Diversity techniques for blind channel equalization in mobile communications

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1 Introduction

Mobile communications operate in a very hostile environment due to multipath propagation and vehicle displacement. Depending on the transmission rate and vehicle speed, either frequency-selectivity or Doppler spreading becomes the major concern ([1]). In this paper, a blind equalization technique is proposed which can be applied to compensate the distortion introduced by the multiplicative and the frequency-selective mobile channels.

The proposed approach relies on the availability of space or time diversity which enables the use of single-input multiple-output formulation (SIMO) of the transmission system. It is based on a criteria which allows for linear equalization of the received data. In fact, the proposed formulation is more general than the application suggested here and could also be applied in other environments. Thus, here it will be shown to be useful for defining a deterministic criteria for blind equalization, but it could also be applied to the problem of channel estimation by means of cyclostationary statistics-based methods (e.g. [2]).

The suggested algorithm has a low computational load and exhibits performance similar to that of one of other deterministic criteria proposed in the literature: it obtains relatively good results for short data sets, it assumes the channel is FIR with known length (this constraint will be relaxed further on) and its original derivation does not take into account the additive noise, although it is of course considered when defining the method final formulation.

As opposed to methods which have appeared earlier in the literature, the proposed algorithm is based on the assumption that the receiver can observe the complete convolution of the transmitted data and the channel response. In the case of convolutive channels, the full channel output is available

\[ Y_i(z) = T(z) \cdot C_i(z) + W_i(z) \quad i = 1, ..., B \quad (1) \]

If the noise term is negligible it follows that:

\[ Y_i(z) = T(z) \cdot C_i(z) \quad i = 1, ..., B \]

and therefore:

\[ T(z) = g.c.d.\{Y_i(z)\} \quad i = 1, ..., B \quad (2) \]

where \( g.c.d. \) stands for the greatest common divisor. The algorithm presented here is based on the estimation of the transmitted data using equation (2). Of course, in order to apply this equation the complete z-transform \( Y_i(z) \) must be

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available and, therefore, a block transmission scheme with a 
guard interval is needed. In the case of OFDM operating, no 
efficiency is lost because no guard interval is needed. Indeed, 
when working in the transformed domain, the OFDM received 
signal could be regarded as the result of a circular convolution 
with the DFT (Discrete Fourier Transform) of the channel 
distortion ([6]):

$$Y'[k] = T[k] \otimes C'[k] \quad i = 1, ..., B$$

where \(\otimes\) stands for circular convolution. Furthermore, over-
sampling the OFDM received signal is equivalent to zero 
padding the transformed domain sequences as long as there 
was no aliasing introduced when they were sampled at the 
symbol rate ([5]). Thus, if the received signal is oversampled, the 
circular convolution can be converted into a linear one:

$$Y'[k] = X_{ZP}[k] \otimes C'_{ZP}[k] = X[k] \ast C'[k]$$

where \(ZP\) stands for ‘zero-padding’. This way, the trans-
mmission of an OFDM signal through a multiplicative channel 
would fit also the model of equation (1).

Figure (1b) shows the linear equalization architecture em-
ployed in this paper. The multiple diversity branches are 
combined by means of FIR filters \(E'[k]\) to generate an output \(R[k]\):

$$R(z) = \sum_{i=1}^{B} Y'(z) \cdot E'[z] = T(z) \cdot \sum_{i=1}^{B} C'(z) \cdot E'[z] \quad (3)$$

Thus, our problem can be stated as that one of designing 
the filters \(E'[k]\) in order to retrieve the transmitted data: 
\(R[k] = T[k]\). Notice that if \(B = 2\) (dual diversity) and the 
optimization criteria is based on forcing \(R[k] = 0\) then the 
algorithm in [7] is obtained. In the present case, the perfect 
equalization (zero forcing) criteria requires \(R(z) = T(z)\) and 
therefore

$$\sum_{i=1}^{B} C'(z) \cdot E'[z] = 1 \quad (4)$$

In the next section a blind algorithm is summarized which 
provides the equalizer coefficients \(E'[k]\).

3 Blind algorithm design

The proposed algorithm is based on the following property 
(Bezout equation):

Given \(B\) polynomials \(\{A'(z)\}\) the equation

$$\sum_{i=1}^{B} A'(z) \cdot a'(z) = 1$$

has a iff the \(B\) polynomials are coprime. Furthermore, the 
solution is unique (up to a multiplicative constant) iff

$$\deg \{a'(z)\} + 1 = \frac{\deg \{A'(z)\}}{B - 1} \quad i = 1, ..., B \quad (5)$$

If the polynomials \(a'(z)\) have a greater degree infinite 
solutions can be found for this equation. When this property 
is applied to equations (3)-(4), it turns out that perfect channel 
equalization can be obtained only when channel responses 
have no factor in common, a result well known in the liter-
ature ([8]). In case this condition is satisfied, the zero-forcing 
equalizer coefficients will be achieved by solving equation (4).

