# NONLINEAR COMPENSATION OF MAGNETIC RECORDING CHANNELS

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## RÉSUMÉ

On considère un modèle non-linéaire, fondé sur les séries de Volterra, d'un système d'enregistrement magnétique. On montre qu'un annuleur non-linéaire d'interférence performe beaucoup mieux qu'un égaliseur linéaire.

## ABSTRACT

A nonlinear Volterra model of a magnetic saturation recording channel is considered, and different readback structures are compared. It is found that nonlinear cancellation with decision feedback performs significantly better than linear equalization both in terms of mean-square error and of error probability.

## 1 Introduction

The digital saturation magnetic recording channel has been widely studied using a linear model (see, e.g., [1, 2]). Various detection methods based on this model have been proposed to compensate for the severe intersymbol interference (ISI) and the presence of noise.

Coding of the information before recording is often used. This can be interpreted as a form of write equalization, which reduces ISI and facilitates the detection independently of noise at the cost of reducing the information recording density [5, 6, 7]. Another solution is readback equalization, aimed at reducing the effects of ISI in the presence of noise (see, e.g., [1] and the references therein). The most common detection structures are inherently based on the linear model of the channel and cease to be optimal when the nonlinear effects – due to high densities and high-speed recording – appear.

Modeling of mildly nonlinear systems using Volterra series is by now a widely used tool [11]. A third-order discrete Volterra series model for a digital magnetic recording saturation channel was recently proposed by Hermann [4], who also provided a method for Volterra kernel identification. The results in [4] show a very good matching between the real channel and its model, especially with high recording density. Hirt [3] has applied to the nonlinear magnetic recording channel the optimal linear receiving filter proposed in [9].

A wide variety of detection methods are known for nonlinear transmission systems modeled by Volterra series. Among them, [10] generalizes the concept of decision feedback equalization, long known to provide a good compromise between benefits and complexity [8], to a nonlinear voiceband data channel. This concept, called "nonlinear cancellation," is applied here to a high-density, highly nonlinear magnetic recording channel. Simulation results show a very good improvement over the simple linear equalizer both in terms of mean square error and symbol error probability. An interesting feature of this nonlinear cancellation method is the possibility of simply adding the canceller device to the existing detection system.

## 2 The magnetic recording channel

We assume here that the magnetic recording channel $\mathcal{H}$ has finite memory length $L = D + D'$, so that we are allowed to write

$$y_k = \mathcal{H}(x_{k-D}, \ldots, x_k, \ldots, x_{k+D'}) \qquad (1)$$

where the symbols $x_k$ take on values $\pm 1$ and $y_k$ are the samples observed at the output of the channel.

Based on the results in [4] we approximate the nonlinear function $\mathcal{H}$ by the third-order Volterra series

$$\begin{aligned} y_{k+D} &= \sum_{n=0}^{M} h_n x_{k-n} + \sum_{n=0}^{M} c_n^{(1)} x_{k-n} x_{k-n-1} \qquad (2) \\ &+ \sum_{n=0}^{M} c_n^{(2)} x_{k-n} x_{k-n-2} + \sum_{n=0}^{M} c_n^{(3)} x_{k-n} x_{k-n-3} \end{aligned}$$

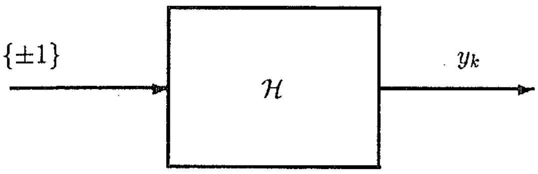

Figure 1: Discrete model of the magnetic recording channel.

| $n$ | $h_n$ | $c_n^{(1)}$ | $c_n^{(2)}$ | $c_n^{(3)}$ | $c_n^{(1,2)}$ |
|---|---|---|---|---|---|
| 0 | 0.05 | −0.005 | 0.005 | 0.001 | −0.001 |
| 1 | 0.10 | −0.010 | 0.007 | −0.002 | −0.002 |
| 2 | 0.20 | −0.003 | 0.005 | 0.010 | −0.005 |
| 3 | 0.60 | −0.145 | 0.070 | 0.050 | −0.125 |
| 4 | 1.00 | −0.100 | 0.120 | 0.037 | −0.080 |
| 5 | −0.60 | 0.100 | −0.020 | 0.001 | 0.120 |
| 6 | −0.80 | 0.040 | −0.010 | 0.000 | 0.070 |
| 7 | −0.40 | 0.020 | −0.005 | 0.004 | 0.030 |
| 8 | −0.20 | 0.010 | 0.000 | 0.000 | 0.020 |
| 9 | −0.10 | 0.005 | 0.000 | 0.000 | 0.000 |
| 10 | −0.05 | 0.001 | 0.000 | 0.000 | 0.000 |

