La Detection et Estimation d'une Impulsion en Utilisant la Transformation Discrete de Fourier
The Detection and Estimation of a Single Impulse Using the Discrete Fourier Transform

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RESUME

Etant donné un vecteur \( x = (x[0], x[1], x[2], \ldots, x[N-1]) \), dont les éléments \( x[i] \) sont des nombres complexes et que la transformation Discrete de Fourier (D.F.T.) du vecteur \( x \) à la propriété suivante: c'est à dire que deux éléments consécutifs également zéro.

Premièrement nous montrons que si \( Y = X + I \delta \) ou

\[
\delta = (0,0,\ldots,0,1,0,\ldots,0)
\]

et \( I \) est un nombre complexe arbitraire, alors le vecteur original \( X \) ne peut être déterminé qu'à partir du vecteur observé \( Y \) (sans

SUMMARY

Given a vector \( x = (x[0], x[1], \ldots, x[N-1]) \) whose components \( x[i] \) are complex numbers. Assume that the Discrete Fourier Transform (D.F.T.) of the vector \( x \) has the property that two consecutive components are identically zero.

First we show that if \( y = x + I \delta \) where \( \delta = (0,0,\ldots,0,1,0,\ldots,0) \) and \( I \) is an arbitrary complex number, then the original vector \( x \) can be uniquely determined from the observed vector \( y \) (without error).
Calculated values of this error probability are given in Figure 1 for the following codes: (N,K) = (4,2), (8,6), (16,14), (32,30) and (64,62). The results of a computer simulation of the (4,2) code are also shown for comparison.

The value of the impulse was estimated using the following estimator

\[ \hat{1} = \frac{1}{2}(W_N^H \bar{y}(n-2) + W_N^H \bar{y}(n-1)) \]

For the case where \( n=m \), we then find that

\[ I = I + Re(n[m]) + j Im(n[m]) + \mathbb{E} \]

where both the real and imaginary parts of \( \mathbb{E} \) are Gaussian with mean 0 and variance \( \left( \frac{N}{2} - 1 \right)^2 \).

Writing the components of our final estimate of the data vector as \( x[j] \), where \( x[j] = y[j] \) for \( j \neq m \) and \( x[1] = y[1] = 1 \) for \( j = 1 \), we find that

\[ \mathbb{E} \frac{1}{N-1} \sum_{j=0}^{N-1} | x[j] - \bar{x}[j]|^2 \mid (x[m]) \mid = (3N-4)c^2. \]

Similar results can be obtained for the case where \( n=m \). We can show that \( (3N-4)c^2 \) serves as a lower bound to the unconditional mean-squared error between the transmitted and decoded vectors. An upper bound to this unconditional mean-squared error is \( (3N-4)c^2 + 21c c^2 \max_{r} r[R(m)]. \)

4. Discussion

One method of generating a discrete-time, continuous amplitude sequence with characteristics similar to those described here is to sample a continuous-time band-limited waveform at a rate faster than the Nyquist rate. For finite \( N \), the \( N \) by \( N \) D.F.T. will not have components which are identically zero but the high frequency components will be small.

Alternatively, we could create a discrete-time, continuous amplitude sequence whose D.F.T. has true zeros by inserting redundant samples in an arbitrary sequence. Starting with \( N \) arbitrary data points, one can always insert \( (N-K) \) additional data points to force \( (N-K) \) consecutive zeros in an \( N \) by \( N \) D.F.T.

This paper only considered the case of the detection and correction of single impulses. If \( T \leq \frac{(N-K)}{2} \) one can use a variant of a B.C.H. decoding algorithm to correct \( T \) or fewer impulses. Furthermore, a "voting" strategy can be used for the case where \( T < \frac{(N-K)}{2} \). These cases are discussed further in reference [1].

References


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