RESUME
TRAITEMENT D'UN RESEAU DANS DES CHAMPS ALÉATOIRES SEMI - HOMOGÈNES

Des modèles de bruit appropriés pour examiner un système Sonar passif pour des sondages goniométriques et pour des mesures du spectre d'une cible dans un milieu bruité, sont les champs aléatoires homogènes consistant d'ondes planes. Du fait que l'hypothèse de stationnarité est fréquemment irréaliste, nous examinons des généralisations selon les procédés semi-stationnaires stochastiques et nous introduisons des champs aléatoires semi-homogènes. Nous démontrons comment les méthodes de Priestley, qui estiment la densité spectrale évolutive peuvent être modifiées pour le faisceau rayonnant et pour l'analyse spectrale des signaux rayonnants, par exemple si en fait un réseau de lignes est utilisé.

Des échantillons des signaux peuvent être traités par l'intermédiaire des algorithmes rapides qui se basent sur les transformations de Fourier rapides et qui sont comparables aux algorithmes déjà connus dans le cas de stationnarité. Finalement, nous indiquons une méthode de poursuite pour des signaux modulés dans un spectre évolutif.

SUMMARY
ARRAY PROCESSING IN SEMI-HOMOGENEOUS RANDOM FIELDS

Useful noise models for investigating a passive sonar system for taking the bearings and measuring the spectrum of a target in ambient noise are homogeneous random fields consisting of plane waves. Since the assumption of stationarity is frequently unrealistic, we investigate generalizations in the sense of Priestley's semi-stationary stochastic processes and introduce semi-homogeneous random fields. We show how Priestley's method for estimating the evolutionary spectral density can be modified for beam forming and spectral analysis of the beam signals, e.g. if a line array is used. Sampled signals can be processed by fast algorithms which base on the fast Fourier transform and which are similar to known algorithms in the case of stationarity. Finally, we indicate a method for tracking modulated lines in an evolutionary spectrum.
ARRAY PROCESSING IN SEMI-HOMOGENEOUS RANDOM FIELDS

Introduction

Passive array processing is, following Liggert's [3] definition, interpretation of noise and noise-like signals received by an array of sensors, i.e., hydrophones in the case of sonar. This paper deals with processing methods for taking the bearings and measuring the spectra of signals which are generated by distant targets and which are disturbed by ambient noise from the far field. Frequently used models for stationary ambient noise at the outputs of the sensors are homogeneous random fields consisting of plane waves, cf. for example [1],[3]. Stationary signals can be incorporated in such models as discrete components of the random field. General properties of homogeneous random fields are proved in Yaglom's book [13].

Spectral estimation and beamforming techniques for estimating the targets have been frequently investigated. Smoothing of short-period spectra of a beam signal is a well known method to estimate the spectrum of noise from a given direction. It was shown in [9], [12] that this can be approximately done for sampled data by a fast method basing on multivariate fast Fourier transform.

The assumption of stationarity of the signals and of noise and moreover of homogeneity of the random field is unrealistic in many applications, especially for fluctuating ambient noise and moving targets. In this paper, we generalize the concept of homogeneity. Homogeneous random fields are composed of uncorrelated plane, elementary waves. The random fields we are interested in are superpositions of similar, but slowly modulated elementary waves. This concept is a direct generalization of Priestley's [6] model of semi-stationary processes. Therefore, we call the corresponding random fields semi-homogeneous. A semi-stationary process possesses evolutionary, i.e., time dependent, spectra which describe the power over frequency and time and which can be estimated under suitable conditions. A semi-homogeneous random field is characterized by evolutionary frequency-wavenumber spectra. We indicate that these spectra can be estimated by careful smoothing of short-period frequency-wavenumber spectrograms. Then, the frequency implemented tracking of the evolution of short-period spectrograms is a reasonable method for analysing a long observation of a semi-homogeneous random field. Similar to the case of homogeneity, short-period spectrograms of the beam signals generated by weighting, delaying, and summing the sensor outputs can be approximated by a variant of the multidimensional FFT-method indicated above. Finally, we mention some special cases, e.g., models with a fixed velocity of propagation, line arrays of sensors in such a model in the plane, and a method for estimating of modulated lines in evolutionary frequency spectra.