Table 1: Volterra kernels for 56 kfci [4].

$$+ \sum_{n=0}^{M} c_n^{(1,2)} x_{k-n} x_{k-n-1} x_{k-n-2}$$

where $M = L - 3$ and $h_n, c_n^{(1)}, c_n^{(2)}, c_n^{(3)}, c_n^{(1,2)}$, for $n = 0, 1 \ldots M$, are the linear, second order, and third order Volterra kernels, respectively. These have been identified in [4] for different recording densities. We have taken these results for the highest density of 56 kfci (kilo flux changes per inch) as a model for the nonlinear channel (Table 1) and we have then applied our equalization technique.

# 3 The detection system

The detection scheme we propose is shown in Fig. 2, and will now be described in some detail. The input symbols $x_k \in \{\pm 1\}$ represent the positive and negative saturation currents which magnetize the recording medium at a normalized rate $T^{-1} = 1$. Since we are interested in "raw" channel performance, we assume that no coding is used so that a binary information source is directly mapped onto the bipolar input according to the following rule: $0 \rightarrow -1$ and $1 \rightarrow +1$.

The channel introduces a fixed delay $D$. From Table 1, column $h_n$, we see that this is equal to four input symbols. The channel output is sampled at the rate $T^{-1}$ symbols per second, and white Gaussian noise samples $n_k$ are added. The samples $w_k = y_k + n_k$ are read back by the detection system. The equalizer is a linear tapped-delay line with $N + 1$ taps ($N$ even). The values of its tap weights were calculated so as to minimize the mean square error

$$E\{|q_k - x_{k-D-\Delta}|^2\}$$

between the input and the output sequence. Here $q_k$ denotes the equalizer output, and $\Delta$ is the delay introduced by the equalizer, which was found to take on the optimal value $N/2$. Since the symbols $w_k$ follow two different routes, these must introduce the same delay $P = \Delta + D$ caused by the equalizer and the canceller, so that a $P$-symbol delay must be introduced.

The 'Sgn' blocks perform a zero-threshold decision on their inputs and produce bipolar outputs, which are the preliminary and final decisions on the corresponding input symbols, denoted $x_{k-P}^{(p)}$ and $x_{k-P-D}^{(f)}$ respectively.

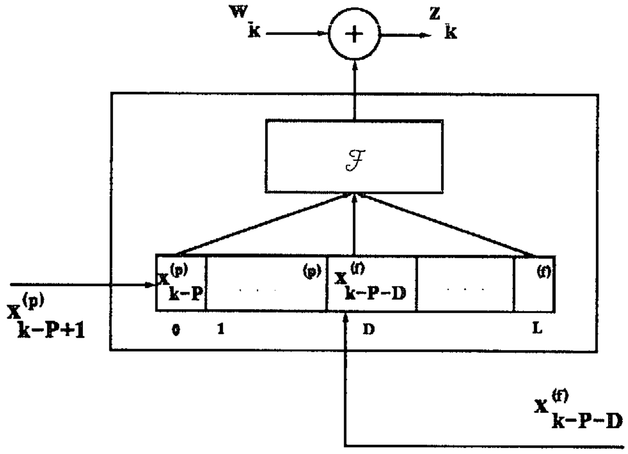

Figure 3: The canceller.

From Fig. 3 we can take a closer look at the structure of the canceller. Its aim is to subtract the ISI terms from the output of the delay line before a final decision on the transmitted symbol is taken.

Eq. (1) can be rewritten in the following form:

$$y_k = h_D x_k + \mathcal{F}(x_{k-D}, \ldots, x_k, \ldots, x_{k+D'}) \tag{3}$$

where

$$\begin{aligned} \mathcal{F}(x_0, \cdots, x_L) &= \sum_{n=0, n \neq D}^{M} h_n x_n + \sum_{n=0}^{M-1} c_n^{(1)} x_n x_{n+1} \qquad (4) \\ &+ \sum_{n=0}^{M-2} c_n^{(2)} x_n x_{n+2} + \sum_{n=0}^{M-3} c_n^{(3)} x_n x_{n+3} \\ &+ \sum_{n=0}^{M-2} c_n^{(1,2)} x_n x_{n+1} x_{n+2} \end{aligned}$$

where, as before, $L$ denotes the channel memory.

