Influence of Noise and Quantization Error in Microwave Holography on Holograms and Image Reconstructions

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RESUME

L'holographie des micro-ondes est une méthode complexe dans la technique radar servant à identifier la forme d'objets métalliques plans et tridimensionaux. La simulation numérique de deuxième ordre (transformation discrète de Fresnel-Kirchhoff) des hologrammes offre l'avantage de pouvoir étudier l'extension appropriée des hologrammes et l'influence de l'analyse discrète afin d'améliorer la précision de la méthode de représentation. L'usage de l'algorithme "Fast Fourier Transform" assure une reconstruction numérique rapide des images avec laquelle une identification sûre des objets est possible. Pour les signaux au bruit les problèmes ne sont pas encore complètement résolus. Cette publication recherche systématiquement l'influence des erreurs de bruit et de quantification (pour amplitude et phase) sur les hologrammes et la reconstruction des images. On démontre qu'avec une superposition numérique appropriée des hologrammes une identification d'objets rectangulaires et circulaires est possible même lorsque le rapport signal-bruit est d'environ 1/5. La publication démontre de plus qu'une définition appropriée d'un critère d'acuité pour l'identification d'objets même à la détermination de la distance sans aucune ambiguïté. Cette détermination peut être réalisée par des procédés traditionnels d'optimisation numérique.

SUMMARY

Microwave holography is a versatile tool in radar to identify the shape of planar as well as of three-dimensional metallic objects. Numerical second-order simulation (discrete Fresnel-Kirchhoff-Transformation) of holograms yields the advantage that the suitable expansion of the holograms and the sampling influence can be systematically investigated in order to improve the accuracy for the imaging method. The use of the Fast Fourier Transform algorithm provides quick computer reconstruction of the images where clear identification of the objects is possible. For noisy signals the problems are not yet completely solved. The paper investigates systematically the influence of noise and quantization error (for amplitude and phase) on holograms and image reconstruction. It is shown that by suitable computer superposition of holograms an identification of rectangular and circular objects is possible even if the signal-to-noise ratio is about 1/5. The paper shows further that a suitable definition of a clearness criterion for the identification of objects leads to an unambiguous range determination which can be carried out by common numerical optimization procedures.
1. INTRODUCTION

Microwave holography\textsuperscript{1-6} is a useful technique for imaging remote or inaccessible metallic objects. As has already been shown, a clear identification of two- and three-dimensional objects is possible, if the necessary conditions for sampling theorem, hologram plane dimension and window functions are held\textsuperscript{7,8,9}.

Although for hologram formation in reality scattered field intensity and phase of an illuminated object are measured, it has turned out to be advantageous\textsuperscript{1,7,10} to investigate the necessary hologram formation conditions by numerical computer simulation using the FRESNEL-KIRCHHOFF's diffraction integral. Further, the computer simulation of holograms, taken from two-dimensional as well as from three-dimensional objects, has been proved to be an appropriate tool to perform complicated experiments, where practical measurements are too expensive or in fact impossible. The image reconstruction step is carried out digitally, where the FRESNEL-approximation enables the use of standard signal processing techniques like the Fast-Fourier-Transform (FFT).

Systematic errors at the hologram formation (like loss of information by underscanning or by band limitation in the hologram plane) have already been extensively investigated\textsuperscript{1,7}. By suitable choice of the hologram formation parameters those errors can largely be eliminated.

Stochastic disturbance of the hologram formation however, like noise caused by incomplete reflection characteristic of the illuminated object and by spurious noise sources in space, or quantization error caused e.g. by the insufficiency of the measuring equipment, have not yet been investigated systematically. This paper is a contribution to remove this leak.

It will be shown, that suitable superposition of noisy complex hologram function values leads to a clear identification of the reconstructed image even if the signal-to-noise ratio is much more less than 1. The dependence of the amplitude and phase quantization error on the information contents in holograms is systematically investigated at the examples of a circular and a quadratic metallic plate.