Semi-Homogeneous Random Fields

Let us assume that the d-dimensional real vector \( x \) describes the position of a sensor in an array of a finite number of sensors and that the sensor outputs are sampled with a period \( \tau \). If we presume wide-sense stationarity in time and space which means homogeneity, the output of the receiver at position \( x \) can be described by a CREAR representation, cf. [1],

\[
Y(x, \omega) = \sum_{k} e^{j \omega t} \mathbf{e}^{j \mathbf{k} \cdot \mathbf{x}} \mathbf{p}(n, \mathbf{k}) \quad (|m| = 0, \ldots, \infty),
\]

where \( \omega/(2\pi) \) is a frequency in Hertz, \( \mathbf{k} \) is a d-dimensional real vector-wavenumber and \( \mathbf{x} \) means the transposed vector of \( x \). \( \mathbf{z} \) describes a zero mean orthogonal random field, i.e., \( \mathbf{z}(\omega, \mathbf{k}) \) is a random function of the intervals \( \omega, \mathbf{k} \) with expectation

\[
E_\omega \mathbf{z}(\omega, \mathbf{k}) = 0,
\]

for disjoint intervals \( \omega_1, \ldots, \omega_M \) and \( \mathbf{k}_1, \ldots, \mathbf{k}_N \),

\[
E_\omega \mathbf{z}(\omega_1, \mathbf{k}_1) \mathbf{z}(\omega_2, \mathbf{k}_2) = 0, \quad \omega_1 \neq \omega_2
\]

and

\[
E_\omega \mathbf{z}(\omega_1, \mathbf{k}_1) \mathbf{z}(\omega_2, \mathbf{k}_2) = \mathbf{K}(\omega_1, \omega_2) \mathbf{K}(\mathbf{k}_1, \mathbf{k}_2),
\]

where \( \mathbf{K}(\omega_1, \omega_2) \) and \( \mathbf{K}(\mathbf{k}_1, \mathbf{k}_2) \) are known constants. In this case, the complex and deterministic modulation \( \mathbf{A}(\omega, \mathbf{k}) \) slowly oscillates in \( m \) and \( \mathbf{x} \) in comparison with the \( e^{j \omega \tau} \mathbf{e}^{j \mathbf{k} \cdot \mathbf{x}} \) and does not modulate the elementary wave. The output of the receiver is therefore

\[
Y(x, \omega) = \frac{j \tau}{\gamma} \int \frac{d^d \mathbf{k}}{2 \pi^d} \mathbf{e}^{j \mathbf{k} \cdot \mathbf{x}} \mathbf{A}(\omega, \mathbf{k}) \mathbf{p}(n, \mathbf{k}) \mathbf{z}(\omega, \mathbf{k}).
\]

We call a random field \( Y(x, \omega) \) semi-homogeneous if there exists a zero mean random field \( \mathbf{Z} \) and a function \( \mathbf{A} \) such that \( Y(x, \omega) \) can be represented by (1), where the complex modulation function \( \mathbf{A} \) has the following properties.

There exists a spectral representation

\[
A(\omega, \mathbf{k}) = \int \frac{d^d \mathbf{k}}{2 \pi^d} \mathbf{e}^{j \mathbf{k} \cdot \mathbf{x}} \lambda(\mathbf{k}, x),
\]

where \( \lambda(\mathbf{k}, x) \) and \( \mathbf{p}(n, \mathbf{k}) \) has its maximum for \( k = 0 \) and \( \mathbf{x} \) and

\[
A(\omega, \mathbf{k}) = \int |\mathbf{k}| \mathbf{p}(n, \mathbf{k}) \mathbf{Z}(\omega, \mathbf{k}) \mathbf{e}^{j \mathbf{k} \cdot \mathbf{x}} | \mathbf{k}| > 0.
\]

A homogeneous random field is of course semi-homogeneous with \( \mathbf{A} = \mathbf{1} \). If a function \( \mathbf{A} \) is independent of \( \mathbf{x} \) and satisfies the condition (2), the product of \( \mathbf{A} \) and a homogeneous random field \( Y(x, \omega) \) is

\[
Y(x, \omega) = \mathbf{Z}(\mathbf{x}, \omega) \mathbf{A}(n, \mathbf{k}) e^{j \mathbf{k} \cdot \mathbf{x}}.
\]

For a special semi-homogeneous random field \( Y(x, \omega) \), we consider the class of all functions \( \mathbf{A} \) as above and define \( B_x = \mathbf{A} \mathbf{B}_{\mathbf{x}} \) as the characteristic width of the random field \( Y(x, \omega) \). The number \( B_x \) can be roughly interpreted as the (d+1)-th power of the diameter of a maximum sphere in which the field can be treated as approximately homogeneous. For the sake of simplicity, we assume in the following that there exists one and only one \( A \) with \( B_x = \mathbf{x} \), and we consider the corresponding natural representation (1). We shall not discuss the problem if there does not exist a maximum or \( A \) is not unique which could be done similar to (6).

