PULSE TRANSMISSION ON COUPLED STRIP LINES

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RESUME

Pour le traitement du signal, des lignes de transmission couplées sont très intéressantes à double égard: premièrement, comme circuit direct de transmission des impulsions, où des perturbations des impulsions par le couplage mutuel sont à éviter autant que possible, et, deuxièmement, comme transformateur ou formateur des impulsions, où la transmission des impulsions d'une ligne à l'autre par le couplage est justement désirée. Cette publication démontre que, dans la première sphère d'application, un propre alignement de la fréquence de répétition des impulsions au temps de propagation entraîne une transmission des impulsions non perturbée le long d'une ligne malgré du couplage avec l'autre ligne, quand même les impédances terminales sont choisies arbitrairement. Dans le deuxième cas, il est déduit que l'octopôle des lignes couplées rend possible la transmission des impulsions sans perturbations, et sans composante de courant continu, d'une ligne à l'autre ligne couplée, si l'on respecte des conditions propres pour la fréquence de répétition des impulsions ou pour les impédances terminales. En outre, l'octopôle des lignes couplées réalise des conceptions d'impulsion désirées quant à la fréquence de répétition et la forme d'onde d'impulsion: augmentation de la fréquence de répétition, et/ou modification de la durée d'impulsion, génération des impulsions en gradins, quasi-triangulaires et quasi-trapezoïdales. Les résultats théoriques sont vérifiés avec deux microbandes couplées le long du côté plat des lignes, présentant un temps de propagation de 21 ns, ou 5 ns, et un coefficient de couplage de 0.55.

SUMMARY

For signal processing, coupled transmission lines are of twofold interest: firstly, as direct pulse transmitting circuits, where undistorted pulse transmission along each line, in spite of mutual coupling, is desired, and, secondly, as pulse transformer and pulse shaping unit, whereby prescribed pulse transmission from one line to the coupled other line is aspired. The paper shows that, for the first application case, suitable line delay time to pulse repetition rate matching leads to undisturbed pulse transmission along each single line, even for close mutual coupling and for arbitrary terminating impedances. For the second case, it is demonstrated that the coupled line four-port enables dc-isolated and undistorted pulse transmission from one line to the coupled other one, if appropriate line delay time or terminating impedance matching conditions are hold. Further, the coupled line four-port enables desired shaping of pulses concerning repetition rate and impulse shape: increase of pulse repetition rate, and/or change of pulse duration, generation of staircase, quasi-triangular, and trapezoidal pulses. Theoretical results are verified at two broadband coupled strip line prototypes of 21 ns, and 5 ns line delay time, respectively, with a coupling factor of 0.55.
1. Introduction

The investigation of pulse transmission along mutually coupled lines (Fig. 1) includes several technical aspects. The objective, e.g., for high-speed digital systems, where commonly several lines are packed closely together, is to avoid pulse distortion along the signal lines 1-0, 2-0 (Fig. 1), caused by mutual coupling effects [1]-[3]. Earlier known investigations show [1]-[4] that this is possible by matching the terminating impedances, Z, through \( Y \). The disadvantage of this type of matching is, however, that a pulse signal is still transmitted to the near-end output-port of the adjacent line as has been shown in [4]. Further, the necessary terminating impedance values are limited to within a small range, which causes problems for many applications. This paper demonstrates that a suitable line delay time matching to the pulse repetition rate leads to undistorted pulse transmission along each single line, whereby the terminating impedance can be arbitrarily chosen, and no pulse crosstalk to the coupled other line exists.

For pulse techniques, e.g., in pulse generators or amplifiers, dc-isolating broad-band pulse transformers are required. As has already been investigated in literature [4]-[7], an alternately shorted coupled line four-port is a very attractive pulse transformer because of its broad-band operation. This paper shows that a variety of terminating conditions yields, by choice, undistorted pulse transmission to port \( IV \) (cf. Fig. 1b) or (delayed by line delay time) to port \( IV \). Moreover, also suitable line delay time matching leads to undistorted pulse transformation without limiting termination conditions.

Further, this paper demonstrates that by a coupled-line four-port pulses may be shaped with regard to repetition rate or impulse shape. The corresponding equations for the pulse repetition rate and terminating impedances are given for the following cases: 1) Pulse repetition rate multiplication with variation of the pulse shape, 2) change of the pulse duration, 3) pulse repetition rate multiplication with simultaneous change of pulse duration, 4) staircase pulses, and 5) pulses with quasi-triangular and quasi-trapezoidal shape.