The paper investigates furthermore suitable criteria for numerical range determination, which can be carried out by common and well known optimization procedures. An appropriate clearness criterion leads to additional information which allows to locate the range up to an error of about 1\% even if the hologram function is noisy. By analysing the real and imaginary part of the reconstructed image function it is possible to separate mutually distant parts of three-dimensional objects, which is demonstrated at the example of two quadratic metal plates of different sizes mounted in different distances.

2. THEORY

The computer simulation of the diffracted wave at the point \( P_0(x_0, y_0) \) in the hologram plane (Fig. 1) is based on the FRESNEL-KIRCHHOFF's diffraction integral, which is written down in a three-dimensional form, where the z-dimension of the object (Fig. 1a) is subdivided into \( I \) planes with the dimensions \( \Delta x_i \times \Delta y_i \) and \( \Delta z_0 \times \Delta y_0 \) and the complex hologram function \( \tilde{W} \) is taken for I-L points with the sample distance \( \Delta x_0, \Delta y_0 \):

\[
\tilde{W}(x_0, y_0) = \frac{1}{2\pi} \sum_{p=1}^{P} \sum_{m=1}^{M} \sum_{n=1}^{N} \Delta x_0 \Delta y_0 \Delta z_0 \exp \left\{ jk \frac{\Delta y_0}{\Delta y_i} \Delta y \right\} e^{-jk \frac{\Delta z_0}{\Delta z_0} \Delta z} \left\{ \cos(\theta_m \Delta x_0) \cos(\theta_n \Delta y_0) \right\} \tag{1}
\]

with

\[
\lambda = \text{wavelength} \\
\nu = \frac{2\pi}{\lambda} \\
\theta = \text{field distribution of the diffracting interface (object planes)} \\
\theta_m = \left[ (x_0, x_0, \Delta x_0, s) \right] \left[ (y_0, y_0, \Delta y_0, s) \right], \left[ (z_0, z_0, \Delta z_0, s) \right] \right\} \tag{2}
\]

\[
\theta_n = \left[ (x_0, x_0, \Delta x_0, s) \right] \left[ (y_0, y_0, \Delta y_0, s) \right], \left[ (z_0, z_0, \Delta z_0, s) \right] \right\} \tag{3}
\]

The computer simulation of the hologram is carried out with the FRESNEL-integral, which is described as two-dimensional discrete FOURIER-transform of the function

\[
\tilde{W}(x, y) = \sum_{m=1}^{M} \sum_{n=1}^{N} \Delta x_0 \Delta y_0 \Delta z_0 \exp \left\{ jk \frac{\Delta y_0}{\Delta y_i} \Delta y \right\} e^{-jk \frac{\Delta z_0}{\Delta z_0} \Delta z} \left\{ \cos(\theta_m \Delta x_0) \cos(\theta_n \Delta y_0) \right\} \sum_{p=1}^{P} \sum_{m=1}^{M} \sum_{n=1}^{N} \Delta x_0 \Delta y_0 \Delta z_0 \exp \left\{ jk \frac{\Delta y_0}{\Delta y_i} \Delta y \right\} e^{-jk \frac{\Delta z_0}{\Delta z_0} \Delta z} \left\{ \cos(\theta_m \Delta x_0) \cos(\theta_n \Delta y_0) \right\} \tag{4}
\]

A computer program [Program A] is used to simulate the hologram formation process. The program calculates the hologram function \( \tilde{W} \) and the reconstructed image function \( W \) for a given set of parameters. The program is capable of simulating holograms for a variety of different objects and viewing angles.

The program is also capable of simulating holograms for three-dimensional objects. The program is used to simulate holograms for a variety of different objects and viewing angles. The program is capable of simulating holograms for a variety of different objects and viewing angles. The program is used to simulate holograms for a variety of different objects and viewing angles. The program is capable of simulating holograms for a variety of different objects and viewing angles. The program is used to simulate holograms for a variety of different objects and viewing angles.
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3. NOISE INFLUENCE

Imperfect back-scattering of the illuminated object leads to a superposition of a complex noise component $\Delta \psi_{\text{mp}}$ to the object function $B_{\text{mp}}$ (equation (1))

$$B_{\text{mp}}' = B_{\text{mp}} + \Delta \psi_{\text{mp}}$$

(4)

