THE ORTHOGONAL EXPANSION METHOD FOR SOLVING ELECTROMAGNETIC SCATTERING PROBLEMS IN FINITE MILLIMETER-WAVE STRUCTURES

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RESUME

Ce document étudie numérique et en partie avec des mesures l'onde électromagnétique répartissant des guides d'ondes rectangulaires et homogènes vers les guides d'ondes avec une section transverse non homogène. Ces transitions jouent un rôle important dans le recent développement de circuits à onde-millimètre dans le domaine des satellites de communication.

La méthode de calcul basée sur l'expansion de champs dans des modes propres orthogonaux appropriés est introduite avec plus de détail pour l'exemple de la transition directe d'un guide d'ondes homogène vers une microligne écrannée. Ce traitement peut être considéré comme général dans ce sens qu'il faut prendre en considération tous les six composants de champ et les caractéristiques des sections transversales avec des plaques diélectriques.

Pour la transition de guide d'ondes vers la microligne il est montré que le plupart de la puissance est transmise par le mode de premier ordre hybride et non par le mode fondamental de la microligne communément désiré. Cela indique qu'une jonction directe des guides d'ondes avec les microlignes n'est pas appropriée.

SUMMARY

This paper investigates numerically and partially by measurements the electromagnetic wave scattering at transitions from homogeneous rectangular waveguides to waveguides with an inhomogeneous cross-section. Such transitions play a significant role in the recent development of suitable millimeter-wave communication circuits.

The calculation method, which is based on the field expansion into suitable orthogonal eigenmodes, is introduced in more detail for the example of the direct transition from a homogeneous waveguide to a shielded microstrip line. This treatment can be regarded as general in the sense that all six possible field components have to be considered and the features of dielectric slab-filled cross sections are taken into account.

For the investigated waveguide transition to the microstrip-line it is shown that the principal part of the transmitted power is transported by the first order hybrid mode and not by the commonly desired fundamental microstrip mode. This indicates that a direct waveguide instrumentation of microstrip lines is inappropriate.

Dielectric-slab structures, however, mounted in the E-plane of rectangular waveguides are very suitable for millimeter wave applications. This is demonstrated by the examples of the waveguide transition to a dielectric slab filled waveguide and to a fin-line, where the dielectric slab is partially metallized. The fin-line structure is very suitable for designing millimeter-wave integrated circuits. A nine step fin-line filter is designed and calculated as an example for 33.7 GHz which exhibits a measured insertion loss of only about 0.5 dB.
1. INTRODUCTION

Recent studies, e.g.\textsuperscript{1,2}, have indicated that there will be a significant increase in satellite communications in the near future. To overcome the problem of spectral crowding, efforts are being made to expand the communications services upward to the millimeter-wave region of the spectrum. Since the millimeter-wave systems are based on waveguide instrumentation, waveguide scattering problems find increasing interest.

Further, because of their known advantages in the lower frequency range, particularly compactness and lower price, there is a growing demand in the actual satellite communication research activities for suitable millimeter-wave integrated circuit (MIC) structures. The ideal transmission line type for MIC's for millimeter-wave application is one which avoids excessive miniaturization, is suitable for waveguide instrumentation, and yet offers the potential for low cost production through batch-processing techniques. The fin-line and the dielectric image line (Fig. 1) are such line types. However, there is still a paucity of suitable theoretical research on such structures. So far only experimental design data and first order design theories are available, e.g.\textsuperscript{3,4}, especially concerning three-dimensional problems.

![Dielectric Substrate](image1)

**Fig. 1** Millimeter-Wave Structures

a) Rectangular Waveguide

b) Shielded Dielectric Image Line

c) Fin-Line

The purpose of this paper is therefore to show that the orthogonal expansion method, principally introduced by Whinnery and Jameson\textsuperscript{5} is an appropriate numerical method for the rigorous solution of millimeter-wave structure discontinuity scattering problems. An infinite set of linear equations for the amplitudes of the reflected and transmitted modes for any incident mode can be obtained by exactly satisfying the boundary conditions. This theory takes into account the fact that higher mode propagation plays a significant role in the effect of those scattering problems.

![Waveguide](image2)

**Fig. 2** Transition from a Rectangular Waveguide to a Microstrip Line

The first example investigated is the direct transition from a rectangular waveguide to a microstrip line (Fig. 2). It is shown, that a significant higher order mode propagation is caused at the discontinuity.

The transition from a waveguide to a dielectric loaded waveguide (Fig. 3), the second example, represents the first step towards the investigation of the transition to a dielectric image line (Fig. 1). The $H_{10}$-mode of the empty waveguide is transmitted almost undisturbed into the dielectric loaded part thus showing the possibility of a direct waveguide instrumentation. This is in clear contrast to the microstrip transition (Fig. 2), where the $H_{10}$-mode is considerably disturbed, so that suitably tapered transitions are required.

