La Detection et Estimation d'une Impulsion en Utilisant
la Transformation Discrete de Fourier
The Detection and Estimation of a Single Impulse Using
the Discrete Fourier Transform

1. Introduction

In this paper we explore a seemingly new method of locating and removing impulses which occur in a time-discrete but continuous-amplitude time series. It is based upon the Discrete Fourier Transform (D.F.T.) and principles previously utilized in algebraic coding theory.

The basic notion used in our system is as follows. We consider a discrete-time sequence whose D.F.T. has zeros in certain positions. After transmission over an additive impulse noise channel, the D.F.T. of the received discrete-time sequence no longer has zeros in those positions. Specifically, the values of the components of the received sequence's D.F.T. in these positions are due only to the noise on the channel. Assuming that the noise on the channel is impulsive in nature, a curve fitting strategy is used to find the location and values of the impulses that caused these spectral components.

Let us assume that for \( T=1 \), the non-zero value of \( e[i] \) is \( e[m] = I \). That is, we assume we have an impulse in the \( m \)-th position and that the value of this impulse is the complex number \( I \). If we take the D.F.T. of the errored data sequence \( y[0], y[1], \ldots, y[N-1] \) to result in the \( N \) spectral components \( Y[0], Y[1], \ldots, Y[N-1] \) we find that since \( X[N-2] = X[N-1] = 0 \), then

\[
Y[N-2] = \frac{1}{\sqrt{N}} e^{m(N-2)}
\]

and

\[
Y[N-1] = \frac{1}{\sqrt{N}} e^{m(N-1)}.
\]

We can solve these equations for \( I \) and \( m \) as follows:

\[
m = -\frac{N}{2} \angle \left( \frac{Y[N-1]}{Y[N-2]} \right), \text{ modulo } N,
\]

\[
I = \left( \frac{Y[N-2]}{Y[N-1]} \right)^{N-1} / (Y[N-1])^{N-2}.
\]

Thus, we see that in the absence of other errors, a single (impulsive) error can be located and its value.
Calculated values of this error probability are given in Figure 1 for the following codes: \((N,K) = (4,2), (8,6), (16,14), (32,30)\) and \((64,62)\). The results of a computer simulation of the \((4,2)\) code are also shown for comparison.

The value of the impulse was estimated using the following estimator

\[
\hat{I} = \frac{1}{2}(W_N^* r[n-2] + W_N^* r[n-1])
\]

For the case where \(m = m\), we then find that

\[
I = I + Re(n[m]) + j Im(n[m]) + \eta
\]

where both the real and imaginary parts of \(\eta\) are Gaussian with mean 0 and variance \((N/2-1)^2\).

Writing the components of our final estimate of the data vector as \(x[j]\), where \(x[j] = y[j]\) for \(j \neq 4\) and \(x[j] = y[j] - I\) for \(j = 4\), we find that

\[
E \left[ \sum_{j=0}^{N-1} |x[j] - y[j]|^2 \right] = (3N-4)\sigma^2.
\]

Similar results can be obtained for the case where \(m = m\). We can show that \((3N-4)\sigma^2\) serves as a lower bound to the unconditional mean-squared error between the transmitted and decoded vectors. An upper bound to this unconditional mean-squared error is \((3N-4)\sigma^2 + 21 \eta^2 \max r[m\eta].

4. Discussion

One method of generating a discrete-time, continuous amplitude sequence with characteristics similar to those described here is to sample a continuous-time band-limited waveform at a rate faster than the Nyquist rate. For finite \(N\), the \(N\) by \(N\) D.F.T. will not have components which are identically zero but the high frequency components will be small.

Alternatively, we could create a discrete-time, continuous amplitude sequence whose D.F.T. has true zeros by inserting redundant samples in an arbitrary sequence. Starting with \(K\) arbitrary data points, one can always insert \((N-K)\) additional data points so as to force \((N-K)\) consecutive zeros in an \(N\) by \(N\) D.F.T.

This paper only considered the case of the detection and correction of single impulses. If \(T \leq \lceil (N-K)/2 \rceil\) one can use a variant of a B.C.H. decoding algorithm to correct \(T\) or fewer impulses. Furthermore a "voting" strategy can be used for the case where \(T \leq (N-K-1)\). These cases are discussed further in reference [1].

References


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