Jamming by noise sources in the measuring space is expressed by an additional complex noise component $\Delta \psi_{\text{c}}$ to the hologram function $H_{\text{11}}$ (equation (1))

$$H'_{\text{11}} = H_{\text{11}} + \Delta \psi_{\text{c}}$$

(5)

where $\psi_{\text{c}}$ are amplitude factors, and $\Delta \psi_{\text{c}}$ complex random variables which may be functions of location and time. The statistical parameters, amplitude density $p_{\psi}$, variance $\sigma_{\psi}^2$, and expected value $\mu_{\psi}$, for magnitude (m) and phase (ph) of the random variables $\Delta \psi_{\text{m}}$ and $\Delta \psi_{\text{ph}}$ are

$$p_{\psi} = \frac{1}{\sigma_{\psi}} \quad \sigma_{\psi}^2 = \frac{1}{36} \quad \mu_{\psi} = 0$$

(6)

$$p_{\psi_{\text{ph}}} = \begin{cases} \frac{1}{2\pi} & \text{for } 0 < \psi_{\text{ph}} < 2\pi \\ 0 & \text{else} \end{cases} \sigma_{\psi_{\text{ph}}}^2 = \frac{1}{3} \mu_{\psi_{\text{ph}}} = \pi$$

(7)

$\Delta \psi_{\text{m}}$ and $\Delta \psi_{\text{ph}}$ are simulated numerically. For generation of the uniformly distributed random phase the (0,1)-algorithm

$$z = (\text{a},z_{\text{mod}})_{\text{random}}$$

$$\tilde{z}_{j+1} = \frac{z_{j} - x_{j} \left( \sum_{k=1}^{12} x_{k} \right)}{\beta} + 4$$

(8)

is used, where $(\text{a},z_{\text{mod}} \in [0,1,2,\ldots,\alpha = 15], z_{\text{mod}} = 8_{\text{b}})$ (for 16 bit computer), $z_{\text{mod}}$ is arbitrary but odd. The normal distribution of the random amplitude is generated by the uniformly distributed $z_{\text{mod}}$ of equation (8)

$$z_{j} = x_{j} + x_{j} \left( \sum_{k=1}^{12} x_{k} \right)$$

(9)

where $x_{j} = 0$ and $c_{2} = 1/6$. Fig. 3a shows a sequence of 32x32 normally distributed random variables for noise amplitude simulation. Fig. 3b shows a sequence of 32x32 uniformly distributed random variables for noise phase simulation.

The influence of noise in the object function $B_{\text{0}}$ (equation (4)) on the hologram function $H_{\text{11}}$ and on the image reconstruction is largely reduced by the transformation process involved in equations (1) and (2), respectively, which leads to an averaging of the perturbances. This effect is demonstrated in Figs. 4a, where an arrangement of a circular disk and a quadratic plate, with a distance of 2m from each other, is considered as an object example (Fig. 4b). Magnitude and phase of the complex object functions are assumed to be disturbed with a signal-to-noise ratio of about $S/N = 0.5$ (Figs. 4b).

4. REDUCTION

**Fig. 4**

Reduction of the influence of object noise by the diffraction and imaging transformation process.

- Plane objects: distance from each other by 2m.
  - 1 circular disk, diameter $d = 0.5m$, centerpoint coordinates $x_1 = 1.25m, y_1 = 0.75m$.
  - 2 quadratic plates, length of side $a = 0.62m, x_2 = 0.8m, y_2 = 1.2m, x_3 = 0m$. y_3 = 0m.
The complex hologram function (Fig. 4d) of the disturbed objects differs only slightly from that of the undisturbed objects (Fig. 4e). Further, the reconstructed images (Fig. 4f) of the disturbed objects are clearly identifiable in spite of the original signal-to-noise ratio S/N=0.5. The above results are numerically confirmed by a separate hologram formation of the noise component \( Y_g \) in equation (4) alone for the disk as object example. The maximum magnitude in the noisy hologram (Fig. 4a) is only about \( 1/13 \) of that one of the undisturbed case (Fig. 4g) in spite of an assumed signal-to-noise ratio S/N=0.5 as before.