![Waveguide](image3)

**Fig. 3** Transition from a Rectangular Waveguide to an E-plane Dielectric Loaded Waveguide

In a fin-line structure, metal inserts ("fins") are printed on a dielectric substrate which bridges the broad walls of a rectangular waveguide (Fig. 1c, Fig. 4). The fin-line combined with RT/duriod substrate material is very suitable for millimeter-wave integrated circuits. Therefore, as the third example, a fin-line filter for about 34 GHz is designed. A new design theory is introduced, based on the orthogonal expansion method, taking into account the higher mode propagation.
2. METHOD

The method of the series expansion into orthogonal eigenmodes is described at the rather general and relatively complicated example of the direct transition from a waveguide to the microstrip (Fig. 2). This example in principle includes all cases investigated in this paper, because all six field components are excited, and it takes into account the aspects of dielectric slab filled cross sections.

The hybrid modes on the shielded microstrip line are derived from the axial z-components of the vector potentials $A_y$ and $A_z$:

$$
\mathbf{E} = \text{rot}(A_{z}^{*}) + \frac{i}{\hbar} \omega \text{rot}(A_{y}^{*})
$$

$$
\mathbf{H} = -\frac{1}{\hbar} \omega \text{rot}(A_{z}^{*}) + \frac{i}{\hbar} \omega \text{rot}(A_{y}^{*}).
$$

The subdivision of the cross section according to Fig. 5 leads to four parallel lines. Their vector potentials can be written as a product of the eigenfunctions $V_{h,v}(x,y)$ with the common propagation expression $e^{-jkz^2}$ and the square roots of the field impedances

$$
Z_h = 1/V_h = \frac{w_0}{k_z}, \quad Z_v = 1/V_v = \frac{k_z}{w_0}
$$

$$
A_{h} = \sqrt{Z_v} V_{h,v}(x,y) e^{-jkz} \quad A_{v} = \sqrt{Z_v} V_{h,v}(x,y) e^{-jkz}
$$

with $z = i, IIa, IIb,$ and III

$$
U_h = \sum_{n=0}^{\infty} \frac{\cos \theta_n}{1 + \theta_n^2} (A_{h} e^{-jkz} + B_{h} e^{-jkz})
$$

$$
\delta_{\omega n} = \text{Kronecker delta}.
$$

The other eigenfunctions can be written in the same manner. This way is analogous to the known Fourier expansion method with the still unknown coefficients $A_n, B_n$, which are determined by the boundary conditions.

The eigenfunctions $V(x,y)$ can be regarded as representing waves traveling in the $y$ direction, with the still unknown propagation constant $k_y$. It is possible to define the amplitudes

$$
I_{mn}^\nu(y) = a_{mn}^\nu e^{-jk_y y} + b_{mn}^\nu e^{jk_y y},
$$

$$
U_{mn}^\nu(y) = \frac{dI_{mn}^\nu(y)}{dy} = jk_y a_{mn} e^{-jk_y y} + jk_y b_{mn} e^{jk_y y},
$$

and in the same manner the corresponding amplitudes related to the vector potential $A_j, B_j$.

The boundary conditions at the partial waveguides

$$
0 < x < g : E_{x}^{III} = E_{x}^{IIa}, \quad H_{x}^{III} = H_{x}^{IIa}
$$

$$
0 < x < g + c : E_{x}^{III} = D, \quad H_{x}^{III} = H_{x}^{IIa}
$$

$$
(a/2 < x < c : E_{x}^{III} = E_{x}^{IIb}, \quad H_{x}^{III} = H_{x}^{IIb}
$$

successively applied to (5) lead finally to the relation between the amplitudes at the lower boundary ($y=1$) and the amplitudes at the upper boundary ($y=1/2$) (Fig. 6):

$$
\begin{bmatrix}
I_{1}^{III} \\
I_{1}^{II} \\
I_{1}^{I} \\
I_{1}^{e}
\end{bmatrix}
= (M)
\begin{bmatrix}
I_{1}^{III} \\
I_{1}^{II} \\
I_{1}^{I} \\
I_{1}^{e}
\end{bmatrix}
$$

The still missing boundary condition at the metallic surfaces at $x = -d/2 + h$ and $y = b - d/2 - h$ leads to the resonant condition:

$$
0 = \begin{bmatrix}
M_{hn} & M_{hr} & 1_{n} \\
M_{hn} & M_{hr} & 1_{r}
\end{bmatrix}
$$

This matrix of the characteristic equation (8) is the upper right quarter of the matrix product of (7). The zeros of the determinant which is a transcendental function of $k_y (w_k)$ provide the interesting dispersion characteristic $k_y (w_k)$ for each mode taken into account.
3. RESULTS

In order to show the convergence of the orthogonal expansion method, Fig. 6 indicates the normalized propagation factors \( \frac{\kappa_{r}}{k_{0}} \) (\( \kappa_{0} = \) free space propagation constant) of the shielded microstrip line for four modes as a function of the number of eigenmodes considered. It can be stated that for a number of about 10 to 12 eigenmodes the propagation factors converge to a corresponding constant value.