In principle, if the canceller were fed with correct decisions on the input symbols, all the ISI terms would be subtracted, and the only source of performance degradation would be additive Gaussian noise. However, if some of the decisions in the canceller are incorrect, then ISI is removed only in part, and some spurious terms with opposite sign appear. As we will see in the next section, the effectiveness of the

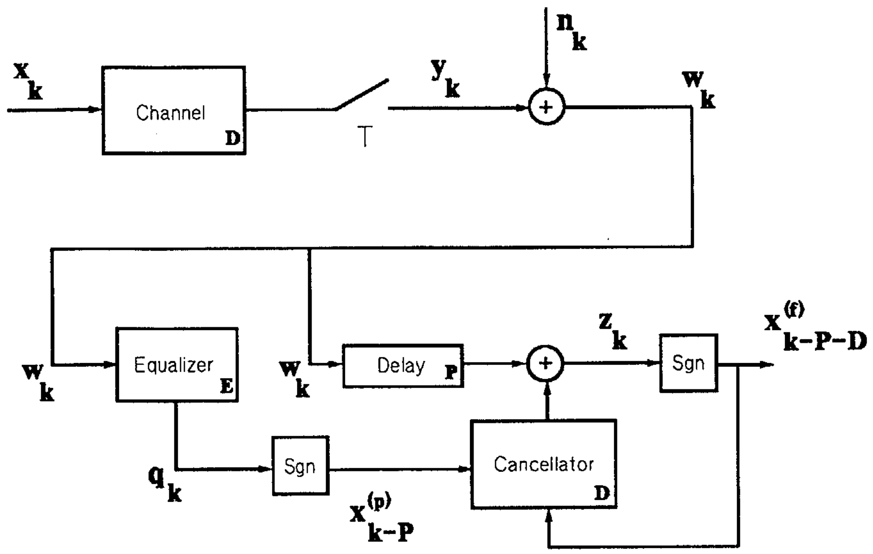

Figure 2: The detection system.

this technique largely depends on the reliability of the preliminary decisions used to feed the canceller. For this reason the canceller will be fed, where possible, by the final decisions, which are more reliable than the preliminary ones. This is the concept of "decision feedback". The canceller actually works as follows: (i) the final decision $x^{(f)}_{k-P-D}$ is fed back at location $D$ of the shift register, and overwrites the old preliminary decision, (ii) the new preliminary decision $x^{(p)}_{k-P+1}$ is shifted into location 0, (iii) the function $\mathcal{F}$ is applied to the content of the register and (iv) the result is subtracted from the delayed readback symbol $w_k$.

## 4 Simulation results

In this section we will discuss the results of the computer simulations that were run based on the above model. The goal is to evaluate the performance of the system in terms of mean-square error (MSE) and error probability $P(e)$ for different values of SNR.[1]

In general, the mean square errors $E\{|q_k - x_{k-P}|^2\}$ and $E\{|z_k - x_{k-P-D}|^2\}$ are calculated more easily than error probabilities: however, they do not provide a true performance measure since the sporadic symbol errors influence very modestly their final value. Nevertheless, MSE gives a first guess of the possible gains in terms of $P(e)$, which should be the ultimate criterion.

Figs. 4 and 5 show the limitations of linear equalization as the number of taps increases. It turns out that increasing the length of the equalizer will produce diminishing gains above a certain limit — this is due in part to the nonlinear behavior of the channel. This limit can be overcome by introducing nonlinear cancellation, which is what we advocate in this paper.

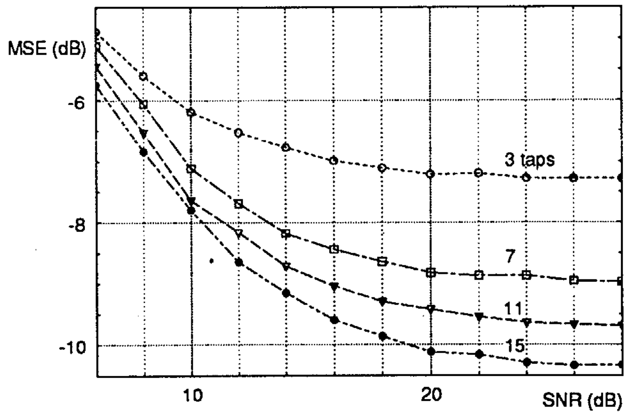

Figure 4: MSE at the output of a linear equalizer with 3,7,11, and 15 taps.