\[ d \] magnitude and phase of the complex hologram function of the object 1 from the hologram formation \( d \).

\[ e \] Image reconstruction of the object 1 from the hologram formation \( d \) - disturbed according to Fig. 4b.

\[ f \] Image reconstruction of the object 2 from the hologram formation \( d \) - disturbed according to Fig. 4b.

\[ g \] Magnitude of the complex hologram function of the undisturbed object 1 alone. Maximum absolute magnitude \( |u|_{\text{max}} = 0.974 \).

\[ h \] Magnitude of the complex hologram function of the object noise of object 1 alone. Maximum absolute magnitude \( |u|_{\text{max}} = 0.074 \).

The influence of noise in the hologram function \( U \) (equation (9)) on the image reconstruction \( B \) is averaged analogously to the object 1. By way of noise by the corresponding transformation in equation (2). This is demonstrated in Figs. 5a,b. Although the signal-to-noise ratio of the noisy hologram (Fig. 5a) is S/N=1, the objects 1 and 2 (cf. Fig. 4a) still can be identified (Figs. 5b). For numerical example, again a separate transformation of noise alone is investigated. Fig. 5c shows the complex hologram noise component \( Y_g \) in equation (9) alone together with the corresponding image reconstruction (Fig. 5d). The maximum magnitude in the noisy image reconstruction (Fig. 5f) is about \( 1/3 \) of that one of the reconstruction (Fig. 5c) taken from the undisturbed hologram (Fig. 5d), in spite of S/N=1. Because of the low amplitude density in Fig. 5c, however, the noise influence for this example is more severe than for example Fig. 4b (\( 1/3 \) instead of only \( 1/13 \)).

\[ \]

**Fig. 5** Reduction of the influence of hologram noise by the imaging transformation process

\[ a \] Magnitude and phase of the noisy hologram according to the object arrangement Fig. 4a - S/N=1.

\[ b \] Image reconstruction of objects 1 and 2 from the noisy hologram (reconstruction data cf. Figs. 4e,f).
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4. QUANTIZATION ERROR

The influence of magnitude and phase quantization in the hologram function, equation (11), on image reconstruction is investigated by introducing \( \tilde{U}_{11} \) in equation (2),

\[
\tilde{U}_{11} = (U_{11} + \Delta U_{11})
\]

where

\[
\Delta U_{11} = \text{error of magnitude quantization},
\]

\[
\Delta \phi_{11} = \text{error of phase quantization}.
\]

The uniform distributed quantization errors in magnitude \((\Delta U)\) and phase \((\Delta \phi)\) show the following properties

\[
\sigma(U, \phi) = \begin{cases} 
\frac{1}{\sqrt{\pi}} & \text{for } -\frac{\Delta U}{2} < U < \frac{\Delta U}{2} \\
0 & \text{else} 
\end{cases} 
\text{, } \sigma(\phi) = \frac{\Delta \phi}{2}
\]

Because of the stochastic character of the quantization errors, like in chapter 3, their influence on image reconstruction is also reduced by the transformation process involved. This effect is shown in Fig. 7, where for better perception the complex hologram function is exhibited perspective. For 10 quantization steps in magnitude, and 8 in phase (Fig. 7a), the objects can still be identified clearly. The limit of identification is reached for 5 quantization steps in magnitude and 4 in phase (Figs. 7c, d). Because of the lower hologram function portion of object 1 in the hologram of the two objects (cf. e.g. Fig. 4g with Fig. 4c), the reconstructed image of object 1 is more disturbed than that of object 2.

