OPTIMUM NONCOHERENT SPACE-DIVERSITY DETECTION
OF WEAK SIGNALS IN NON-GAUSSIAN NOISE

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RESUME

Le problème ici étudié rentre dans le cadre de la
synthèse des algorithmes asymptotiquement optimaux
de détection des signaux noyés dans des bruits non
gaussiens. Plus précisément on utilise la diversité
spatiale pour la détection non cohérente d'un signal
faible à bande étroite affecté de fluctuations d'am-
plitude ("fading") lentes.

La performance asymptotique (i.e. dans l'hypothèse
d'un nombre élevé d'échantillons dans l'intervalle
d'observation) de la structure envisagée est évaluée
et comparée avec celle d'un détecteur sous-optimal
récemment proposé par les auteurs dans le but de sem-
plifier l'implémentation.

SUMMARY

The theory of asymptotically optimum detection in
non-Gaussian noise is considered to synthesize a spa-
ce-diversity structure for the noncoherent detection
of a bandpass signal subject to slow amplitude fluc-
tuations.

The large-sample performance of the proposed de-
tection structure is evaluated and numerical results
are presented and discussed. Moreover, a comparison
between the asymptotically optimum detector and a sub-
optimum structure, operating according to a simple
scheme recently proposed by the authors, is made.

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1. INTRODUCTION

The Gaussian noise assumption has been largely adopted to synthesize and analyze optimum receiver structures for both coherent and non-coherent reception. Such an assumption, although greatly simplifies the implementation and the analysis of the receiver structures, is not always justified because there is a variety of natural and man-made electromagnetic sources that exhibit highly non-Gaussian characteristics [1-3]. Such non-Gaussian background noise can drastically degrade [4-6] the performance of the conventional systems (i.e., those optimized with respect to the Gaussian noise).

The weak signal assumption allows one to simplify the problem of the reception in non-Gaussian noise environments. Such an assumption, on the other hand, is reasonable because usually the detection algorithms must assure near-optimum behaviour just when it is necessary, i.e., when the signal is weak compared to the arbitrary noise added by the channel.

The locally optimum detectors (LOD's), which approach optimum performance as the signal becomes vanishingly small (0), maximize the slope of the power function at the zero signal level while keeping a fixed false-alarm probability.

The design of the LOD is based [7,8] on the leading terms of a power series expansion for the generalized average likelihood ratio (GLR) about zero signal level.

The LOD structures have been analysed extensively [9,10] in the case of lowpass and bandpass signals. Moreover, recently, the authors have carried out [11], with reference to the coherent reception, the synthesis and the performance analysis of a LOD that, making use of the space-diversity technique, effectively counteracts short-term fading effects.

In weak signal situations a large sample size is generally required to achieve a satisfactory system performance. Such an assumption, however, leads to an increase in the effect of higher-order terms which are dropped in the synthesis of the LOD's. In other words, for a fixed small signal level, the LOD's become increasingly suboptimum as the sample size increases [12].

In order to assure optimum performance in situations of practical interest, it is more suitable to consider the asymptotically optimum detectors (AOD's), whose operating characteristics approach the optimality as the sample size increases and the signal amplitude vanishes.

The synthesis of the AOD's is based [13] on the asymptotic expansion of the GLR for sample sizes \( N \to \infty \) and for signal amplitude decreasing as \( 1/\sqrt{N} \).

The present paper deals with the synthesis (Section 2) of the asymptotically optimum space-diversity detector for the noncoherent detection of a bandpass signal in non-Gaussian noise.

For the proposed structure the performance analysis is carried out and numerical results and comments are presented (Section 3). Moreover, a comparison between the performances of the AOD and a suboptimum structure, easier to implement, operating according to a scheme recently proposed by the authors in [11], is made.

2. ASYMPTOTICALLY OPTIMUM DETECTOR STRUCTURE

The detection problem under consideration can be represented by the following hypothesis test:

\[
H_0: \quad \hat{R}_p = \hat{R}_{p1}^1, \quad \hat{R}_{p1}^1 = A_{p1} + \bar{B}_p^1, \quad \bar{B}_p^1 = \sum_{i=1}^{M} \sqrt{A_{p1}^2 + \sigma_p^2} \quad p = 1, 2, \ldots, N
\]

where \( \hat{R}_{p1}^1 \) and \( \hat{R}_p^1 \) denote the complex amplitudes of the received signal and the noise at the ith instant on the pth diversity branch. \( \bar{B}_p^1 \) is the ith sample of the complex envelope of the bandpass signal. The random variable \( (r.v.) A_p^1 \), which assumes nonnegative real values, takes into account the presence of a slow fading in the pth channel. The r.v. \( \bar{B}_p^1 \) is assumed uniformly distributed over a 2\( \pi \) interval (noncoherent reception) and, finally, \( \gamma \) is a positive constant proportional to the SNR (after the signal processing).

