds to the Terzaghi’s consolidation) and ii) creeping process regarded as the motion of the solid phase of the medium, opposed by the viscous resistance, resulting from the assumption that moving grains “are sliding” on the film water bound to its surface, Strzelecki et al.(2007).

As it was mentioned earlier, the classical Biot’s model does not consider the viscosity behaviour of the soil matrix, therefore Zener’s body was additionally introduced. In the papers of Kisiel and Lysik (1966), Kisiel (1967,1982), Derski (1975), Sawicki (1994) many other more or less complicated rheological models can be found, for example: Kelvin’s, Maxwell’s, Goldstejn’s and Tan’s models. Hence, section two gives an explanation to why the Zener’s model was used in the present paper. It also deals with the fundamental equations of consolidation, the rheological Zener’s skeleton model, and the definition of numerical problem.
Section three contains the definition of the boundary value problems (classical Biot’s model, Biot’s model with Zener’s skeleton) and the results. The discussion of the results and conclusions are presented in the fourth section.

2 Biot’s model with rheological Zener’s skeleton

According to what has been already said in the introduction, the analysis of one-dimensional consolidation of soil medium is presented. The starting point of the considerations is the Biot’s fundamental equation of poroelasticity, complemented by the Biot-Darcy’s filtration law. One-dimensional equations of the inner equilibrium of a two-phase medium, omitting the body forces, are denoted by:

\[
\begin{align*}
N_2 \nabla^2 u_1 + (N_2 + A_2 - \frac{Q^2}{R}) \varepsilon_{11} &= -\frac{H}{R} \sigma_{11} \\
\frac{k}{f^2} \nabla^2 \sigma &= \frac{1}{R} \sigma - \frac{H}{R} \dot{\varepsilon}
\end{align*}
\]

where \(Q=H-R\), \(N_2, A_2, H, R\) are Biot’s constants (Biot (1941), Strzelecki et al. (2007)), \(k\) is Darcy’s filtration coefficient and \(f\) stands for medium porosity. This model assigns elasticity properties to the soil skeleton (through parameters \(N, A\)) and to the water that fills it’s pores (through parameters \(H, R\)). However, it is well known that, under stress the soil medium does not behave as “Hooke’s body” and hence the curve illustrating the soil settlement in time obtained from equations (2.1) is not consistent with the empirical results. Biot’s model generates too large instantaneous settlements (just after the imposing of load), whereas the values observed in the laboratory increase gradually. In other words: in an edometrical experiment the surface of the soil sample under load does not “deflect” instantaneously to a considerable value, but settles in time with a certain velocity, in the direction consistent with the direction of acting load. Therefore it can be assumed that this motion is a manifestation of the viscosity of the soil medium. The introduction of the rheological element, with the property of viscosity (dash-pot in Fig. 1), to the classical Biot’s model can result in (dependent on the configuration) the suppression of the instantaneous settlements. Such an action makes model solutions considerably closer to the real behaviour of the medium.

Theoretically there are an infinite number of rheological models that can correctly describe various soil properties. Nevertheless, according to the opinion of the already cited authors, the choice of an adequate model is always a compromise between the accuracy of the description of the examined soil properties - on one hand and on the other - the possibility of determining the parameters of rheological elements and calculation possibilities.

In the search of a rheological model to attain the objective specified in the introduction the Zener’s body was adopted (Fig. 1a and 1b). It consists of two springs: 1), 2) and a dash-pot 3), where the configurations a) and b) of rheological elements are equivalent.

According to Kisiel and Lysik (1966) the most versatile in describing the general properties of soil is the simple Tan’s model, which is actually the Zener’s model incorporated into Terzaghi’s column. The versatility originates from the fact, that after removing spring 1) we obtain Kelvin’s body, after removing spring 2) we obtain Maxwell’s body, whereas after removing elements 2) and 3) we obtain Hooke’s body.
It seems possible to interpret the particular elements of the Zener’s body by referring to the course of the consolidation process. And hence, spring 1) represents „structural elasticity”. It relates to the situation when under load the soil pores are closing, the water is squeezed out and the particles making up the skeleton change configuration. Next the load is being transferred onto the skeleton grains which deform elastically – this is represented by spring 2). When the whole load is transferred onto the solid phase, a long-lasting process of slow viscous flow of the medium (creeping) begins. This can be regarded as the grains “sliding” on the layers of the film water bound to their surfaces, which is represented by dash-pot 3). As shown in Fig. 1 each rheological element is characterized by two parameters. \( A \) represents bulk modulus of the soil, \( N \) is the shear modulus, whereas \( \lambda \) and \( \mu \) stand for volumetric and dynamic viscosity of the skeleton, respectively. The stress-strain relationship for Zener’s body that takes the stress transferred by the fluid into account is denoted by:

\[
\sigma_{ij} + \sigma \delta_{ij} = 2N_2 \psi_p \varepsilon_{ij} + A_2 \psi_o \varepsilon \delta_{ij} + \frac{Q}{R} \sigma \delta_{ij} - \frac{Q^2}{R} \varepsilon \delta_{ij} + \sigma \delta_{ij}
\]

where:

\[
\psi_p = \frac{1 + \Gamma \cdot T \cdot s}{1 + T \cdot s}, \quad \psi_o = \frac{1 + \Gamma_1 \cdot T_1 \cdot s}{1 + T_1 \cdot s}, \quad \Gamma = \frac{N_1 + N_2}{N_2}, \quad \Gamma_1 = \frac{A_1 + A_2}{A_2},
\]

\[
T = \frac{\mu}{N_1}, \quad T_1 = \frac{\lambda}{A_1}, \quad \text{whereas } s \text{ is Mikusinski’s operator, } s = \partial/\partial t.
\]

