La Dtection et Estimation d'une Impulsion en Utilisant
la Transformation Discrete de Fourier

The Detection and Estimation of a Single Impulse Using
the Discrete Fourier Transform

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RESUME

Etant donne un vecteur \( \mathbf{x} = (x[0], x[1], x[2], \ldots, x[N-1]) \), dont les elements \( x[i] \) sont des nombres complexes et que la transformation Discrete de Fourier (D.F.T.) du vecteur \( x \) a la proprieté suivante: c'est à dire que deux elements consecutifs egalent zero.

Premièrement nous montrons que si \( Y = \mathbf{x} + \mathbf{I} \delta \) ou
\[ \delta = (0,0,\ldots,0,1,0,0) \]

et \( I \) est un nombre complexe arbitraire, alors le vecteur original \( \mathbf{x} \) ne peut être déterminé qu'à partir du vecteur observé \( Y \) (sans erreur). Puis nous examinons le cas où \( Y = \mathbf{x} + \mathbf{I} \delta + \mathbf{n} \) où \( \mathbf{n} \) est un vecteur dont les éléments sont i.i.d.* variables aleatoires Gaussiennes. Nous trouvons la règle de détermination optimale pour déterminer la localité de l'impulsion \( I \) (c'est à dire pour déterminer la valeur de \( I \)). Finalement nous déterminons le rendement de ce systéme optimal.

*i.i.d.: (indépendante équิ-répartie.)*

SUMMARY

Given a vector \( \mathbf{x} = (x[0], x[1], \ldots, x[N-1]) \) whose components \( x[i] \) are complex numbers. Assume that the Discrete Fourier Transform (D.F.T.) of the vector \( \mathbf{x} \) has the property that two consecutive components are identically zero.

First we show that if \( \mathbf{y} = \mathbf{x} + \mathbf{I} \delta \) where \( \delta = (0,0,\ldots,0,1,0,0) \) and \( I \) is an arbitrary complex number, then the original vector \( \mathbf{x} \) can be uniquely determined from the observed vector \( \mathbf{y} \) (without error). Then we investigate the case where \( \mathbf{y} = \mathbf{x} + \mathbf{I} \delta + \mathbf{n} \) where \( \mathbf{n} \) is a vector whose components are i.i.d. Gaussian random variables. We find the optimal decision rule for determining the location of the impulse \( I \) (that is, for determining the value of \( I \)). Finally we determine the performance of this optimal system.
1. Introduction

In this paper we explore a seemingly new method of locating and removing impulses which occur in a time-discrete but continuous-amplitude time series. It is based upon the Discrete Fourier Transform (D.F.T.) and principles previously utilized in algebraic coding theory.

The basic notion used in our system is as follows. We consider a discrete-time sequence whose D.F.T. has zeros in certain positions. After transmission over an additive impulse noise channel, the D.F.T. of the received discrete-time sequence no

Let us assume that for $T=1$, the non-zero value of $e[i]$ is $e[m] = I$. That is, we assume we have an impulse in the $m$-th position and that the value of this impulse is the complex number $I$. If we take the D.F.T. of the errored data sequence $y[0], y[1], \ldots, y[N-1]$ to result in the $N$ spectral components $Y[0], Y[1], \ldots, Y[N-1]$ we find that since $X[N-2] = X[N-1] = 0$, then $Y[N-2] = \frac{I}{N}$ and $Y[N-1] = \frac{I}{N}$.

We can solve these equations for $I$ and get following
Calculated values of this error probability are given in Figure 1 for the following codes: \( (N,K) = (4,2), (8,6), (16,14), (32,30) \) and \( (64,62) \). The results of a computer simulation of the \((4,2)\) code are also shown for comparison.

The value of the impulse was estimated using the following estimator:

\[
\hat{I} = \frac{1}{2}(\text{Re}(N-2) + \text{Im}(N-1))
\]

For the case where \( \hat{m}m \), we then find that

\[
I = I + \text{Re}(n[m]) + j \text{Im}(n[m]) + \hat{z}
\]

where both the real and imaginary parts of \( \hat{z} \) are Gaussian with mean 0 and variance \((N/2-1)^2\).

Writing the components of our final estimate of the data vector as \( \hat{x}[j] \), where \( \hat{x}[j] = y[j] \) for \( j \neq \hat{m} \) and \( \hat{x}[j] = y[j] - \hat{I} \) for \( j = \hat{m} \), we find that

\[
\frac{1}{N-1} \sum_{j=0}^{N-1} |x[j] - \hat{x}[j]|^2 = (3N-4)c^2.
\]

Similar results can be obtained for the case where \( \hat{m}m \). We can show that \((3N-4)c^2\) serves as a lower bound to the unconditional mean-squared error between the transmitted and decoded vectors. An upper bound to this unconditional mean-squared error is \((3N-4)c^2 + 2|H|_\infty^2 \max \mathbb{P}[\hat{m}m]\).

4. Discussion

One method of generating a discrete-time, continuous amplitude sequence with characteristics similar to those described here is to sample a continuous-time band-limited waveform at a rate faster than the Nyquist rate. For finite \( N \), the \( N \) by \( N \) D.F.T. will not have components which are identically zero but the high frequency components will be small.

Alternatively, we could create a discrete-time, continuous amplitude sequence whose D.F.T. has true zeros by inserting redundant samples in an arbitrary sequence. Starting with \( K \) arbitrary data points, one can always insert \( N-K \) additional data points so as to force \((N-K)\) consecutive zeros in an \( N \) by \( N \) D.F.T.

This paper only considered the case of the detection and correction of single impulses. If \( T \leq (N-1)/2 \), one can use a variant of a P.C.W. decoding algorithm to correct \( T \) or fewer impulses. Furthermore, a "voting" strategy can be used for the case where \( T \leq (N-1) \). These cases are discussed further in reference [1].

References


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