Acoustic Signaling in the Undersea Channel

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RESUME


En ce rapport, nous examinons la dispersion des signaux, en termes de temps et de fréquence, créés par la propagation à plusieurs voies et par l'interaction avec le surface. Pour ce canal de double-dispersion, des mesures qualitatives des données sont obtenues en fonction de distance et fréquence.

SUMMARY

The transmission of information via acoustic propagation in the undersea channel is subject to several signal distortion effects. These distortions appear as temporal, frequency, and spatial fluctuations of the signal. The fluctuations arise primarily from multipath interference, and from modulation effects caused by scattering from the surface, bottom and volume.

In this paper, we examine the signal dispersion effects in time and frequency caused by multipath propagation and by surface intersection. For this doubly dispersive channel, qualitative measures of data rates are obtained as a function of range and frequency.
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1.0 INTRODUCTION

The transmission of information via acoustic propagation in the undersea channel is subject to several signal distortion effects. The distortions appear as temporal, frequency, and spatial fluctuations of the signal. The fluctuations are caused by (1) multipath interference; (2) surface, bottom, and volume scattering; and anisotropic refraction influences.

Here we are concerned with signal fading effects caused by multipath propagation and frequency smearing caused by time varying surface modulation. In particular, some issues in signal design for the frequency selective fading channel are examined. Qualitative limits on information transfer rates are determined as a function of range, frequency, and signal mismatch to the channel.

1.1 DOUBLY DISPERSIVE CHANNEL

The undersea acoustic channel is generally characterized as being dispersive in both time and frequency. An impulse transmitted in time will appear at the receiver to spread over $L$ seconds. This time dispersion is caused by multipath propagation, which is governed primarily by the geometry of the channel. Similarly, an impulse in frequency, i.e., a continuous wave (CW) tone, will appear to be spread by a value, $B$, in frequency as the result of interaction with the moving boundaries. The boundaries are the surface in the case of fixed terminals and the bottom and surface when the terminals are in motion. The time and frequency smears ($L$ and $B$, respectively) are useful descriptors of the undersea environment as an acoustic communications channel.

In particular, $L$ and $B$ characterize the fading properties of the channel. A diversity model of the channel establishes a frequency-time grid with dimension $1/B$ in time and $1/L$ in frequency, as shown in Figure 1. The implication is that every cell of length $1/B$ and width $1/L$ is a region of constant fading behavior and the signal within any cell may be considered coherent over the cell dimensions. Independent samples may be taken at time intervals of $1/B$ sec. Independent fades occur at frequencies separated by $1/L$. Therefore, $B$ is the fading rate and $1/L$ is the coherent fading interval.

The signal design should be matched to the channel such that the frequency and time resources $(W$ and $T$, respectively) at the transmitter are partitioned into bauds corresponding to the channel partitioning in $B$ and $L$. Since a transmitted baud of length $T$ and bandwidth $W$ will appear at the receiver as a signal of length $T + L$ and bandwidth $W + B$, then the frequency-time $(f-t)$ space must be such that sequential or simultaneous bauds (or chips) are separated by these distances in $t$ and $f$, respectively.

![Figure 1. Channel Partitioned f-t Space](image)

1.2 INFORMATION TRANSFER

Consider a single tone of duration $T$ sec transmitted through a dispersive channel that has a multipath delay spread of $L$ sec and produces a Doppler smear of $B$ Hz. Since the pulse bandwidth is $2/T$ Hz, the waveform observed at the receiver will have a bandwidth $BW_R$ and a duration $T_R$, given by

$$BW_R = 2/T + B$$

and

$$T_R = L + T.$$  \hspace{1cm} (1)

If the available signaling bandwidth is $W$, then the number of tones, $N_T$, that may be transmitted per second is

$$N_T = W/(2/T + B)(L + T).$$ \hspace{1cm} (2a)

The value for $N_T$ is maximized when $T$ is chosen ($\partial N_T/\partial T = 0$) to be

$$T_p = \sqrt{2L/B}.$$ \hspace{1cm} (2b)

In Figure 2, $T$ is plotted versus $L$ for several values of $B$.

