Impulse response function analysis of pore pressure in earthdams

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Abstract:

Effective control of dam safety requires that the measured pore-pressure data be interpreted in the shortest possible time following the readings. Direct resolution based on partial differential equations are not appropriate. We present a relevant formalism for analysing pore-pressure monitoring data: the Impulse Response Function Analysis (IRFA) method. The model based on approximations for the impulse response of the dam gives the variations in the pore-pressure measurement resulting from changes in the reservoir levels. An expression for the explicit estimation of in situ hydraulic diffusivity is derived. The model were applied to the analysis of monitoring data obtained at a zoned earthdam. Obtained results proved that essential aspects of the observed phenomenon in most cells data can be described in this linear framework, and that they are taking into account.

Résumé:


Key-words:

pore pressure ; monitoring ; earthdams

1 Introduction

The analysis of dam monitoring data have to be carried out periodically at short time intervals. Direct resolution based on partial differential equations, like the Finite Element Method or the Finite Volume Method, are therefore not appropriate. In order to quantify the changes occurring under constant conditions (i.e., the ageing, the trends, the drift), it is necessary in the first place to be able to account for any time-independent changes due solely to external constraints, such as variations in the reservoir and precipitation levels.

The most common tools employed for dam monitoring data analysis are the statistical methods of the Hydrostatic-Season-Time (HST) type. They were developed in the 1960s for analysing the displacements resulting from the pendulum effects occurring at arch dams (Ferry and Willim, 1958). These methods are used nowadays to analyse measurements of other kinds.
The experience acquired on several hundreds of dams has confirmed what an excellent tool this approach can be for interpreting monitoring data.

These methods are however unable to deal with dissipative processes like seepage. Indeed, it is necessary to look at the loading history responsible for the levels occurring at a given moment, rather than simply taking the values of all the loads measured at the same moment.

Recently a very simple delay model has been designed to analyze the pore pressures measured in and around dams, which are influenced by the reservoir and rainfall levels (Bonelli and Royet, 2001; Bonelli, 2003). This model was based on the approximation of the Impulse Response Function (IRF) of the dam works with an exponential decay. In this paper, we focus on the influence of the water level on pore-pressure measured in the core of an earthdam.

2 IRFA model for pore-pressure analysis

When the temperature and the porosity are supposed to be constant, flow through porous media is usually described by the Richards equation obtained by combining the equation of mass conservation and the momentum equation, expressed by Darcy law. It is a well known fact that Richards equation is non linear.

In a first order approach, Dams can be assumed to behave in an approximately linear manner in normal operation, within a limited range of loadings. The description is therefore taken here to be linear: the water pore pressure is described by a linear parabolic equation. The solution can then be expressed as a function of the initial condition and the boundary conditions in terms of the Green's function associated with the boundary problem.

This representation amounts to an external description in which the impulse responses of the system can be expressed in terms of the Green's function. This approach has a well-established theory (Beck et al., 1992). However, the Green’s function is unknown. A classical method in signal processing is then to construct models which represent these impulse response functions, at least approximately.

The time origin $t = 0$ is the relevant date between the completion of the dam and the first impounding. We propose a model for analysing monitoring data corresponding to dam operation, starting at a date $t_0$ such as influence of initial pore-pressure can be neglected. The pore-pressure $P(t)$ is assumed to be influenced by the water level, and the time (ageing). The obtained IRFA model is therefore:

$$P(t) = C + H(t) + T(t),$$

where $t > t_0$, $C$ is a constant, $H(t)$ is the water level effect, and $T(t)$ is the time effect.

The simplest approximation of the impulse response function is given by the two-parameters $(\alpha, \eta)$ exponential decay:

$$H(t) = \frac{\alpha}{\eta} \int_0^t \exp \left( -\frac{t - \tau}{\eta} \right) \Delta Z(\tau) d\tau,$$

where $\Delta Z(t) = Z(t) - Z_{\text{min}}$, $Z(t)$ is the reservoir water level, $Z_{\text{min}}$ is the minimum water level (e.g. that of the drainage blanket), $\alpha$ is the static damping factor, and $\eta$ is the characteristic diffusion time of the located point. The variable $T(t)$ accounts for the other non stationary effects (e.g. ageing), of which the formulation is beyond the scope of the present paper.
Taking a ramp expression for the water level time series, we obtain the following ARMA(1,2) model for \( H(t) \):

\[
H^{n+1} = (1 - \theta_1) H^n + \theta_1 \left[ \theta_2 \Delta Z^{n+1} + (1 - \theta_2) \Delta Z^n \right],
\]

with

\[
\theta_1 = 1 - e^{-\Delta \theta / \eta} \quad \text{and} \quad \theta_2 = \frac{1}{\theta_1} - \frac{\eta}{\Delta \theta},
\]

where \( \Delta \theta = t^{n+1} - t^n \). It can be noted that \( 0 < \theta_1 < 1 \) and \( 1/2 < \theta_2 < 1 \). In addition, \( \theta_1 \to 1 \) when \( \eta \to 0 \): this model includes the special case consisting of an instantaneous response.

### 3 Interpretation of the parameters

Coefficient \( \eta \) is a characteristic diffusion time: the system has some memory of the previous values of the loading time series. The role of this parameter is given by harmonic analysis: if \( \Delta Z(t) = \sin(\omega t) \), then \( H(t) = \alpha \sin(\omega(t - \eta)) \) under slowly varying loading conditions \((\omega \eta)^2 \ll 1\). The characteristic time \( \eta \) quantifies the time elapsing between the onset of the loading and the response, and the dimensionless parameter \( \alpha \) characterises the damping.