2. Coupled Strip Lines

The electromagnetic coupled line four-port (Fig. 1) is calculated by its (A)-matrix [3], [4]

\[
\begin{bmatrix}
U_L \\
U_W \\
I_L \\
I_W
\end{bmatrix}
= \begin{bmatrix}
U_I \\
U_W \\
I_I \\
I_W
\end{bmatrix}
\]

with

\[
(A) = \begin{bmatrix}
\cos \theta & -\frac{1}{V} \sin \theta & \frac{1}{V} & 0 \\
-\frac{1}{V} \cos \theta & \cos \theta & \frac{1}{V} & -\frac{1}{V} \\
-\frac{1}{V} \cos \theta & \frac{1}{V} & \cos \theta & -\frac{1}{V} \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

where

\[
\theta = \arctan \frac{T_L}{n},
\]

\[n = \text{number of Fourier coefficient},
\]

\[\omega_0 = \frac{2\pi}{\lambda_0} \text{ pulse repetition frequency},
\]

\[\tau_L = \frac{1}{\nu} \text{ line delay time},
\]

\[\lambda_0 = \text{geometrical line length},
\]

\[\nu = c/\sqrt{\epsilon_r},
\]

\[c = 1/\sqrt{\mu_0 \epsilon_r},
\]

\[\epsilon_r = \text{relative dielectric constant},
\]

\[k = \text{coupling factor} \text{ (cf. Figs. 1,2)},
\]

\[Z_L = \text{characteristic impedance} \text{ (cf. Figs. 1,2)}.
\]

Since for coupled TEM wave transmission lines the relation

\[
(L') (C') = \frac{T_1}{k^2}
\]

holds [3],[4], where \((L')\) and \((C')\) are the matrices of the coupled line inductances, and capacitances, per unit length, respectively, the coupling factor \( k \) and the characteristic impedance \( Z_L \) may be determined from the line capacitances per length (cf. Fig. 1). Therefore, known methods for capacitance coefficient calculations, like e.g., the moment method [7],[8], can be applied. Fig. 2 shows calculated results for the coupled strip line prototype (Fig. 1) chosen for the experimental investigations in this paper. Commercially available copper-clad Polyguide (Polyolefin Laminate, \( t_r = 3.2 \) mm) with substrate thicknesses of 1/16" (1.887 mm), and 1/32" (= 0.783 mm), is used for strip line material. The copper-clad thickness is 35 \( \mu \)m.

For experimental investigations, broadside coupled strip lines (Fig. 1) are very appropriate, because relatively high coupling factors can easily be achieved. It will be shown that measurements are in close accordance with the calculations.
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The output signal \( u_2(t) \) at the port I (Fig. 1b) for a given input signal \( u_1(t) \) is calculated using the Fast Fourier Transform algorithm. The voltage transmission coefficients \( A_{12} \) between the inner generator port 0 and the port I is calculated from the A-matrix (2) together with the terminating impedances and phase relations chosen. For the measurements at the coupled strip lines a digital pulse generator hp 8082 and a sampling oscilloscope hp 180 D (with 1911 A, 1430 C) are used.

3. UNDISTORTED PULSE TRANSMISSION

For direct pulse transmitting circuits no pulse distortion along the coupled lines from port I to port II (Fig. 3a) is desired. For arbitrary coupling factors

\[
\begin{align*}
\text{input} & \quad Z_a \quad \text{arbitrary} \quad u_1(k) \\
\text{output} & \quad u_2(k)
\end{align*}
\]

The output signal \( u_2(t) \) is obtained by matching of the pulse period \( T_0 \) to the line delay time \( T_L \).

4. PULSE TRANSFORMER

A pulse transformer enables dc-isolated and undistorted pulse transmission. This is achieved by a coupled-line four-port (Fig. 4), where suitable choice of pulse period \( T_0 \) or terminating impedances leads to undisturbed pulse transmission with or without phase inversion. At output port III the pulse is not delayed, at port IV the delay is \( T_L \) (line delay time).

4.1 Matching of the pulse period

All pulses symmetrical to \( T_0/4 \) (Fig. 4b), with only odd Fourier coefficients \( (2n-1)^{th} \) may be transmitted without distortion to ports III or IV, respectively, if the relation line delay time \( T_L \) to pulse period \( T_0 \) is chosen to be

\[
T_L = \frac{2p-1}{k} \quad T_0, \quad p = 1, 2, 3, \ldots
\]