We assume that the noise samples on the pth branch are identically distributed and statistically independent of each other and of the r.v.'s \( A_{p1}^1 \) and \( \bar{B}_p^1 \). Moreover, we suppose that the fadings, the noise processes and the phases \( \theta_p^1 \) on the different branches are mutually independent. Therefore, the logarithm of the GLR can be easily related to the single GLR's in each channel:

\[
\ln \left( \hat{R}_{p1}^1, \bar{R}_p^1, \ldots, \bar{R}_N^1 \right) = \frac{1}{N} \sum_{i=1}^{N} \ln \left( \frac{1}{A_{p1}^1} \prod_{i=1}^{N} \left( \hat{R}_{p1}^1 A_{p1}^1 + \bar{B}_p^1 \gamma \sqrt{A_{p1}^2 + \sigma_p^2} \right) \right)
\]

where \( \bar{B}_p^1 = (\bar{B}_1^1, \ldots, \bar{B}_N^1) \) is the vector of the samples at the input of the pth branch; \( \hat{R}_{p1}^1 \) is the joint probability density function (pdf) of the inphase and quadrature components of each noise sample at the input of the pth branch; \( R_{p1}^1 \) and \( R_{p1}^1 \) represent the inphase and quadrature components on the output of the inphase and quadrature pth channels, respectively; \( \Re(\cdot) \) and \( \Im(\cdot) \) denote the real and imaginary part (respectively); the asterisk identifies the complex conjugate: \( E_{A_{p1}^1} \) denotes the statistical expectation with respect to \( A_{p1}^1 \) and \( \bar{B}_p^1 \).

On the assumption that the joint pdf of the noise components on each branch possesses circular symmetry, i.e.,

\[
f_{R_{p1}^1 R_{p1}^1} = f_{R_{p1}^1} (R_{p1}^1) = f_{R_{p1}^1} (\sqrt{R_{p1}^1} \bar{R}_{p1}^1) \triangleq f_{R_{p1}^1} (R_{p1}^1)
\]

an asymptotic expansion of the conditional GLR \( \ln (\hat{R}_{p1}^1, \bar{R}_p^1) \) can be derived by the approach followed in [13, Ch.3]. Therefore, from (2) it follows that

\[
\ln \left( \hat{R}_{p1}^1, \bar{R}_p^1, \ldots, \bar{R}_N^1 \right) = \frac{1}{N} \sum_{i=1}^{N} \ln \left( \exp \left( -\kappa_0^2 \bar{B}_p^1 \right) \sqrt{2\pi} \frac{1}{\sigma_p^2} \right)
\]

where:

\[
\kappa_0^2 = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} \bar{B}_p^1
\]

(4)

provides a measure of signal power, \( \sigma_p^2 \) is the common variance of the inphase and quadrature noise components; \( \kappa_0^2 (\cdot) \) is the modified Bessel function of the first kind.

1 The comparison between the conditions of applicability [12] of the asymptotically optimum and locally optimum approaches shows that, in spite of their similar construction, the AOD and LOD are directed to different classes of signals.

2 Unlike AOD structure, such a detection scheme does not require the evaluation of the mean (aver fading) signal-to-noise ratios (SNR's) at the input of the local processors.

3 The assured law of decrease for the signal amplitude assures that the SNR is finite and not zero for any value of \( N \).
and zero order; $E_A^p$ denotes the statistical expectation with respect to $A_p$ and

$$I_p = \frac{1}{N} \sum_{i=1}^{N} \text{Re}(\hat{R}_p s_i^*) g_p(R_p)$$

(6)

$$Q_p = \frac{1}{N} \sum_{i=1}^{N} \text{Im}(\hat{R}_p s_i^*) g_p(R_p)$$

(7)

$$g_p(R_p) = \frac{1}{R_p} \frac{d}{dR_p} \log P(R_p)$$

(8)

$$F_p = \int_0^{R_p} R g_p(R) f_p(R) dR$$

(9)

Equation (4) shows that the asymptotically optimum space-diversity detector consists of $K$ local processors that evaluate the statistics $I_p^2 + Q_p^2$ and of a central processor. It evaluates the logarithms of the statistical expectations (with respect to the r.v.'s $A_p$) of the quantities enclosed within square brackets, performs their summation and compares the result with a threshold chosen to satisfy the false-alarm probability requirement.

