CONSTRAINT BASED IMAGE RECONSTRUCTION
IN DIFRACTIVE TOMOGRAPHY

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Résumé
Cet article traite le problème de la restauration d'un objet dégradé par un système passe-bande ideal. Nous examinons ici le cas dans lequel l'image a été obtenue par l'utilisation d'une particulière technique de tomographie cohérente basée sur le traitement du champ rétrodiffusé.

Nous étudions l'application de ce problème d'une technique de restauration itérative basée sur l'obligation que l'objet soit positif.

En particulier nous considérons le cas dans lequel l'objet est formé par un ou deux points irradiateurs.

Summary
This paper concerns the problem of restoring a two-dimensional object that has been distorted by an ideal band-pass imaging system. The problem is here focused to the case in which the image is obtained by using a particular tomographic technique that consists in irradiating the body to be explored by a C.W. source and processing the backscattered field. The effectiveness of constrained iterative restoration algorithms using positivity constraint is investigated with reference to the case in which the object is constituted by one and two pointform scatterers.

1. Introduction

Recently a particular tomographic technique [1] has been proposed for the reconstruction of two-dimensional images of sections of a body B using coherent radiation (ultrasound or microwaves). This technique consists in irradiating the body B to be explored (see Fig. 1) by a C.W. source with different wavelengths $\lambda_i$ and suitably processing the backscattered field received from different angular positions $\theta_j$.

It can be proved that the received field, for each angular position $\theta_j$ and each wavelength $\lambda_i$ is the Fourier Transform of the scattering strength density function $g(x,y)$ inside B evaluated at a point $P$ of polar coordinates $(2\pi/\lambda_i, \theta_j)$.

Supposing to perform the measurements for a suitable number of $\theta_j$ and $\lambda_i$, the straightforward way to reconstruct the image is to compute the Inverse Fourier Transform of the partially known spectrum of $g(x,y)$.

In [1,2] it has been shown that this imaging technique provides resolution performance exceeding that predicted by the Rayleigh criterion; however, as a limit of the technique, the Point Spread Function of the system shows very high sidelobes that prevent the detection, in the image, of objects with small magnitude near objects with large magnitude.

In [3,4] the performances of this technique were examined in the case that several complete observations $(0 \leq \theta_j \leq 2\pi)$ with different wavelengths were accomplished; the performed computer simulations showed that the resolution of the reconstruction method increases as the bandwidth and the mid-band frequency values used in the exploration increase; in particular in order to achieve satisfactory resolution performance very large exploration bandwidth is required.

Fig. 1 - Diagram of the tomographic technique examined.

From an experimental point of view the use of large exploring bandwidth is difficult and cumbersome as it implies wide band instrumentation and expensive data acquisition systems.

The aim of this paper is to study if, by means of suitable restoration algorithms, it is possible to obtain satisfactory results for the reconstructed image also in the case where a narrow exploration bandwidth is used.
Among the several proposed restoration methods, we chose a particular class of iterative algorithms that, using some priori information about the object, try to extend the known spectrum.

Reference was made to the case where the explored object was constituted by a single pointform scatterer or by two equal pointform scatterers at distance 2D from each other inside a homogeneous medium matched to the exploring radiation. Performance parameters were defined and their improvement due to the restoration algorithm was investigated.

2. The imaging method

With reference to Fig. 1 the backscattered field received at the angular position for a particular wavelength \( \lambda \) is

\[
G(\theta) = \int_{-\infty}^{\infty} g(x,y) e^{-4\pi j/A \{x \sin \theta + y \cos \theta\}} \, dx \, dy
\]

(2.1)

where \( g(x,y) \) = scattering strength density function (object).

By defining

\[
f_x = 2 \sin \theta / \lambda \quad f_y = -2 \cos \theta / \lambda
\]

eq (2.1) becomes

\[
G(f_x,f_y) = \int_{-\infty}^{\infty} g(x,y) e^{2\pi j \{f_x x + f_y y\}} \, dx \, dy
\]

(2.2)

i.e. the received signal from direction \( \theta \) may be seen as the two-dimensional Fourier Transform of the scattering strength density function \( g(x,y) \) evaluated at a point \( P \) of polar coordinates \( (2\lambda/\theta, \theta) \).

In the case of a continuous exploration using a radiation with frequency values inside \( \Delta F = f_2 - f_1 \), the \( G(f_x,f_y) \) on an annular region of inner and outer radii \( R_1 = 2f_1 / c \) and \( R_2 = 2f_2 / c \) is obtained.

