A STATISTICAL ESTIMATOR OF TIME
DELAY AND DOPPLER SHIFT FROM
MULTI-SAMPLED RANDOM SIGNALS

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RÉSUMÉ

L’estimation du retard et du coefficient
doppler est un sujet important en plusieurs
applications du traitement du signal. L’effet
du doppler pour les signaux à bande large
peut être regardé comme une compression
temporelle. Après une esime primaire basée
sur les signaux discrets en temps et doppler,
nous considérons ici une approximation
parabolique pour l’estimation fine. Les
résultats numériques ont été référe à signaux
aléatoires gaussiens perturbés par bruits
gaussiens indépendants. La performance
obtenue montre les capacités potentielles de
cette méthode.

ABSTRACT

Time delay and doppler shift estimation
is an important issue in many signal
processing areas [1-2]. These include the
direction of arrival and trajectory in
underwater acoustics, sonar and radar range
and speed estimation in a multisensor
environment, inter-satellite communications,
timing acquisition in a spread spectrum
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compensation in moving images, stereo
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instantaneous time scale compression or
expansion (or, for duality, an expansion or
compression of the frequency scale).

Minimization of the ambiguity
function based on generalized cross-
correlation allows to find ML-optimal
estimates of the unknown parameters [1].
Efficient estimation methods are based on a
two-steps algorithm [4]: a coarse estimate is
obtained from some unambiguous smoothed
function; a subsequent fine estimate works on
a wide-band ambiguity function starting from
the coarse estimate. This allows to estimate the
absolute minimum of the ambiguity function
avoiding the wrong convergence on a
relative one. In a recent paper [5], a parabolic
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sample) estimation of the time delay from
sampled signals. The method is herein
extended to doppler estimation.

1. INTRODUCTION

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2. CHOICE OF A SUITABLE MODEL

One preliminary question arises, namely whether it is possible to separate the estimation procedures of time delay and doppler shift. The answer provided by the theory is that one may use two separate estimators if the relative estimation errors are not correlated. In fact, this fact actually depends on the particular model chosen to represent a dopplered signal. Starting from a reference signal $s(t)$ defined for sake of simplicity in the time interval $[-W/2, W/2]$, we have assumed for the scaled, delayed and dopplered received signal $r(t)$ a model [6] for which the above condition applies, i.e.:

$$r(t) = a \cdot s \left( \frac{t-D}{1+F} \right)$$  \hspace{1cm} (1)

where $D$ and $F$ are the actual time delay and the doppler velocity, respectively.

Moreover, the time delay error is usually less relevant than the doppler error. In practice [2], 30-60 Nyquist samples are enough for a good estimation performance of time delay, while at least 2000 samples need to be employed to achieve a similar performance for the doppler shift. As a consequence, working on the largest window, we will focus on the estimation of the doppler coefficient.

A very simple and efficient way to obtain dopplered versions of a received signal is to sample it with different rates. For our purposes, only three measurements of the dopplered signal are needed, other than the reference one. The only requirement is that the actual doppler belongs to an interval determined by the lowest and the highest employed sampling rates. This can be implemented by driving the reference rate with some a priori value of the doppler coefficient (for example, obtained by a past measurement), while the other two rates depends on a given maximum of acceleration of the moving object. If no a priori information is available, an alternative scheme uses a parallel grid of samplers, tuned at different speeds. This is equivalent to sample the ambiguity function in the doppler domain.