The proposed AOD structure depends on the fading statistics on the different diversity branches. In the single channel case ($K=1$), vice versa, the fading, as it results immediately from (4), does not affect the receiver structure.

If one assumes the fading r.v.'s are Rayleigh distributed, from (4) the following decision statistic results:

$$T^{AOO}_p = \sum_{p=1}^{K} \frac{\rho_p R_p}{2\rho_p R_p} (I_p^2 + Q_p^2) = \sum_{p=1}^{K} w_p T^{AOO}_p$$

(10)

where

$$w_p = \rho_p \rho_p / (2 + \rho_p \rho_p)$$

(11)

$$\rho_p = E(A_p^2) r_p^2 / P_0$$

(12)

is the mean (over fading) SNR at the input of the $p$th local processor and

$$I_p^2 = I_p / \sqrt{2P_F / \rho_p} \Delta D_{I_p}$$

(13)

and

$$Q_p^2 = Q_p / \sqrt{2P_F / \rho_p} \Delta D_{Q_p}$$

(14)

are normalized versions of $I_p$ and $Q_p$ such that the variance of $I_p^2 + Q_p^2$ is unity. In (10) $T^{AOO}_p$ is a sufficient statistic for the AOD corresponding to the single channel case.

The space-diversity detector structure (Fig.1) consists of $K$ local processors that evaluate the $T^{AOO}_p$'s (Fig.2) and of a central processor that performs weighting and compares the result of the linear combination (10) with a threshold $T$.

Equation (10) shows that the weight $w_p$ is an increasing function of the product $\rho_p \rho_p$, where $P_F$ can be readily recognized [10] to be the asymptotic relative efficiency [7,8] of the $p$th local processor (i.e., that implementing $T^{AOO}_p$) with respect to the corresponding optimum detector for Gaussian noise.

Finally, we note that, in the particular case of Gaussian noises, from (8) it follows that $g_p(\cdot) = 1/\sigma_p^2$ and, therefore, (6), (7) and (10) lead to the well-known receiver structure optimized against Gaussian noise.

Fig.1 - Asymptotically optimum space-diversity detector.

Fig.2 - Structure of the $p$th local processor.

3. PERFORMANCE ANALYSIS AND NUMERICAL RESULTS

The AOD performance analysis for large sample size $N$ is carried out by means of the characteristic function (CF) approach [14] in that the decision variable $T^{AOO}$ stated in (10) is a linear combination of statistically independent r.v.'s. Therefore, it follows that [11]

$$1 - 2 - 2\pi \int_{-\infty}^{0} C_{1A00}(\omega) \frac{e^{j\omega t}}{\mu} dt = \begin{cases} P_{FA} & t = 0 \\ P_{D} & t = 1 \end{cases}$$

(15)

where $C_{1A00}(\cdot|H_1)$ denotes the conditional CF and $P_{FA}$ and $P_D$ are the false-alarm rate and the detection probability, respectively.

Under the hypothesis $H_0$ the central limit theorem allows one to state that $I_p$ and $Q_p$ are independent zero-mean Gaussian r.v.'s with variance $P_F / \rho_p^2$.

Under the hypothesis $H_1$ and the conditioning upon $A_p$ and $\eta_p$, the r.v.'s $I_p$ and $Q_p$ are asymptotically Gaussian with the same variance $P_F / \rho_p^2$ and statistical expectations $\rho_p \cos \theta_p P_F / \rho_p^2$ and $\rho_p \sin \theta_p P_F / \rho_p^2$, respectively.

Therefore, from (15) it follows that

$$P_{FA} = \frac{1}{2} - \frac{1}{2\pi} \sum_{p=1}^{K} e^{j\mu \eta_p} \int_{-1}^{1} (1-\mu \eta_p) \mu^{-1} d\mu$$

(16)

$$P_D = \frac{1}{2} - \frac{1}{2\pi} \sum_{p=1}^{K} e^{j\mu \eta_p} \int_{-1}^{1} \frac{1}{\mu} \exp (\frac{j\mu \eta_p}{1-\mu \eta_p} P_F / \rho_p^2) d\mu$$

(17)

Equations (16) and (17) allow one to evaluate, for a fixed false-alarm probability $P_{FA}$, the detection rate $P_D$ as a function of the products $\rho_p P_F$, in correspondence of any fading law on each diversity branch.