Since \( Y(x, \omega) \) has expectation zero and variance \( \text{Var} \ Y(x, \omega) = f(\mathbf{A} \mathbf{B}_{\mathbf{x}} \omega, \mathbf{k}) \mathbf{F}(\omega, \mathbf{k}) \), we define the evolutionary frequency-wavenumber spectrum with respect to \( \mathbf{A} \) as

\[
F_{\mathbf{B}_x}(\omega, \mathbf{k}) = \mathbf{A}(\omega, \mathbf{k}) \mathbf{F}(\mathbf{B}_x, \omega, \mathbf{k})
\]

and the corresponding evolutionary density in case \( F \) has a density \( f \) as

\[
f(\omega, \mathbf{k}) = \mathbf{A}(\omega, \mathbf{k}) f(\mathbf{B}_x, \omega, \mathbf{k}).
\]

Spectral estimation

In this section, we sketch a method for estimating the evolutionary power of a semi-homogeneous random field in an \( (\omega, \mathbf{k}) \)-band or the evolutionary density, if it exists and is sufficiently smooth, from one observation.

Priestley [6] showed for semi-stationary processes that careful time smoothing of the time evolution of short-period spectrograms yields reasonable estimates if the width of the data window is small in comparison with both the width of the smoothing window and the characteristic width of the process and if both are much smaller than the range of observation.

The complex demodulation which means the short-period time-space Fourier transform of \( Y(x, \omega) \) is first investigated. With respect to the intended applications, let us presume a finite time-space window \( g(x, \omega) \), \( x \) ranges over the finite set of sensor positions. The complex demodulation of \( Y(x, \omega) \) is defined as

\[
Y_{\mathbf{B}_x}(x, \omega, \mathbf{k}) = \frac{1}{2\pi} \int e^{-j \mathbf{k} \cdot \mathbf{x}} \int g(x, \omega) Y(x, \omega) e^{-j \omega \tau} e^{j \mathbf{k} \cdot \mathbf{x}}
\]

for given frequency \( \omega \) and wavenumber vector \( \mathbf{k} \). Using (1)
and (2), the right hand side equals
\[
\int_{-\infty}^{\infty} e^{i\omega x} \left(\varphi(x) - \varphi(x+z)\right) dx
\]
resulting in
\[
U(m,x,u,k) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathcal{F}\{\varphi (x)\}(\omega) e^{-i\omega u} e^{-i\omega kx} d\omega dx
\]
where
\[
(5) \quad G(\omega,k) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathcal{F}\{\varphi (x)\}(\omega) e^{-i\omega kx} d\omega dx.
\]

The approximation is specified in the following sense. Let
\[
R_1 = \hat{f}(\hat{g}\hat{r}^{-1})
\]
where
\[
\hat{r}(\omega,k) = \mathcal{F}\{\varphi (x)\}(\omega) e^{-i\omega kx}
\]
and
\[
\hat{g}(x) = \int_{-\infty}^{\infty} \hat{r}(\omega,k) e^{i\omega x} d\omega dk.
\]
Then,
\[
|\hat{r}(\omega,k)| \leq \sup_{|\xi| = 1} |\mathcal{F}\{\varphi (x)\}(\omega)|
\]
for an e > 0 B e, e = 0, it can be shown similar to the proof of Theorem 2.1 in [5] that
\[
|\hat{r}(\omega,k)| \leq \frac{e}{\pi}.
\]

Assuming the time of the short-time spectrograms of the beam signal with a window w as in the last section,
\[
(11) \quad V_{\omega}(m,x,w) = \mathcal{F}\{\mathcal{F}\{\varphi (x)\}(\omega) e^{-i\omega kx}\}(\omega,k)\mathcal{F}\{\varphi (x)\}(\omega) e^{-i\omega kx} \mathcal{F}\{\varphi (x)\}(\omega) e^{-i\omega kx}
\]

The estimator P has similar properties as P in (9) and (10). We only have to substitute \(\varphi (x)\) with \(\hat{r}(\omega,k)\) for the above restrictions. We compute
\[
(12) \quad \mathcal{F}\{P\} = \mathcal{F}\{\hat{r}(\omega,k)\} e^{-i\omega kx} \mathcal{F}\{\varphi (x)\}(\omega) e^{-i\omega kx} \mathcal{F}\{\varphi (x)\}(\omega) e^{-i\omega kx} \mathcal{F}\{\varphi (x)\}(\omega) e^{-i\omega kx}
\]