Fig. 7 shows the scattering parameters as a function of the normalized frequency \( \frac{f}{\lambda_{0}} \) (\( \lambda_{0} = \) wavelength in air) of a direct waveguide to microstrip transition (Fig. 2). If a \( H_{01} \)-mode is incident in the rectangular waveguide, up to \( \frac{f}{\lambda_{0}} = 1 \) the reflection coefficient in the waveguide is only related to the backward \( H_{01} \)-wave. The other modes (dotted lines) do not yet propagate, since their frequency is below the corresponding cut-off frequency.

At the microstrip line (lower figure), however, higher order modes play a significant role. It can be seen that the principal part of the transmitted power is transported by the first higher order \( HE_{0n} \)-mode and not by the commonly desired fundamental microstrip mode \( EH_{01} \). This is because of the incoherence of the waveguide \( H_{01} \)-mode with the \( HE_{01} \)-mode. The figure indicates that a direct waveguide instrumentation of microstrip lines is inappropriate. Since suitable transitions (e.g., rigid waveguide taper geometry are relatively complicated (the state of the art are insertion losses of about 2.5 dB per transition), other structures, e.g., the two following examples, are much more appropriate for millimeter-wave MIC's.

1) The \( EH_{01} \)-mode is the hybrid-(EH)-mode with the first cut-off frequency and a purely \( H \)-mode at cut-off frequency. The other hybrid modes on the microstrip line are indicated in the corresponding manner.
If the dielectric slab mounted in the waveguide E-plane is partially metallized, a fin-line structure (Fig. 4) is obtained, which is also very adequate for direct waveguide instrumentation. Further it has the advantage that suitable MIC structures, such as filters, can easily be constructed by photolithography techniques. A low insertion loss fin-line filter is designed with the orthogonal expansion method which consists of a fin-line structure with alternatively metallic bridges and gaps in the total height of the rectangular waveguide. Fig. 9 shows the calculated value of the insertion loss in dB as a function of the frequency for a three-step fin-line filter (four metal inserts, Fig. 10), designed for a mid-band frequency of 33.71 GHz, mounted into an R-band waveguide (a=7.112 mm, b=3.556 mm). The dielectric substrate is RT/duriod 5880 with $\varepsilon_r = 2.22$. The theoretical insertion loss in the pass-band is 0.1 dB, the measured value is about 0.5 dB. The slight frequency displacement of the measured values is due to the copper cladding thickness (0.025 mm) which has not been taken into account in the calculations.

Line structures with circuits suspended in the E-plane of rectangular waveguides are very well compatible with the $H_{10}$-mode of the waveguide. Thus for the direct waveguide transition to a waveguide filled with a dielectric slab in the E-plane (Fig. 3) it can be stated (Fig. 8) that below the $H_{10}$-mode cut-off frequency practically the total energy is transmitted into the desired fundamental $H_{10}$-mode of the filled waveguide. This is also confirmed by measurements (Fig. 8).

In the designed high-Q fin-line filter.
Fig. 10 Photograph of the designed Fin-Line Filter for 33.7 GHz

CONCLUSION

Since there is a growing demand for communication systems at higher frequencies, millimeter-wave structures, such as waveguide transitions, dielectric slab filled waveguides, and fin-lines, are of great interest. For these finite structures an orthogonal expansion method is described which takes into account the higher order mode propagation. This method can be interpreted as a generalized Fourier expansion method, where the series terms are eigenmodes of suitably subdivided structures. The orthogonal expansion method allows one to calculate numerically the propagation constant as well as the reflection and transmission coefficients of rather complicated wave structures up to a desired degree of accuracy, depending on the chosen number of eigenmodes. A number of about 10 to 12 eigenmodes has turned out to be sufficient for an error of only about 1 %.

For millimeter waves, waveguide instrumentation is commonly used. There is therefore a great interest in direct waveguide transition scattering problems. As has been calculated a direct transition from waveguide to a microstrip-line, the usual centimeter-wave integrated circuit type, is not appropriate, because the energy of the waveguide is mainly transmitted to an undesirable higher order mode. Dielectric slab structures, mounted in the E-plane of the waveguide, lead, however, to millimeter-wave circuits for which a direct waveguide instrumentation causes no problems. This is demonstrated by the example of the transition waveguide to a dielectric slab E-plane filled waveguide, where the calculated and measured transmission coefficient to the fundamental H_{10} mode is almost equal to one below the cut-off frequency of the higher order mode. A second very excellent millimeter-wave structure is the fin-line, where the dielectric slab is partially metallized. This circuit structure allows low-cost photolithographic techniques to be used and is very suitable for designing millimeter-wave integrated circuits. A fin-line filter is calculated, designed and measured for about 33.7 GHz. The measured insertion loss is only about 0.5 dB.

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