Some measures of $B$ and $L$ will be examined here in order to obtain qualitative measures of the data capacity for the undersea acoustic communications channel. These limits will then used to estimate the signaling rate as a function of range (i.e., frequency) that can be supported in the channel.

![Figure 2. Band Length for Maximum Number of Tones/Second](image)

2.0 TIME SPREAD AND MULTIPATH FADING

Propagation over multiple paths in the ocean produces signal fading effects because of the coherent addition of out-of-phase signal arrivals. The degree of signal suppression caused by destructive interference is critically dependent upon the specific propagation geometry and the characteristics of the medium. In particular, signals propagating over different paths are subject to different attenuation, reflection, and absorption losses. Both the signal intensity and the propagation delay differ for various paths. The coherent addition of these signals produces an attenuation pattern having specific nulls and peaks determined by the paths providing the dominant energy and by the relative delay times along those paths. Diversity gains against such fading can be realized by incoherent combining of resolved multipaths. The resolution of these arrivals requires sufficient signal bandwidth to match the impulse response duration of the medium. Theoretically, the signal bandwidth must be larger than the inverse of the multipath delay ($W>1/L$).

The delay differences are likely to be very small (on the order of milliseconds) for the shallow water channel ($<300$ m), especially over ranges for which no direct path arrivals occur. This suggests large diversity bandwidths on the order of hundreds of Hertz. For deep water channels, the multipath delays are on the order of 2 to 10 sec for ranges out to 1000 nm. Generally, 1 sec of multipath dispersion occurs for each 100 nm of range.

Insight into the significance of the fading bandwidth can be obtained by examining a simple two-path model. Consider the geometry shown in Figure 3, which consists of a direct path of length $d_0$ and a boundary-reflected path of length $2d$ in an isovelocity medium.

For $d<<d_0$, the difference in path lengths between $T$ and $R$ is

$$\Delta t = (2d_0 - 2d) = d^2/d_0^2.$$ \hspace{1cm} (3)

At a given frequency, for example, $f_\text{m}$, the separation in range between nulls (out-of-phase arrivals) is

$$\Delta r \approx 2f_\text{m}^2/d^2.$$ \hspace{1cm} (4)
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![Figure 3. Two-Path Propagation Model (Isovelocity)](image)

Note that the null spacing, ΔR, increases as range, 2L₀, increases and decreases as the multipath delay difference (Δd) increases.

To determine the frequency shift (f₁-f₂) necessary to produce a peak response at the range where the null for a given frequency, f₁, occurs, we must assume that an attenuation fade exists at 2L₀ (the difference is effectively a measure of the fading bandwidth). It is then necessary that f₂ be selected such that the null is shifted in range by ΔR/2 (one-half the null spacing of f₁). Then

\[ Δf(f₁) = \frac{d}{d₂/L₀} = nλ₁/2 \]

and

\[ Δf(f₂) = \frac{ΔR}{2L₀} = nλ₂, \]

where

\[ f₂/f₁ = λ₂/λ₁ = (L₀ + ΔR/2)/L₀ = 1 + ΔR/2L₀ \]

with f₂ = f₁ - Δf; then,

\[ Δf = ΔR/2L₀. \]

Then substitute for ΔR from (4) (using (3) and c = fλ) to yield

\[ Δf = \frac{cL₀}{d²} = 1/4AT, \]

where ΔT is the travel time difference between the two paths. Note that the frequency separation required to produce maximum separation in range for the fades (nulls) is given by the inverse of the delay difference between the two arrivals.

In order to obtain a qualitative understanding of fading effects, an idealized six-path propagation model was developed to produce steady-state single-frequency attenuation patterns in a shallow channel for ranges of 1 to 20 km. The frequency was varied to find the separation necessary to produce patterns having independent fades (i.e., fades that do not overlap in range). Figures 4a, 4b, and 4c show the attenuation patterns for 10, 10.01, and 10.1 kHz for three range intervals when the six paths were equally weighted.

**3.0 FREQUENCY SPREAD ANALYSIS**

Analysis of the frequency smear induced by the time varying random surface follows from an examination of references 3 and 4 and the consequent extension of scattering models to the rough surface case that is of interest in undersea acoustic communications. Reference 3 provides experimental evidence that the sea surface imposes phase modulation on an incident monochromatic tone and spreads the energy into sidebands displaced from the carrier by multiples of the surface frequency.