After differentiating formula (2.2) and putting the terms in order, the one-dimensional equation of Biot’s consolidation with Zener’s rheological skeleton is denoted by:

\[
N_2 \nabla^2 u_1 + (N_2 + A_2 - \frac{Q^2}{R}) \varepsilon_{,1} + \frac{H}{R} \sigma_{,1} = \frac{k}{f^2} \nabla^2 \sigma = \frac{1}{R} \sigma - \frac{H}{R} \dot{\varepsilon}
\]

As the solution of the equation set shown above, in the case when the relaxation time \( T \neq T_1 \) proves difficult (second time derivative appears), the calculations were performed assuming that \( T = T_1 \). The other terms are adopted as follows:

- \( N_1 = 10^7 \) [Pa], \( A_1 = 5 \times 10^7 \) [Pa],
- \( N_2 = 10^7 \) [Pa], \( A_2 = 5 \times 10^7 \) [Pa],
- \( \mu = 10^{11} \) [Pa*s], \( T = T_1 = \mu / N_1 = \lambda / A_1 = 10^4 \) [s]
- \( k = 10^{11} \) m/s, \( f = 0.35 \)
- the length of the soil sample (region of consolidation) is 10 m.

It has to be underlined that the rheological values mentioned above are approximate – established from the literature (Emmrich (1984)).

The given boundary value problems and their solutions are presented in the next - and third - section.

3 The boundary value problems and solutions

The problem of soil medium consolidation was solved using the finite element method implemented in code FlexPDE5. Non-dimensional values were used in the calculations and the reference values were assumed as follows: displacement of medium \( U_0 = 10^{-2} \) m, fluid stress \( S_0 = 10^5 \) Pa. The geometry of the region and the given boundary conditions are shown in Fig. 2.

\[
\sigma = -10^5/S_0 \\
\sigma = -10^5/S_0 \\
\begin{align*}
& u_p = 0 \\
& \text{pointload}(u_p) = 10^{-3}/U_0
\end{align*}
\]

Fig. 2 – Geometry of the consolidation region and boundary conditions

Firstly, the problem of Biot’s body consolidation was solved. In this case only spring 2) is active (Fig. 1) and filtration (equation (2.1)). Hence, it can be assumed that the settlements (displacements) obtained, correspond to maximal values of instantaneous settlements (Fig. 3a). (In the example shown instantaneous settlements are nearly identical to the final settlements).

Fig. 3 – The settlements values obtained from models: a) Biot’s \((t = 10^9 \) s), b) Zener’s \((t = 10^9 \) s)
In the second step however, in Zener’s model, the dash-pot 3) was disconnected and the problem of Biot’s body consolidation was solved again. This time with springs 1), 2) and filtration (equation (2.1)). This situation corresponds to the minimal value of instantaneous settlement, in this case the value obtained was -9.6e-3 m.

In the two cases described above, creeping of the medium appears, apart from the mentioned instantaneous settlements. However, this time it results from the viscous resistance of the filtrating fluid.

**FIG. 4** – The distribution of the stress after time $t = 10^3$ s transferred by: a) water, b) soil skeleton

Next, the problem of medium consolidation with Zener’s skeleton was solved. In this case the boundary value problem was complemented with an initial condition: fluid stress $\sigma(t=0) = -10^3/S_0$ and displacement $u_p(t=0) = -9.6e-3*x/U_0$, where variable $x$ stands for the
point’s instant coordinate. The resulting values of settlements are shown in Fig. 3b. It can be seen that the maximal values, from the classical Biot’s model and from the model that considers rheological properties of the skeleton, agree.

The process of consolidation was observed at times: \(t=0.2s\), \(t=1s\), \(t=10s\), \(t=100s\), \(t=10^3s\), \(t=10^4s\), \(t=10^7s\), but only some results are presented in this paper.

The distribution of the stress transferred by water and the soil skeleton after time \(t=10^3s\) and \(t=10^7s\) are shown in Fig. 4 and Fig. 5, respectively.

4 Discussion of the results and conclusions

The results of the numerical analyses presented in this article seem to agree with the engineering intuition. Even by analyzing the pressure distribution in the soil medium and in the fluid, we observe that during the first stages of consolidation \((t=10^3s)\) the stress in the fluid is similar to that in the soil skeleton, whereas at the end of this process \((t=10^7s)\) the stress in the skeleton is much larger. The maximal settlements obtained in both cases, that is for the classical Biot’s model and for Zener’s body, take the same values. The essential difference is between the instantaneous settlements, which in the case of the evaluated rheological model are smaller. This statement - however expected - provides an answer to the question of the influence of „viscosity” of the soil skeleton on the stress-strain process.

It should be emphasized that in the numerical experiment with Zener’s body, adopting adequate initial conditions remains crucial.

Considering that informatization and computer technology development provide better and better computational tools, it seems advisable to search for models and solutions that most accurately describe the natural processes. In this context the present paper is only a minor step in distinguishing which of the simple rheological models describes well enough the general properties of the soil medium.

References


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