La Detection et Estimation d'une Impulsion en Utilisant la Transformation Discrete de Fourier
The Detection and Estimation of a Single Impulse Using the Discrete Fourier Transform

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RESUME

Étant donné un vecteur $x = (x_0, x_1, \ldots, x_{N-1})$...
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Calculated values of this error probability are given in Figure 1 for the following codes: \( (N,K) = (4,2), (8,6), (16,14), (32,30) \), and \( (64,62) \). The results of a computer simulation of the \( (4,2) \) code are also shown for comparison.

The value of the impulse was estimated using the following estimator
\[
\hat{I} = \frac{1}{2} \left( W_N^\ast y[n-2] + W_N^\ast y[n-1] \right)
\]

For the case where \( \hat{m} = m \), we then find that
\[
I = I + \Re(n[m]) + j \Im(n[m]) + \zeta
\]
where both the real and imaginary parts of \( \zeta \) are Gaussian with mean 0 and variance \( \left( (N/2)-1 \right)^2 \).

Writing the components of our final estimate of the data vector as \( \hat{x}[j] \), where \( \hat{x}[j] = y[j] \) for \( j \neq \hat{m} \) and \( \hat{x}[j] = y[j] - \hat{I} \) for \( j = \hat{m} \), we find that
\[
\sum_{j=0}^{N-1} \left| \hat{x}[j] - x[j] \right|^2 \left| (\hat{m} = m) \right| = (3N-4)\sigma^2
\]

Similar results can be obtained for the case where \( \hat{m} \neq m \). We can show that \( (3N-4)\sigma^2 \) serves as a lower bound on the unconditional mean-squared error between the transmitted and decoded vectors. An upper bound to this unconditional mean-squared error is \( (3N-4)\sigma^2 + 2\max \sum P_r(m) \).

4. Discussion

One method of generating a discrete-time, continuous-amplitude sequence with characteristics similar to those described here is to sample a continuous-time band-limited waveform at a rate faster than the Nyquist rate. For finite \( N \), the \( N \) by \( N \) D.F.T. will not have components which are identically zero but the high frequency components will be small.

Alternatively, we could create a discrete-time, continuous-amplitude sequence whose D.F.T. has true zeros by inserting redundant samples in an arbitrary sequence. Starting with \( K \) arbitrary data points, one can always insert \( (N-K) \) additional data points so as to force \( (N-K) \) consecutive zeros in an \( N \) by \( N \) D.F.T.

This paper only considered the case of the detection and correction of single impulses. If \( T \leq (N-K)/2 \), one can use a variant of a B.C.H. decoding algorithm to correct \( T \) or fewer impulses. Furthermore, a "voting" strategy can be used for the case where \( T \leq (N-K-1) \). These cases are discussed further in reference [1].

References


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