The environment of interest to acoustic communications is characterized by a surface bounded channel. The surface boundary is considered to be rough when the root mean square (rms) surface wave height, o, is larger than the wavelength, λ, of the acoustic radiation (i.e., o/λ>1).

A meaningful parameter for expressing degree of surface roughness is the Rayleigh parameter, which is defined here as

\[ γ = (4π)(o/λ) \sin ψ, \]

where ψ is the angle of incidence of the acoustic wavefront at the surface. Typically, the dominant surface reflected rays are incident to the surface at angles of within ±5°. Surface roughness depends upon sea state and varies from an rms waveheight of ≈1 m for sea state 2 to ≈3 m for sea state 6.

Figure 5 shows the value of γ versus frequency for several combinations of incident angle and wave height, o. The bi-frequency formulation is used to describe the spectrum of surface reflected energy. For our purposes, an order-of-magnitude measure of the channel properties is sufficient. The bi-frequency spreading model, which was successfully applied to low-Rayleigh-number cases γ<1.4, was extended to enlarge its applicability for γ>1.4 to derive these measures.

**Figure 4a. Six-Path Isovelocity Multipath Model**

![Figure 4a](image)

**Figure 4b. Multipath Interference at 9 to 11 kyd**

![Figure 4b](image)

**Figure 4c. Multipath Interference at 19 to 21 kyd**

![Figure 4c](image)

**Figure 5. Rayleigh Number versus Frequency**

![Figure 5](image)

The principal results of that effort indicate that:

1. In the limit of high Rayleigh numbers (i.e., increasing frequency and/or waveheight), the spreading bandwidth for a single tone reflected from a rough surface is at best directly proportional to γ and certainly no worse than γ².

2. The bandwidth measure, (W²-W) is proportional to γ and is sensitive to the assumed probability distribution function of the surface wave process, p(x, y, t), in the wide beam case (i.e., several
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Fresnel zones ensonified.

3. As surface roughness, \( a/\lambda \), increases, the spectrum of the scattered energy tends to fill-in and become continuous. The shape of the spectrum is determined by the characteristic function of the surface height process. The spreading characteristics of typical rough surface channels are summarized in Figure 6.

Generally, it may be stated that

1. the time varying rough surface produces both angle and amplitude modulation on an incident carrier;

2. at low Rayleigh numbers, angle modulation is the dominant effect and the discrete sidebands generated are displaced from the carrier at multiples of the surface frequency; and

3. at higher Rayleigh numbers, amplitude modulation effects become important and the angle modulation index (a) increases to produce higher order sidebands and (b) introduces sufficient sideband smearing to create a smooth, suppressed carrier spectrum.

![Figure 6. Frequency Spread versus Rayleigh Number](image)

Estimates of the spreading bandwidth fit asymptotic behavior bounded by linear and quadratic dependence on \( \gamma \) (see Figure 6). It has been noted as \( \gamma \to \infty \) in the limit, the spectrum defined by the bi-frequency function, \( f(x) \), is similar in form to Middleton's expression for a high-index phase modulated carrier. In fact, if the tail effects of the spectrum are ignored, the results are identical with \( \gamma = \mu \) (see references 5 and 6), i.e.,

\[
\gamma(x) = a/\gamma \sqrt{R} f\left(\alpha/\gamma\right)
\]

and

\[
W(x) = A_2^{1/2} w(y/\mu) w_{1/2}(x/\mu),
\]

where \( x = (W \cdot W) \) and \( w(x) \) and \( f(x) \) are distribution functions of the time derivative of the (surface) modulation waveform. It is possible, using this analogy, to employ the classic knowledge concerning the bandwidth spread of an FM modulation carrier. The curve A-A on Figure 6 illustrates the total bandwidth relationship computed from phase modulation bandwidth relationships, where \( \gamma \) is treated as modulation index \( \mu \). At best, the frequency smear increases linearly with frequency over the audio band.

4. PERFORMANCE CONSIDERATIONS

The principal measures of communication performance are range, message reliability, and data rate. They are interrelated parameters whose upper bounds dictated by channel characteristics, available power, and environmental noise.