In this case the PSF of the considered imaging system may be written in the form

\[
h(x,y) = R_2 J_1(2\pi R_2 \sqrt{x^2 + y^2}) / \sqrt{x^2 + y^2} - R_1 J_1(2\pi R_1 \sqrt{x^2 + y^2}) / \sqrt{x^2 + y^2}
\]

(2.3)

where \( J_1 \) is the Bessel function of first kind, order one. Expression (2.2) becomes the Dirac impulse when \( R_1 = 0 \) and \( R_2 = \infty \). A qualitative shape of a cross section of the PSF is shown in Fig. 2. In [3] a set of parameters is chosen to quantify the performance of the imaging system with reference to its PSF. These parameters are

\[
\rho = A_0/A_1 = \text{main lobe amplitude/first sidelobe amplitude}
\]

\[
\omega = \text{main lobe width}
\]

Moreover a global "quality factor" \( \rho/\omega \) was introduced to quantify the performance of the PSF. It was proved that for any assigned \( f_0 = (f_1 + f_2) / 2 \) the ratio \( \rho/\omega \) increases as \( \Delta f = f_2 - f_1 \) increases.

In [3, 4] the case of two pointform scatterers at distance 2D was also considered, in order to investigate the resolution performance of the imaging method.

It was proved that the resolution increases as \( f_m \) increases and scarcely depends on the increase of \( \Delta f \).
paper reference will be made to noise free simulated data [6].

The implemented algorithm is given by:

$$F\{\hat{G}_s(x,y)\} = 1 - \gamma H(x,y)$$

(3-2)

where $F$ indicates the Fourier Transform operator, C is the operator of the imposed constraints such that $g = C [g]$, $\gamma$ is a real number chosen on the basis of convergence rate considerations. If the constraint operator C is the support limiting operator and $\gamma = 1$ the proposed algorithm coincides with that first proposed by Gerchberg [5] for low-pass filtered images and by Mensa [2] for band-pass filtered images.

4. Simulation results

In order to investigate the performance of the algorithm (3-2) in restoring images two experimental cases were investigated: in the first one the object was supposed to be a single point form scatterer inside a homogeneous medium; in the second the object was supposed to be constituted by two equal point form scatterers at distance $2D$ from each other. In both cases the Fourier Transforms were sampled using a frequency interval $\Delta f = 1/(\Delta s,N)$ with $\Delta s$ spatial sampling step equal to $10^{-3}m$ and $N$ number of samples equal to 512; $2D$ was chosen equal to 2 cm.

The iterative algorithm (3-2) was implemented taking into account the positivity and finite extent of the object; with regard to the finite support constraint it was assumed an overestimated circular object bound with radius equal to $N\Delta s/2$; $\gamma$ value was assumed equal to 1.

With reference to microwave tomography several experimental simulations were accomplished for different values of $f_1$ and $f_2$ in the range 1-10 GHz.

a. single scatterer case

In Tab. 1 some results are summarized with reference to the "quality factor" $r = F/\omega$. This ratio was evaluated both for the image $\hat{G}_{sp}(x,y)$ and the restored image $\hat{G}_{150}(x,y)$ and called respectively $r_0$ and $r_{150}$. Some general observations can be made:

1. in all the examined cases a significant improvement is obtained for $r_{150}$ with respect to $r_0$;
2. The use of restoration method with 150 iterations is equivalent to use an experimental exploration bandwidth about four times wider.
3. Looking at the shape of the reconstructed images before and after the restoration process, it is always possible to appreciate the noticeable reduction of the spurious sidelobes produced by the restoration algorithm (see Fig. 3).

b. two scatterers case

Also for this case several experimental conditions in the range 1-10 GHz were simulated. On the basis of the obtained results the following considerations can be drawn:

1. In all cases a noticeable improvement in resolution and spurious side lobes amplitude reduction was achieved (see fig. 4);

2. The comparison between the shape of restored and unrestored images evidenced again that the use of the restoration algorithm is equivalent to the use of a wider experimental exploration bandwidth; however it is to note that to obtain now the performance associated to an equivalent band four times wider a number of iterations larger than that relative to the single scatterer case is required (about 600).

5. Conclusions

The problem of restoring two-dimensional objects distorted by ideal band-pass imaging systems has been investigated with reference to a backscattering based microwave tomographic technique.

The use of constrained iterative algorithms has been considered and the cases in which the object is constituted by one or two point form scatterers have been analyzed.

The results obtained, although relative to noiseless simulated data, evidence the good performance of the iterative algorithm when the positivity and limited extent constraints are imposed. In particular, the analysis showed that the use of the restoration algorithm is equivalent to broaden the bandwidth of the imaging system.

This result seems to be very interesting because the experimental implementation of the considered tomographic technique is possible without expensive and cumbersome procedure only in the case of maximum relative bandwidth values of about 30-40%, for which the performance of the imaging system are normally too poor.
Fig. 3 - Sections of reconstructed images for a single scatterer where $f_m = 3.6 \text{GHz}$ and $f = 0.9 \text{GHz}$

a : before the restoration
b : after the restoration

Fig. 4 - Sections of reconstructed images for two scatterers with $f_m = 3.6 \text{GHz}$ and $f = 1.44 \text{Hz}$

a : before the restoration
b : after the restoration

References