With \( \ell = n = (2n-1) \cdot \frac{T_L}{T_0} = (2n-1) - 2 \cdot \frac{T_0}{T_L} = 2(2n-1) (2p-1) \cdot \frac{T_0}{T_L} \)

and equation (2) the voltage transmission coefficients \( A_{12} \) and \( A_{14} \) to ports III and IV, respectively are found to be

\[
A_{12}^{(I)} = \frac{Z_{12}^{(II)} (Z_{12}^{III} + Z_{14}^{IV})}{2}, \quad A_{14}^{(II)} = \frac{Z_{14}^{III} (Z_{14}^{II} + Z_{12}^{IV})}{2}, \quad \ell = n = \frac{2p-1}{k}.
\]
with the abbreviations
\[ z_1 = z_L + \frac{z_{II}}{z_L}, \quad z_3 = \frac{z_{III}^2 z_{IV}}{z_L}, \]
\[ z_2 = z_L + \frac{z_{III} z_{IV}}{z_L}, \quad z_4 = \frac{z_{III}^2 z_{IV}}{z_L}. \]

By suitable choice of the terminating impedances the output-pulse may be inverted, if desired. Fig. 4 shows the arrangement of the ports and the theoretical pulse shapes. The measured pulse at port III, for example, confirms the theoretical investigations.

4.2 Matching of the terminating impedances

For undistorted transmission to port III the terminating impedances have to be chosen
\[ z_{II} < z_L', \quad z_{III} < z_{IV} < z_{L}^2, \quad z_{III} > z_L', \quad z_{IV} = \frac{z_L^2}{z_L}. \]

With equation (2) the voltage transmission coefficient \( A_{III} \) to port III is then
\[ A_{III} \approx k \quad (8) \]

if \( \sin \hat{t} \neq 0 \), i.e. \( \hat{t} \neq (p-1)\pi \), \( p = 1, 2, 3, \ldots \); this is warranted for irrational ratios \( T_s/T_0 \). The output-pulse at port III is the undistorted input-pulse multiplied with \( k \) (without dc-portion).

Undistorted and delayed pulse transmission to port IV is achieved by the following two conditions 1) or 2) for the terminating impedances:

1) \( z_L < z_L', \quad z_{II} = z_L/k_0, \quad z_{III} < z_L, \quad z_{IV} = z_L^2/s_1', \)
or

2) \( z_L < z_L', \quad z_{II} < z_L \) and \( z_{III} = k_0 z_L \) or \( z_{IV} = k_0 z_L'. \)

The corresponding transmission coefficients are given by
\[ A_{IV} \approx k e^{-k_1} \quad (9) \]
\[ A_{IV} = \frac{k_0}{(1 + k_0 s_{IV})(1 + k_0 z_{IV}) + \frac{z_{III}}{z_L} - \frac{k_0 z_{IV}}{z_L}} \quad (10) \]
if \( \sin \hat{t} \neq 0 \).

The output-pulse at port IV is the undistorted input-pulse 1) multiplied with \( k \), or 2) multiplied with \( e^{-k_1} \), and delayed by \( T_L' \).

5. PULSE SHAPING

With the four-port of coupled lines, pulses may be shaped with regard to impulse shape and repetition rate. This is achieved by a suitable choice of the ratio line delay time to pulse period together with appropriate terminating impedances, which yields the necessary frequency behaviour of the corresponding transmission coefficient.

5.1 Pulse repetition rate multiplication without variation of the impulse shape

By suitable frequency behaviour of the transmission coefficient \( A \), only the parts \( p \neq 0 \), \( p = 1, 2, 3, \ldots \), of a pulse spectrum \( C_p \) are transmitted; the result is a pulse \( u_{out}(t) \) with a pulse repetition rate \( p \) times as large. The principle of this pulse shaping is illustrated in Fig. 5, special cases are investigated in chapters 5.1.1 and 5.1.2.

Illustration of pulse repetition rate multiplication by suitable frequency behaviour of the transmission coefficient \( A \)
a Input-pulse
b Spectrum of the input-pulse
c Transmission coefficient
d Spectrum of the output-pulse
e Output-pulse

5.2 Pulse repetition rate doubling at port II

Pulse repetition rate doubling at port II
a Arrangement of the ports
b Measured input- and output-pulse (theory of Fig. 5)
5.1.1 Pulse repetition rate doubling at port II

With the following terminating impedances

\[ Z_I = Z_{II} = Z_L, \quad Z_{III} = Z_{IV} = \infty \]

or

\[ Z_I = Z_{II} = Z_L, \quad Z_{III} = Z_{IV} = 0 \]

the transmission coefficient to port II is zero for \( t = (2p-1) \cdot T/2 \) (p = 1, 2, 3, ...). If the line delay time to pulse period is chosen to be

\[ T_L = \frac{1}{c}, \]

odd Fourier coefficients of the input-pulse are not transmitted (cf. Fig. 5). Fig. 6 shows the measured results which verify the theory illustrated in Fig. 5.