In practice, after defining $A(d,f)$ as the ambiguity function of time delay ($d$) and doppler shift ($f$), we must estimate it on a proper discretized grid in the $(d,f)$ space. The coarse estimate is implemented by searching for the minimum value of $A(d_i,f_j)$, say $A(d_1,f_1)$. In order to find a fine (sub-sample) estimate of $(D,F)$, we interpolate $A(d,f)$ around $A(d_1,f_1)$ by a two-dimensional Taylor expansion after retaining the only terms up to the second order. Since the estimation errors are not correlated for the assumed model, such interpolation reduces to two separable one-dimensional ones. In other words, two distinct parabolic interpolations, based on other four measurements $A(d_i,f_j)$ placed in a cross around $A(d_1,f_1)$, need for estimating time delay and doppler shift:

$$d = d_1 - \frac{\Delta_d}{2}$$

$$f = f_1 - \frac{\Delta_f}{2}$$

where $\Delta_d$ and $\Delta_f$ are the difference between two subsequent values of the quantized parameters $d_i$ and $f_j$. 
3. DISCUSSION OF NUMERICAL RESULTS

The numerical results have been obtained for random Gaussian signals with a Gaussian-shaped auto-correlation function, i.e.:

\[ R_{ss}(\tau) = e^{-\frac{\tau^2}{2a^2}} \] (4)

corrupted by uncorrelated Gaussian white noises with several Signal-to-Noise Ratios (SNRs).

We assume to know the reference signal \( x(t) = s(t) + n_1(t) \) and a given number of delayed and dopplered versions of the received signal \( y(t;d_1,f_j) = r[(1+f_j)t+d_1]+n_2(t) \).

The doppler compensation can be implemented at low cost by sampling the received signal at several rates \((1+f_j)T\). In practice, while we know the maximum value of acceleration of the object, we can state the number of the rates to be considered. On the other hand, we can use only three rates by choosing the current speed estimate and the two limit future ones.

Different discrete-time correlators have been used for estimating the sampled ambiguity function \( A(d_1,f_j) \). For our purposes, three discrete-time cost functions have been used. The first one seeks to maximize the direct cross-correlation:

\[ D_{xy}(d_1,f_j) = \frac{1}{N} \sum_{k=1}^{N} x(kT) y(kT;d_1,f_j) \] (5)

while the ASDF method [5] minimizes:

\[ S_{xy}(d_1,f_j) = \frac{1}{N} \sum_{k=1}^{N} [x(kT) - y(kT;d_1,f_j)]^2 \] (6)

as far as the digitally faster AMDF method [5]:

\[ M_{xy}^{1/2}(d_1,f_j) = \frac{1}{N} \sum_{k=1}^{N} |x(kT) - y(kT;d_1,f_j)| \] (7)

In practical applications, the available a priori information on timing and speed may be very different. In fact, in the presence of a good prediction, we can make the estimation by multi-sampling the received signal with a central value of the timing and doppler coefficient \((d_1,f_j)\) very close to their actual values.

Conversely, if the movement of the object is unpredictable, we do not know where the sampling grid should be placed. The achievable performance is then a periodical function of the distance from the ideal sampling rate of the closest parameter values in the actual sampling grid.

As a consequence, both the two limit situations of perfect and wrong sampling have been analysed, namely the ideal case of perfect sampling and an intermediate situation of wrong sampling.

The practical example considered in our analysis refers (just like in [21]) to an underwater object moving with a radial speed of 7 knots (corresponding to a time scaling factor on the order of \( F=2.4 \times 10^{-3} \)) and uses an observation time of 0.1 sec to obtain the timing and doppler estimates. The central sampling time is \( T=5 \times 10^{-5} \) sec, while the autocorrelation standard deviation in eq. (4) has been assumed \( a=T \). The sampling grid resolution is one sampling period for the timing and 2 knots for the speed.

In particular, since the bias is strongly dependent on the autocorrelation assumed for the signals, the variances of the speed estimator (3) using the three discrete-time correlators (5)-(7) has been evaluated by 1000
independent runs of computer simulations and is reported in the figures versus the SNR in the range [0,40] dB. They refer to the case of perfect sampling (Fig. 1) and an intermediate wrong sampling (viz: 0.25 T of delay and 0.5 knots of speed) for the actual parameter values (Fig. 2).

The achieved performance shows the potential capability of such an open-loop algorithm and suggests useful guidelines for further investigations.

References