Figs. 6 and 7 show the performance improvement obtained by a nonlinear canceller with respect to 9-tap linear equalization. The linear equalizer was used to generate the preliminary decisions. It is interesting to note that the two curves cross over at a certain point. This is due to the negative effects of error propagation for low SNRs. To evaluate more carefully where this cross-over takes place, it is instructive to examine error probability curves. They show that cross-over takes place only at relatively large values of $P(e)$, and consequently in normal conditions the introduction of a canceller actually improves performance.

## 5 Conclusions

Preliminary simulations, based on the Volterra model introduced in [4], have shown that the introduction of a nonlinear ISI canceller in a magnetic recording channel can substantially improve its performance with respect to simple linear equalization. The complexity of the canceller increases with the memory length of the channel $L$, but highly parallel structures could make this detection method a practical

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[1] Since the sampling rate and the signal power are normalized to 1, the SNR coincides with the inverse of the variance of the Gaussian random variable $n_k$.

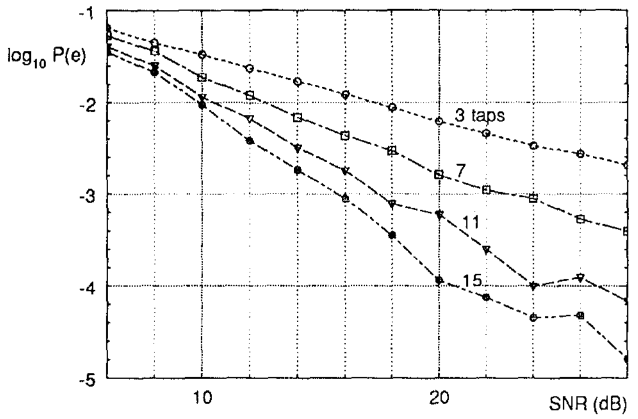

Figure 5: $P(e)$ at the output of a linear equalizer with 3,7,11, and 15 taps.

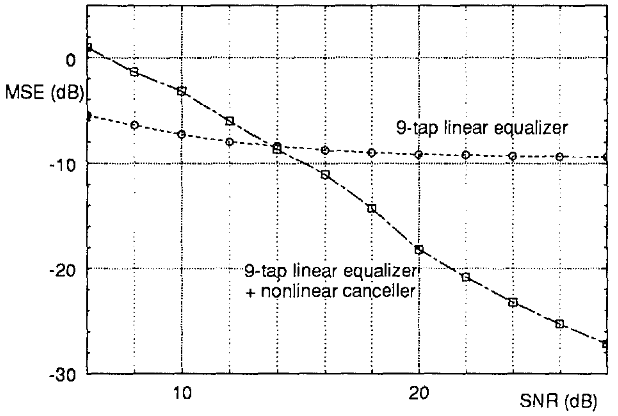

Figure 6: MSE before preliminary and final decisions with a 9-tap linear equalizer.

solution to the problems encountered when increasing the recording density.

It is important to observe that the nonlinear canceller can be added to an existing detection scheme to improve its performance. This feature is a substantial advantage, since it may be used for upgrading existing recording systems, after identifying a nonlinear channel model for a desired recording density.

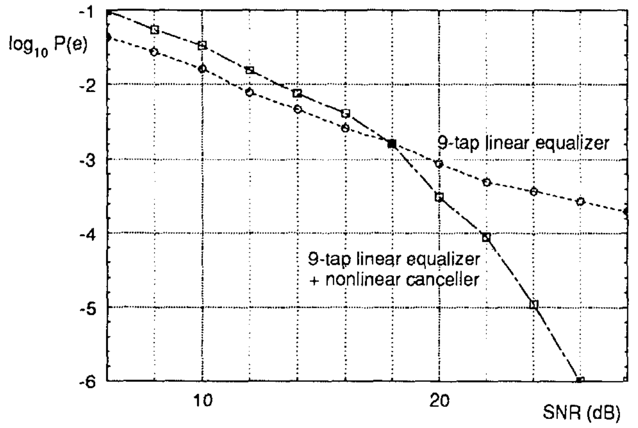

Figure 7: $P(e)$ after preliminary and final decisions with a 9-tap linear equalizer.

Various other structures could be considered, with the aim of increasing the reliability of the decisions to be fed to the canceller. However, their consideration goes beyond the scope of this work and will be examined in further work.

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