The integral corresponds with the mean square error in the case of homogeneous random fields up to the replacement of f by \(\hat{r}\). Consequently, the approximation for wavenumber-bandlimited occurs if the better the smaller the frequency bandwidth of \(\hat{r}\) is chosen. We conclude that
\[
P_{\omega}(m,x,w) = \hat{P}(\mathcal{F}\{\hat{r}(\omega,k)\} e^{-i\omega kx} \mathcal{F}\{\varphi (x)\}(\omega) e^{-i\omega kx} \mathcal{F}\{\varphi (x)\}(\omega) e^{-i\omega kx} \mathcal{F}\{\varphi (x)\}(\omega) e^{-i\omega kx})
\]

where \(\mathcal{F}\{\hat{r}(\omega,k)\} e^{-i\omega kx} \mathcal{F}\{\varphi (x)\}(\omega) e^{-i\omega kx} \mathcal{F}\{\varphi (x)\}(\omega) e^{-i\omega kx} \mathcal{F}\{\varphi (x)\}(\omega) e^{-i\omega kx})

We now define the corresponding term in (12). This bound is not greater than the supremum of \(E|\mathcal{F}\{\hat{r}(\omega,k)\} e^{-i\omega kx} \mathcal{F}\{\varphi (x)\}(\omega) e^{-i\omega kx} \mathcal{F}\{\varphi (x)\}(\omega) e^{-i\omega kx} \mathcal{F}\{\varphi (x)\}(\omega) e^{-i\omega kx})

In Section 3, we deal with the computation of the estimate P(m,x,w) in form of (3), where the point of observation is \(\mathcal{F}\{\hat{r}(\omega,k)\} e^{-i\omega kx} \mathcal{F}\{\varphi (x)\}(\omega) e^{-i\omega kx} \mathcal{F}\{\varphi (x)\}(\omega) e^{-i\omega kx} \mathcal{F}\{\varphi (x)\}(\omega) e^{-i\omega kx})

The exponential factor can be automatically obtained if the transformation of the window data piece suitable rotated is taken. Because of the band limitation of the window, the spectrum of \(\hat{r}(\omega,k)\) is highly over-
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sampled with respect to m and n. We conjecture that, as in Welch's method [11], overlappings of 50 per cent in each direction of the windowed data could suffice for smoothing of $|U(m_1m_2m_3)|^2$. Then, the spectrogram is only computed for $mN/2$ and $nN/2$ with the above restrictions and even M and N. Since the smoothing window is much more frequency bandlimited than the data window, a similar argument shows that $P(m_1o_m, k)$ should be computed only for $m=IN/2$, where IN describes the time duration of w. We use formula (7) with the modification that the sum is only taken over points $mN/2$ and $nN/2$. P is then required for $\pi^2x(N \times 0, \ldots, N-1)$ and $\omega_m$ for given vectors $m=1, \ldots, N$. As usual, we have to interpolate. Space does not permit a discussion.

Special Cases

1) Let us consider semi-homogeneous random fields consisting of plane waves with fixed velocity $c$ of propagation. The model (1) simplifies to

$$Y(m,x) \sim n^{-1/2} \int_x \exp \left( \frac{i m x}{2 \pi a} \right) \sum_{l=0}^{N-1} \sum_{k=0}^{N-1} \int_\mathbb{R} e^{-i \omega_0 t} \sin(\omega t) \sin(\omega t) \int_\mathbb{R} e^{-i \omega_0 t} \sin(\omega t) \sin(\omega t)$$

where $S$ is the sphere with radius $r$, and $A_\omega$ must have a spectral representation $A_\omega \sim n^{-1/2} \sum_{l=0}^{N-1} \int_\mathbb{R} e^{-i \omega_0 t} \sin(\omega t) \sin(\omega t)$.

2) A distant target radiating noise in a model (13) is characterized by a discrete point in the distribution over $S$. The noise generated by the target alone and received at position x can be described by

$$Y(m,x) \sim n^{-1/2} \int_x \exp \left( \frac{i m x}{2 \pi a} \right) \sum_{l=0}^{N-1} \sum_{k=0}^{N-1} \int_\mathbb{R} e^{-i \omega_0 t} \sin(\omega t) \sin(\omega t) \int_\mathbb{R} e^{-i \omega_0 t} \sin(\omega t) \sin(\omega t)$$

with the known problems induced by the argument of $G$. The evolution of the frequency-angle spectral density is $f_{\omega}(w) \sim n^{-1/2} \sum_{l=0}^{N-1} \int_\mathbb{R} e^{-i \omega_0 t} \sin(\omega t) \sin(\omega t)$

3) If the noise received from a point target is represented by (14) and $Z \sim n^{-1/2} \int_x \exp \left( \frac{i m x}{2 \pi a} \right) \sum_{l=0}^{N-1} \sum_{k=0}^{N-1} \int_\mathbb{R} e^{-i \omega_0 t} \sin(\omega t) \sin(\omega t) \int_\mathbb{R} e^{-i \omega_0 t} \sin(\omega t) \sin(\omega t)$

References