5.1.2 Pulse repetition rate multiplication at port IV

Pulses with multiple repetition rate of the input pulse may be generated by the choice

\[ Z_I < Z_{II} < Z_L, \quad Z_{III} < Z_{IV} \ll Z_L. \]

The transmission coefficient \( A_{DIV} \) to port IV is then

\[ A_{DIV} = a \cdot k, \quad a = 1 \quad \text{for} \quad t = m \cdot T, \]

\[ A_{DIV} = 0 \quad \text{else} \quad m = \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots \]

If the relation line delay time \( T_L \) to pulse period \( T_0 \)

\[ T_L = \frac{1}{p} T \]

\[ p = 1, 2, 3, \ldots, \quad i = 1, 2, 3, \ldots \]

holds, where \( p/i \) includes no common divisor, only the

Fourier coefficients \( a = \pm p, \pm 2p, \pm 3p, \ldots \) are trans-
mittance. This is an output-pulse of \( p \)-fold repetition rate. Fig. 7 shows the pulse repetition rate trebling for example.

5.2 Change of pulse duration

By suitable choice of the pulse duration \( T_0 \) to line de-
delay time \( T_L \), the ratio \( T_0/T \), together with appropriate condi-
tions for the terminating impedances, the pulse dura-
tion \( T \) of the output-pulse may be increased or decreased.

5.2.1 Double pulse duration at port III

The terminating impedances are chosen to be

\[ Z_I = Z_L/k_1, \quad Z_{II} > Z_L, \quad Z_{III} \text{ and } Z_{IV} > Z_L \]

the transmission coefficient to port III \( A_{DIV} \) is then
given by

\[ A_{DIV} = k \left( 1 + e^{-32k} \right) \]

if \( \sin \theta \neq 0 \). For adjustment of the pulse duration of
the input-pulse to the line delay time according to

\[ T_L = Z_L/T \]

at port III an output-pulse with the double pulse dura-
tion is generated. Fig. 8 shows the theoretical and
measured results.
5.2.2 Half pulse duration at port IV

With

\[ z_I < z_{II} < z_{III} < z_{IV} < z_L \]

and the condition

\[ \frac{T}{T_L} = 4 \, p \]

the output-pulse at port IV is a pulse group of \( p \) pulses per period with pulse duration \( \tau_1 = \tau / 2p \) each and inverse sign. Fig. 9 shows the pulse duration bisecting at port IV for example.

Further, for the conditions

\[ z_I < z_{II} < z_{III} < z_{IV} < z_L \]

or

\[ z_I < z_{II} < z_{III} < z_{IV} < z_L \]

and

\[ \frac{T_0}{T_L} = 4 \, p \text{, with } \frac{\tau}{T_L} = 2(2q-1), \]

\[ p = 1, 2, 3, \ldots \quad q = 1, 2, 3, \ldots \]

at port III an output-pulse is obtained with \( p \)-fold repetition rate and the pulse duration

\[ \tau_1 = 2 \, T_L \]

Fig. 10 shows for example the pulse repetition rate quadruplication with simultaneous tripartition of pulse duration at port III.

5.3 Pulse repetition rate multiplication with simultaneous change of pulse duration

For pulse shaping case 5.1 (repetition rate multiplication) the input- and output-pulses only hold the same pulse duration if \( p \tau < T_0 \). Otherwise the pulses overlap to the new pulse duration

\[ \tau_1 = \frac{T_0}{p} \left( p \right) \quad \text{entire} \left( \frac{p}{T_0} \right), \]

where "entire" means the highest whole number of the decimal fraction \( \left( p \tau / T_0 \right) \) (e.g. entire 3.9 is 3).

Pulse repetition rate quadruplication with simultaneous tripartition of pulse duration at port III

a. Arrangement of the ports
b. Theoretical results
c. Measured input- and output-pulse
5.4 Staircase pulses

Change of pulse duration (chapter 5.2) but with rational pulse duty factor $T_0/D_0$ (instead of an irrational factor, chapter 5.2) leads to staircase pulses. Further, for

$$Z_I = Z_L, \quad Z_{II} = \frac{Z_L}{2}, \quad Z_{III} = \frac{Z_L}{3}, \quad Z_{IV} = \frac{Z_L}{4}$$

staircase pulses at port III may be generated. Fig. 11 shows the measured staircase pulse at port III, for example.

![Staircase Pulse Diagram](image)

5.5 Pulses with quasi-triangular and -trapezoidal shape

By suitable choice of the line delay time to pulse period ratio (e.g. $T_L/T_0 = 1/6$) and of the terminating impedances (e.g. $Z_{II} < Z_L, Z_{III} < Z_L, Z_{IV} < Z_L$), pulses with quasi-triangular and quasi-trapezoidal shape may be generated. The sides of the pulse are multi-step staircases. Fig. 12 shows a measured quasi-triangular pulse at port IV.

![Pulse Diagram](image)

REFERENCES