Furthermore, from the previous property also follows that the 
zero-forcing equalizer is unique when the equalizer lengths are 
selected according to (5) and are non-unique if their filters are 
longer, being the difference among the possible solutions their 
performance in front of the additive noise ([11]). Thus, design-
ing longer equalizers allowed for performance improvements 
in the BER.

Besides, notice that equation (3) says that the equalizer 
output \(R(z)\) will always be a multiple of the transmitted data 
\(T(z)\), and that

$$\deg \{R(z)\} = \deg \{T(z)\} + \deg \left\{ \sum_{i=1}^{B} C'(z) \cdot E'[z] \right\}$$

Therefore, asking for an output of minimum length (\(R(z)\) of 
minimum degree) is equivalent to asking for perfect channel 
equalization: \(R[k] = T[k]\). This is the design criteria in which 
the proposed method is based: design \(E'[k]\) so that \(R[k]\) has 
minimum length, then \(R[k] = \alpha T[k]\), being \(\alpha\) an unknown 
complex constant. The matrix formulation for the method can 
be found in [5] and is briefly summarized here in order to 
introduce the new method.

As shown in [5], equation (3) can be written using matrix 
notation as

$$Z = Y E'$$

where \(Z\) is a generalized Sylvester matrix. Besides, the 
perfect equalization case in (4) can be written as

$$Y = \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} \quad \begin{bmatrix} \alpha T \\ 0 \end{bmatrix} = \begin{bmatrix} Y_1 E' \\ 0 \end{bmatrix} \quad (6)$$

where the received data matrix \(Y\) has been split in two parts. 
The minimum length criteria can be described then as finding 
those equalizer coefficients \(E'\) such that

$$\begin{bmatrix} \alpha T \\ 0 \end{bmatrix} \quad (7)$$

Thus the method proposed in [5] can be considered as a one 
based on the noise subspace of matrix \(Y\). Once the equalizer 
has been estimated, the received data can be filtered to yield 
an estimation of the transmitted data:

$$\hat{T} = Y \hat{E} \quad (8)$$

4 Proposed algorithm formulation

The previous method has two main drawbacks which are 
solved by the new approach proposed here:

- The previous algorithm can only be applied when equal-
izer lengths satisfy (5), for if they were overdimensioned 
the algorithm might converge to a non-useful solution 
where constant \(\alpha=0\) and, therefore, \(R[k] = 0\). Hence, 
the advantages of long equalizers in terms of noise can-
not be exploited.

- The previous algorithm does not fully exploit the avail-
able data. Both \(Y_1\) and \(Y_2\) contain information on 
the channel and the transmitted data, but the algorithm de-
scribed in [5] designs the equalizer taps based on \(Y_2\) 
only.
According to these considerations, the new algorithm formulation tries to maximize the Signal-to-ISI-plus-noise-ratio (SINR) at the equalizer output. This SINR can be approximately estimated (see eq. (6) ) as

$$\tilde{\text{SINR}} = \frac{E[H R Y]}{E[H Y^H R Y]}$$

(9)

This is the new cost function to be optimized. Notice that this new criteria is coherent with the algorithm in [5], given that it aims to find the solution which maximizes the mean power of detected symbols under the constraint of minimum length equalizer output and noise level reduction.

The covariance matrices associated to $Y$ and $Y_d$ are non-negative defined and thus the quotient in equation (9) corresponds to a typical Rayleigh quotient form (\cite{10}). Therefore, it satisfies:

$$\lambda_{\text{min}} \left[ Y^H Y \right] \leq \frac{E[H R Y]}{E[H Y^H R Y]} \leq \lambda_{\text{max}} \left[ Y^H Y \right]$$

That is, the equalizer output SINR is bounded by the minimum and maximum eigenvalues of the data matrix $Y$ in the norm of $Y$. Thus, the equalizer that maximizes (9) can be obtained as the maximum generalized eigenvector:

$$Y^H Y E = \lambda_{\text{max}} Y^H Y E \quad \tilde{\text{SINR}} = \lambda_{\text{max}}$$

(10)

Notice that this new cost function integrates the information contained in $Y$ and $Y_d$. Furthermore, the solution $R[k] = 0$ would yield a very poor SINR compared to the other solutions and, therefore, it can be rejected as a solution of the new cost function. Once the possibility of converging to this solution has been discarded, the length constraint in (5) can be released and longer equalizers can be employed. Simulations will show the performance obtained by increasing the equalizer length.