The achievable data rate (in bits per second) is governed by the available transmission bandwidth, \( W \), and the signal-to-noise ratio (SNR) at the receiver. The available bandwidth is governed by carrier frequency, \( f_c \), which (in turn) critically affects the communication range. A higher transmission frequency permits a wider signaling bandwidth but is subject to greater attenuation and, therefore, is limited to shorter range communication. The signal level at a given range is limited by the available source level.

Typically, the available bandwidth is about 25 to 50 percent of \( f_c \). Although the transducer bandwidth increases linearly with frequency, the frequency smear bandwidth also increases linearly over a large portion of the acoustic signaling band (i.e., 2 to 3 percent of \( f_c \) from a few hundred Hertz to a few kilohertz). Consequently, the number of signaling channels in this band is fixed at a few hundred (i.e., transducer bandwidth \( \sim 0.5f_c \)/smear bandwidth \( \sim 0.0025f_c \)).

4.1 CHOICE OF CARRIER FREQUENCY

The choice of carrier frequency for acoustic communications is governed by two opposing influences. First, propagation loss caused by absorption, \( a \), in the media increases with frequency according to frequency-square law, \( a \sim f^2 \). This influence dictates the use of lower frequencies. However, source level and available bandwidth increase with frequency while ambient noise decreases with frequency. These considerations suggest the use of higher frequency transmission. Other factors such as dispersion and bottom loss are also frequency sensitive, but the absorption influence dominates.

Consider absorption to be the dominant influence on the choice of transmitting frequency. In order to examine the variation in data rate versus range, we impose the constraint that the SNR, \( E/N_0 \), be held constant. The frequency will then be selected as a function of range in accordance with

\[
10 \log R + aR/1000 = \text{Constant}
\]

(10)

We have assumed cylindrical spreading (10 log \( R \)) and low frequency absorption, \( a \sim \beta f^2 \). From (10) we have the required condition

\[
f^2 = f_c^2 R_0^2 \gamma^2/R,
\]

(11)

where \( f_0 \) and \( R_0 \) are the reference frequency and range, respectively. If \( R_0 \) is chosen to be 2000 nmi, the extent of the largest deep ocean basin, and it is assumed that absorption losses at this range are negligible at \( f_0 = 10 \text{ Hz} \), then (11) provides the data in Table 1 concerning the choice of carrier frequency, \( F \), for ranges from 2 to 2000 nmi.

<table>
<thead>
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<th>Frequency (Hz)</th>
<th>Range (nmi)</th>
<th>Bandwidth (Hz)</th>
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<tr>
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</tr>
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</table>

Table 1. Choice of Carrier Frequency

Since the acoustic sources have a bandwidth of approximately \( 1/3f_c \) (i.e., \( Q = 3 \)) over this range, the available signaling bandwidth, \( W \), is computed directly as a function of range. The theoretical limit on data rate, therefore, varies linearly with range for fixed \( E/N_0 \), i.e.,

\[
C = W \ln(1 + E/N_0)
\]

(12)

Figure 7 shows this limit as plotted versus range for four values of \( E/N_0 \).

![Figure 7. Data Rate versus Range](image)
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4.2 CAPACITY OF THE CHANNEL

Figure 7 shows a relationship between data rate and communication range. Note the weak dependence between range and data rate over the intermediate ranges from a few tens to a few hundreds of nautical miles. There is a stronger inverse relationship between the two parameters at very short and very long ranges.

Increasing the transmission bandwidth permits a higher signaling rate only if the number of signal channels is increased and the total energy increased. Because the bandwidth increases with carrier frequency at the same rate as the frequency smear increases with frequency, the number of channels remains relatively fixed, except at very low (100 Hz) and very high (10 kHz) frequencies. Further, since the source is peak-power limited, the only way to increase the signal energy at the receiver is to shorten the communication range.

The major observation to be made from figure 7 is that there is weak data rate versus range relationship. The data rate may be exchanged for range or reliability. The trade-off however, is constrained by practical limitations such as (1) source level, (2) array gain, (3) ambient noise, (4) Doppler errors caused by platform motion, (5) channel dispersion, and (6) source/receiver geometry. A general rule relating data rate to range or carrier frequency may be inferred from figure 7.

REFERENCES