In order to eliminate the mutual influence of the two objects, in Fig. 8 only object 1 (Fig. 4a) is investigated. For 5 quantization steps in magnitude and 4 in phase (Fig. 8a) the object 1 (in comparison to Fig. 7a) can still be identified clearly.
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5. RANGE DETERMINATION

As has already been reported in reference, the sum of the magnitudes of the complex reconstructed object function on the hologram plane is equal to the real distance \( z_0 \) of the measuring plane. This is used to define an error function for numerical range determination, which yields a minimum for \( z_0 \), real.

\[
\text{Error Function: } E(z) = \sum_{m=0}^{N-1} \sum_{n=0}^{K-1} \left| \frac{R_{mn}(z)}{H_{mn}} \right|^2 \quad \text{MIN}
\]

where 
\[
E(z) = \sum_{m=0}^{N-1} \sum_{n=0}^{K-1} \left[ |aR(n)| \right]^2 \quad \text{MIN}
\]

\[
E(z) = \sum_{m=0}^{N-1} \sum_{n=0}^{K-1} \left[ |aR(n)| \right]^2 \quad \text{MIN}
\]

It is advantageous to define a second criterion for range determination, which uses the fact, that the reconstructed object shows its maximum picture sharpness if also \( z_0 \) is real. This leads to a second error function:

\[
E_2(z) = \left( \sum_{m=0}^{N-1} \sum_{n=0}^{K-1} \left[ |aB(n)| \right] \right)^{1/2} \quad \text{MIN}
\]

Fig. 8 Influence of hologram quantization on image reconstruction. Object 1 alone, magnitude and phase of the complex hologram function (magnitude: 5 quantization steps: 4 quantization steps) and the corresponding image reconstruction.

Fig. 9 Range determination a: with the amplitude density criterion, error function \( E(z) \); b: with the phase density criterion, error function \( E_2(z) \); c: with both density criteria, error function \( E(z) \).
In Fig. 9a, error function $EF_1$ is investigated within a longer distance range than in reference. For a simple object (quadratic plate) $EF_1$ is plotted versus distance $z$ for several numerical image reconstruction techniques. Curve 5 is an error function appropriate for range determination within a suitable $z$ region of about 7 to 15m.

An investigation of the reconstruction techniques, similar to those in Fig. 9a, for error function $EF_2$ (the sharpness criterion) yields the result that a suitable global minimum is accessible for range determination in a wide distance range of about 2 to 22m (Fig. 9b). Fig. 10 shows the error function $EF_3$ versus distance for several parameters $\theta$, cf. equation (13).

**Fig. 9a** Range determination with the image sharpness criterion, error function $EF_2$, cf. equation (13). $\gamma=4, \Delta R_{\min}=0.35, \lambda=0.03m$

The influence of the hologram bandwidth, which affects the image sharpness, on the accuracy of the range determination is demonstrated in Fig. 11a. Triple bandwidth (Fig. 11b) increases the accuracy of error function $EF_2$ (the sharpness criterion) from $1.25\%$ (Fig. 11a) to $0.4\%$ (Fig. 11b) while error function $EF_3$ (amplitude density criterion) holds its accuracy within $1.25\%$.

**Fig. 11 Influence of the hologram bandwidth on accuracy of range determination**
- $EF_1$ amplitude density criterion
- $EF_2$ sharpness criterion
- $\gamma=3, \beta=2, \Delta R_{\min}=0.4, \lambda=0.03m$
- $b \lambda=0.01m$(triple bandwidth)
Fig. 12 shows that with the two error functions chosen a range determination to within 71.5% is possible even if the signal-to-noise ratio is $S/N = 2$.

Fig. 12 Range determination at noisy signals. Signal-to-noise ratio $S/N = 2$

$\lambda = 0.015m$

For three-dimensional objects the real part of the complex image reconstruction function can advantageously be used to display separately distinct object parts. This is demonstrated in Fig. 13, where a quadratic object consisting of two plates of different sizes located in different distances is investigated (Fig. 13a). Fig. 13b shows that a clear separation of the distinct object parts 1 and 2 is possible.

Fig. 13 Separation of distinct object parts in image reconstruction of three-dimensional objects

- a quadratic plate behind the remaining frame at a distance $\Delta z = (2n+1)\lambda/4$.
- magnitude of hologram and image reconstruction regarding the real parts

REFERENCES