The equalizer performance can be further improved if a delay is allowed in the equalized signal. Many sets of equalizers can be obtained for different delays:

$$\sum_{i=1}^{B} C_i(z) \cdot E_i(z) = z^{-d} \quad 0 \leq d < \deg \{ C_i(z) \cdot E_i(z) \}$$

(11)

providing different delayed estimates $R(z) = z^{-d} \hat{T}(z)$. Although in average terms some delays will provide better estimates than others ([9]), the simulations performed showed that all delays are useful for noise impairment reduction due to the reduced set of data available. Notice that, if the eigenvalue is taken as an estimate of the SINR, in order to decide which of the delays yields the better estimate of $\hat{T}(z)$ only the largest eigenvalue must be computed for the different values of the delay $d$. Unfortunately, this estimate is only reliable in high SNR scenarios, otherwise the full computation of the equalizer output must be carried out to find out which delay value is preferred.

5 Relation with other algorithms

In this section the algorithm in [5] (equations (7)-(8) ) is compared with the extension of the deterministic method proposed in [12] to the block transmission case, rather than the continuous transmission case analyzed in the original paper.

Equation (6) can be written using matrix notation as

$$\mathbf{Z} = \mathbf{Y} E = \mathbf{T} C E + \mathbf{W} E$$

where the vectors and matrices are associated to the polynomials with the same letters. In order to compare both methods a singular value decomposition (SVD) must be performed to the generalized Sylvester matrix $Y$:

$$Y = U \Sigma V^H$$

Then, it can be seen that the proposed method is based on the signal subspace column vectors of matrix $U$, whereas the method proposed in [12] was based on its noise ones.

Furthermore, the algorithm proposed here has a computational load much lower that one of [12], even if several values of the delay $d$ in equation (11) are used to reduce variance. Both methods have in common that they require SVD computation. However, the method proposed in this paper only one SVD must be computed and the matrix involved in it has the same size as the channel length, whereas the algorithm in [12] requires two SVD of matrices about the same size as the transmitted signal length. Since the frame duration must be chosen so that

$$\deg \{ T(z) \} >> \deg \{ C_i(z) \}$$

in order to keep efficiency high, the computational load of the proposed algorithm is much lower than that one of [12]. The dimension of the matrix involved, as well as the fact of working with the noise subspace singular vectors, has a second consequence: the algorithm in [12] is more sensitive to noise than the one proposed here.

The advantage of the method [12] in front of the one proposed here relies in the fact that the estimate provided by the former one doesn’t need to be the result of a linear equalization of the received data, whereas the one proposed here does. This means that, in principle, better results can be obtained in ill-conditioned channels where the linear equalization can have noise enhancement problems (even though in the SIMO case they are not as bad as in the single channel case ([11]) ).

6 Simulations

Fig.2 and 3 illustrate the performance of the algorithm proposed in this paper. Both plots display the percentage of realizations (500 and 1000 were averaged) for which the equalizer output EbNo was higher than the value indicated in the x-axis. In all cases the transmitted data consisted of 128 QPSK symbols. Notice that the output EbNo depends on each run due to the algorithm sensitivity to the channel, data and noise realizations caused by the limited amount of data available for estimation.

Figure 2 shows the performance of the algorithm in its application to an OFDM transmission in a frequency-flat fading channel corresponding to a 25Kb/s transmission at 1GHz with a mobile moving at 100Km/h. In that case two antennas were used ($B = 2$) and EbNo=20dB. This figure
shows the improvement obtained when the equalizer design criteria in (7) (I) is replaced by that one of equation (10) (II). Notice the algorithm performs correctly even though the multiplicative Rayleigh channel is a very difficult environment for the blind algorithm, for the multiplicative channel does not fulfill perfectly the finite length channel hypothesis.

Figure 3 shows the performance obtained when the algorithm is applied to a TDMA block transmission in a frequency selective channel. In this case, four antennas were simulated ($B = 4$) and channel responses:

$$C_1(z) = (1+j)z^{-1} + 0.4z^{-1} + 0.5z^{-4}$$
$$C_2(z) = 0.1 + z^{-1} - 0.4z^{-1} + 0.2z^{-1} - 0.5z^{-4}$$
$$C_3(z) = 0.1 + 2z^{-1} - 0.4z^{-1} + 0.2z^{-1} + z^{-4}$$
$$C_4(z) = (1+0.8j)z^{-1} - 0.4z^{-1} + 0.2z^{-1} + (1-0.5)z^{-4}$$

This figure illustrates the improvement obtained by increasing the equalizer length. In this case, $EbNo = 15dB$ and the equalizers of length 2 (I) and 4 (II) were designed using equation (10).

References