Closed-form solution of a relevant boundary value problem yields (Bonelli, 2003):

\[
\alpha = 1 - \frac{x}{L}, \quad \eta = \frac{x}{6L} \left( 2 - \frac{x}{L} \right) T, \quad T = \frac{L^2}{D},
\]

where \( T \) is the characteristic time of diffusion, \( D \) is the hydric coefficient of diffusion, \( x \) is the mean distance between the instrument (e.g. a pore pressure cell) and the loading surface (the upstream face), and \( L \) is the mean seepage path between the loading point, and the outlet point (chimney drain, drainage blanket, downstream face), see Fig. 1.

A coefficient \( \alpha \) around the unit value means either that the instrument is located near the upstream (and \( x \) is small) or that the outlet point is located far from the loading point (and \( L \) is large). A very large time \( \eta \) will reflect the presence of either a highly impermeable soil or a very long drainage distance \( L \).

The characteristic time \( \eta \) makes it possible to assess the diffusion coefficient:

\[
D = \frac{(1 - \alpha^2)L^2}{6\eta}.
\]

The diffusion coefficient is \( D = \lambda / (\mu_c c) \) where \( \lambda \) is the intrinsic permeability, \( \mu_c \) is the dynamic water viscosity and \( c \) is the hydric capacity of the soil.

Unsaturated soils have a non null hydric capacity coefficient \( c = n \partial S / \partial s \) where \( n \) is the porosity, \( S \) is the saturation degree and \( s \) is the suction. In this case, the pore pressure arises almost entirely from suction forces and capillaries and \( c \) is in fact the moisture capacity. This is the explanation of delayed responses of the water level effect observed in pore pressure measured with cells located in the drawdown zones of the dam.

However, in zones located below the free surface, the hydric capacity can also be non null. In these so-called saturated zones, the pore fluid may be a mixture of incompressible water and
compressible gas bubbles. The compressibility of the interstitial fluid is likely to be the most decisive factor involved. In this case, \( c = n / \chi \) is the specific storage (Bear, 1972), where \( \chi \) is the pore fluid bulk modulus.

St-Arnaud (1995) suggested in particular that one should take into account the fact that dam water not only has its own natural air content, but also contains air which was imprisoned during the first impounding, which is partly compressed and partly dissolved.

The water permeability is \( k = \rho_w g \lambda / \mu_w \) where \( \rho_w \) is the water density and \( g \) is the gravitational constant. It may be estimated from the diffusivity by \( k = \rho_w gcD \), provided that \( c \) is known.

![Diagram](image)

**Fig. 1** – Parameters \( \alpha \) and \( \eta \) as a function of \( x / L \) (a). Interpretation for a homogeneous earthdam (b), and for a zoned earthdam (c).

### 4 Application

The measurements obtained on a zoned earthfill dam 42 meters in height with a horizontal drainage blanket were analysed with the IRFA model. The dam rests on a good rock foundation with a 25 m shallow grout curtain under the central clay core. A plan and three cross section are shown in Fig. 2. Elevations are in meters above normal sea water level. A total of 14 electrical pore-pressure cells were installed on three vertical section of the core.

The water level data cover a heigh years period (3175 days). The level of the reservoir underwent cyclic raising and lowering, with a period of approximately one year (\( \omega = 2\pi / 365 \) in days'). The analysis period cover a three years period dam operation (1175 days), starting at a date \( t_0 = 2000 \) days sufficiently long after the dam was first filled, so that influence of initial condition can be neglected. Measurements were done approximately every five days and represent 167 numbers for each serie for the analysis period.

The numerical results are summarized in Table 1. These results show two features: 1) the amplitude of the response \( \alpha \) decreases with distance from the upstream face, 2) the time delay \( \eta \) increases in the downstream direction. The characteristic diffusion time \( T \) ranged from 48 days to 506 days. The diffusivity Eq. (6) ranged from 2 to \( 20 \times 10^{-6} \) m\(^2\)/s.

A good knowledge of the dam is necessary to be able to interpret the parameters \( \alpha, \eta \), and to estimate the water permeability \( k \). However, this is beyond the scope of the present paper.

A typical fit shows the response lag, which was of the same order of size as the characteristic time \( \eta \) (Fig. 3a), as well as the hysteresis occurring during increasing/decreasing water level cycles (Fig. 3b). Some measurements can be taken to mean that an increase in the interstitial pressure has occurred during a decrease in the reservoir level, and vice-versa.

This well-know phenomenon has been observed in situ (Kjaernsli et al., 1982; Myrvoll et al., 1985) as well as being simulated under laboratory conditions (Windisch and Høeg, 2000).
Fig. 2 – Lay-out, cross-sections locations and pore-pressure cells locations.

<table>
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<tr>
<th>Data</th>
<th>α</th>
<th>η (days)</th>
<th>T (days)</th>
<th>L (m)</th>
<th>D (10^{-6} m²/s)</th>
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Table 1 – IRFA results of cells data analysis located in the core.
5 Conclusion

We propose an Impulse Response Function Analysis (IRFA) method for interpreting pore pressures measurements in earthdams. This model is appropriate for flows showing fairly constant diffusivities, subjected to slowly varying loads with in comparison with the characteristic diffusion time. It accounts for some of the main aspects of delayed effects: dissipation, accommodation (delay and damping) under cyclic loading, and influence of the previous loading history. The simplest IRFA model with an exponential decay involves a recurrence equation with which it is possible to apply convolution products using a simple numerical method. The discrete time formulation used is similar to that of a model of the ARMA(1,2) type. This study opens new perspectives as regards the potential use of modern methods of this kind for analysing dam monitoring data.

References

Windisch, E. & Hoeg, K. 2000 Pore pressure in the till core of Oddatjorn dam. 53rd Canadian Geotechnical Conference, Montreal, pp. 231-238.