In the following we present and discuss numerical results on the assumption that the amplitude fluctuations are Rayleigh distributed.

In Fig.3, with reference to dual diversity, $P_D$ is
Fig. 3 - Detection probability $P_D$ versus $\omega_1 F_1$ for some values of $\omega_2 F_2$ and for a fixed value of $P_{FA}$.

Fig. 4 - Detection probability $P_D$ versus $\phi F$ in correspondence of some values of the diversity order $K$ and a fixed false-alarm rate $P_{FA}$.

Fig. 5 - Contours delimiting regions of operating conditions (with reference to the case of two local processors) that satisfy the requirements: $P_D > 80\%$, $P_{FA} = 10^{-6}$, ringing to the curve for the case of absence of diversity.

The performance as a function of the diversity order can be assessed by considering $K$ local processors having the product $\omega_2 F_2$ independent of the processor considered (i.e., the same for any value of $P_D$).

In Fig. 4, $P_D$ is plotted as a function of $\phi F$ for $P_{FA} = 10^{-6}$ and with the number $K$ as a varying parameter. The curves show the transition from critical to optimum operating conditions is increasingly sharper as the diversity order increases.

Figures 5 and 6 allow one to make a comparison between the proposed AOD (Fig. 1) and a suboptimum structure operating according to the scheme proposed by the authors in [11]. The structure consists of peripheral detectors that perform local independent decisions based on the previously introduced statistics $T^AOD_K$ (see [10] and Fig. 2) and of a central processor that, working according to the OR scheme, performs the global decision based on the local ones.

With reference to dual diversity, Fig. 5 presents for both detection strategies the contours in the plane $(\omega_1 F_1, \omega_2 F_2)$ delimiting regions of operating conditions specified in terms of overall detection probability for a fixed global false-alarm rate (the straight line $\omega_1 F_1 = \omega_2 F_2$ is an axis of symmetry for both reported curves). The results show that the performance degradation of the OR scheme with respect to the AOD, in terms of power, is bounded to at most 1.5 dB and becomes more and more negligible as the SNR over one of the channels approaches $-\infty$ (in such a limiting case the corresponding local processor is not operating).

Finally, Fig. 6 presents the loss (in decibels) in terms of $\phi F$ (the same value of $P_{FA}$ is assumed for all). The choice of an OR or AOD scheme leads one to a very simple implementation of the central processor. In general it is not possible to state [11] which scheme performs better. However, the OR strategy is considered here because it was found that OR performs such better in all examined cases.
Fig. 6 - Loss (in terms of $\alpha$) of the suboptimum OR scheme with respect to the AOD structure versus number $K$ of peripheral processors.

local processors) of the OR scheme with respect to the AOD. The performance degradation does not exceed a few decibels for any realistic value of the diversity order $K$.

4. CONCLUSIONS

The theory of the asymptotically optimum detection in non-Gaussian noise environments is considered to synthesize a space-diversity structure for the noncoherent detection of a bandpass signal subject to amplitude fluctuations.

An asymptotic sufficient statistic is derived for arbitrary fading laws on the different diversity branches. It shows that in the AOD structure two processing steps can be recognized: a local processing and a central processing. In the former, which does not depend on the fading law, each observed signal (in terms of inphase and quadrature components) is first processed by a narrowband nonlinearity which depends only on the joint pdf of the noise components and, then, by an envelope correlation detector. In the latter, vice versa, the knowledge of the fading statistics on the diversity branches is required.

In the interesting case of Rayleigh fading the central processing weights the output of any local processor by emphasizing the contribution of those more reliable, performs the summation of such weighted outputs and, finally, compares the result with a threshold.

The analysis of large-sample performance of the proposed asymptotically optimum space-diversity detector is carried out. Moreover, numerical results are presented and discussed and a comparison between the performances of the AOD and of a suboptimum detector (easier to implement) recently proposed by the authors is made. It turns out that the performance degradation does not exceed a few decibels for realistic values of the diversity order.

REFERENCES
